Myller configurations in Finsler spaces.
Applications to the study of torse forming vector fields

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Abstract. In this article we define the Myller configurations in Finsler spaces and the new notion of torse forming vector fields in the sense of Myller in a Myller configuration, with respect to the Cartan Finsler connection. We apply the results to study vector fields tangent to a given Finsler submanifold, which are torse forming vector fields with respect to the induced Finsler connection.

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1 Preliminaries and notations

All the geometric objects used in this material are of class C^∞.

Let M be a real, differentiable manifold of dimension n, (TM, π, M) the tangent bundle. We denote by $(\widetilde{TM}, \tilde{\pi}, M)$ the vector bundle of non vanishing vectors, tangent to M, where $\tilde{\pi} = \pi/_{\widetilde{TM}}$. Let $(\pi^*TM, \pi^*, \widetilde{TM})$ be the pull-back bundle of the tangent bundle by $\tilde{\pi}$, and $\Gamma(\pi^*TM)$ its $\mathcal{F}(\widetilde{TM})$- module of sections. In this material the sections of the pull-back bundle will be called $\tilde{\pi}$- vector fields [9]. We consider local system of coordinates on M, TM and π^*TM, denoted like usual by $(x^i)_{i\in\overline{1,n}}$, $(x^i, y^i)_{i\in\overline{1,n}}$ and respectively $(\bar{x}^i)_{i\in\overline{1,n}}$.

Any local section of the tangent bundle determines a local section of the pull back bundle: for any $X \in \chi(U)$, let $\bar{X} \in \Gamma(\tilde{\pi}^{-1}(U), \pi^*TM)$ be defined by: $\bar{X}(\tilde{x}) = (\tilde{x}, X(\tilde{\pi}(\tilde{x})))$, $\forall \tilde{x} \in \tilde{\pi}^{-1}(U)$. $\bar{X}$ is the lift of the vector field X on M to a local section of π^*TM and is called a $\tilde{\pi}$ - vector field. Particularly, $\{\overline{\frac{\partial}{\partial x^i}}\}_{i\in\overline{1,n}}$ is a local basis in $\Gamma(\tilde{\pi}^{-1}(U), \pi^*TM)$.

We suppose that $F^n = (M, F(x,y))$ is a Finsler space:[1] It means a pair $F^n = (M, F(x,y))$, where $F : TM \to \mathbb{R}$ is a scalar function such that: 1) $F(x,y)$ is differentiable on $\widetilde{TM}$ and continuous on the null section; 2) $F(x,y) > 0$ on TM ; 3) F is positively homogeneous of order 1 on the fibers of the tangent bundle: $F(x, \lambda y) = \lambda F(x,y)$, $\forall \lambda > 0$. 4)the distinguished tensor field $g_{ij}(x,y) = \frac{1}{2}\frac{\partial^2 F^2}{\partial y^i \partial y^j}(x,y)$ is positively defined $\Rightarrow rank[g_{ij}(x,y)] = n$ on $\widetilde{TM}$.

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The fundamental tensor $g_{ij}(x,y)$ determines a Riemannian natural metric on π^*TM: $\bar{g} = g_{ij}d\overline{x}^i \otimes d\overline{x}^j \in \Gamma(\otimes^2\pi^*T^*M)$, where $\{d\overline{x}^i\}$ is the basis in π^*T^*M, dual to $\{\overline{\frac{\partial}{\partial x^i}}\}$. In this paper w'ill work with the Cartan Finsler connection on F^n. We also use the next morphism of vector bundle: $\rho : T(\widetilde{TM}) \to \pi^*TM$, $\rho(\tilde{X}) = (\pi_{\widetilde{TM}}(\tilde{X}), d\tilde{\pi}(\tilde{X}))$, that induces a morphism between the $\mathcal{F}(\widetilde{TM})$- modules of sections, denoted also by ρ and called the lift of a vector field on $\widetilde{TM}$ to a $\tilde{\pi}$-vector field.

2 Myller configurations $\mathcal{M}(\tilde{C}\,,\,\bar{\xi}_1\,,\,\bar{T}^m)$ and torse forming vector fields in the sense of Myller

In this section we generalize the notion of Myller configuration from Riemann spaces [6] to Finsler spaces. The first generalisations to Finsler spaces are made by Khu Quoc Anh [5]. Here, we introduce in a new way the Myller configurations in a Finsler space. For a more detailed study of Myller configurations in Finsler spaces please consult [3].

Let $\tilde{C}$ be a regular curve on $\widetilde{TM}$, differentiable of class C^∞, locally given by $x^i = x^i(s)$, $y^i = y^i(s)$, with s the arc - length parameter of the projection $C = \pi \circ \tilde{C}$. We also consider a regular distribution of class C^∞ and dimension m, $1 < m < n-1$, restricted to $\tilde{C}$:

$$\bar{T}^m \,:\, \tilde{C}(s) \to \bar{T}^m(\tilde{C}(s)) \subset \pi^*_{\tilde{C}(s)}TM \tag{2.1}$$

and $(\tilde{C}, \bar{\xi}_1)$ a $\tilde{\pi}$- versor field from the given distribution:

$$\bar{\xi}_1(\tilde{C}(s)) \in \bar{T}^m(\tilde{C}(s))\,,\ \forall s \in I\,,$$

$$\bar{\xi}_1 = \xi_1^i \frac{\bar{\partial}}{\partial x^i}\,, \qquad \bar{g}(\bar{\xi}_1, \bar{\xi}_1) = g_{ij}\xi_1^i\xi_1^j = 1\,. \tag{2.2}$$

So, we obtain the triplet $\mathcal{M}(\tilde{C}\,,\,\bar{\xi}_1\,,\,\bar{T}^m)$ and we call it *a Myller configuration in the Finsler space* F^n.
The case $m = n-1$ is treated in [4].

Let $FC = (HTM, \nabla)$ be the Cartan Finsler connection of F^n and $\frac{\nabla}{ds}$ the operator of covariant differentiation along $\tilde{C}$.

For any $s \in I$, we consider the orthogonal complement (with respect to $\bar{g}(\tilde{C}(s))$) of the linear subspace $\bar{T}^m(\tilde{C}(s))$ in $\pi^*_{\tilde{C}(s)}TM$, and we denote it by $\bar{T}^p(\tilde{C}(s))\,,\ p = n-m$.

We decompose $\frac{\nabla\bar{\xi}_1}{ds}(\tilde{C}(s))$ in $\bar{T}^m(\tilde{C}(s)) \oplus \bar{T}^p(\tilde{C}(s))$, putting in evidence the lengths and the versors of the sections:

$$\frac{\nabla\bar{\xi}_1}{ds}(\tilde{C}(s)) = G_1(\tilde{C}(s))\bar{\eta}_2(\tilde{C}(s)) + N_{11}(\tilde{C}(s))\bar{n}_1(\tilde{C}(s))\,,\ s \in I\,, \tag{2.3}$$

$$\bar{g}(\bar{\eta}_2, \bar{\eta}_2) = \bar{g}(\bar{n}_1, \bar{n}_1) = 1\,,\ \bar{g}(\bar{\xi}_1, \bar{\eta}_2) = \bar{g}(\bar{\xi}_1, \bar{n}_1) = \bar{g}(\bar{\eta}_2, \bar{n}_1) = 0\,,$$

$$G_1 \geqslant 0 \text{ for } m \geqslant 3 \ \text{ and } N_{11} \geqslant 0 \ \text{ for } p \geqslant 2\,.$$

We remark that $\bar{\eta}_2, \bar{n}_1$ have geometric character and G_1, N_{11} are invariants. If $G_1 \neq 0$, we decompose $\frac{\nabla \bar{\eta}_2}{ds}(\tilde{C}(s))$ with respect to $\bar{\xi}_1(\tilde{C}(s))$, $\bar{\eta}_2(\tilde{C}(s))$ and to their orthogonal complement in $\bar{T}^m(\tilde{C}(s))$, also with respect to $\bar{n}_1(\tilde{C}(s))$ and its orthogonal complement in $\bar{T}^p(\tilde{C}(s))$. Using the fact that $\bar{\xi}_1, \bar{\eta}_2, \bar{n}_1$ are orthogonal unitary sections and that the Cartan connection is metrical with respect to $\bar{g}$, we obtain the unique decomposition:

$$\frac{\nabla \bar{\eta}_2}{ds} = -G_1 \bar{\xi}_1 + G_2 \bar{\eta}_3 + N_{21} \bar{n}_1 + N_{22} \bar{n}_2 \,, \tag{2.4}$$

$$G_2 \geqslant 0 \, (m \geqslant 4) \,, \ N_{22} \geqslant 0 \, (p \geqslant 3) \,,$$

$(\bar{\xi}_1, \bar{\eta}_2, \bar{\eta}_3)$ an orthogonal triplet in $\bar{T}^m$ and $(\bar{n}_1, \bar{n}_2)$ an orthogonal pair in $\bar{T}^p$.

If $G_2 \neq 0$ the method can be continued and, by induction, one can prove the next result:

Theorem 2.1. *The fundamental formulae of the $\tilde{\pi}$-versor field $(\tilde{C}, \bar{\xi}_1)$ in the Myller configuration $\mathcal{M}(\tilde{C}\,,\, \bar{\xi}_1\,,\, \bar{T}^m)$ of the Finsler space F^n are:*

$$\frac{\nabla \bar{\eta}_a}{ds} = -G_{a-1} \bar{\eta}_{a-1} + G_a \bar{\eta}_{a+1} + \sum_{\beta=1}^{p} N_{a\beta} \bar{n}_\beta \,, \quad a \in \overline{1,m} \,, \ \bar{\eta}_1 = \bar{\xi}_1 \,, \tag{2.5}$$

$$\frac{\nabla \bar{n}_\alpha}{ds} = -\sum_{b=1}^{m} N_{b\alpha} \bar{\eta}_b + \sum_{\beta=1}^{p} M_{\alpha\beta} \bar{n}_\beta \,, \quad \alpha \in \overline{1,p} \,, \tag{2.6}$$

and its invariants verify :

for $p \leqslant m$:

$$\begin{aligned} &G_a > 0 \,, \quad a \in \overline{1, m-2} \,, \quad G_0 = G_m = 0 \,, \\ &N_{\alpha\alpha} > 0 \,, \quad \alpha \in \overline{1, p-1} \,, \quad N_{a\beta} = 0 \,, \, a < \beta \,, \\ &M_{\alpha\beta} + M_{\beta\alpha} = 0 \,, \end{aligned} \tag{2.7}$$

for $p > m$:

$$\begin{aligned} &G_a > 0 \,, && a \in \overline{1, m-1} \,, && G_0 = G_m = 0 \,, \\ &N_{\alpha\alpha} > 0 \,, && \alpha \in \overline{1, m} \,, && N_{a\beta} = 0 \,, \, a < \beta \,, \\ &M_{\alpha\beta} + M_{\beta\alpha} = 0 \,, && M_{i,m+i} > 0 \,, \, i \in \overline{1, p-m-1} \,, && M_{i,m+j} = 0 \,, \, j > i \,. \end{aligned} \tag{2.8}$$

We obtained an orthonormal, positively orientated frame along $\tilde{C}$,

$$\mathbb{R}_{\mathcal{M}} = \{\bar{\eta}_1, \cdots, \bar{\eta}_m, \bar{n}_1, \cdots, \bar{n}_p\}, \ \bar{\eta}_1 = \bar{\xi}_1 \,,$$

and $G_a, N_{a\beta}, M_{\alpha\beta}$, some functions that are invariant at changes of coordinates on $\pi^* TM$ and at changes of natural parameter $s \to s + a$ of C. The sections of the frame $\mathbb{R}_{\mathcal{M}}$ are geometrically associated to $\bar{\xi}_1$ in $\mathcal{M}(\tilde{C}\,,\, \bar{\xi}_1\,,\, \bar{T}^m)$.

Another invariants are given by the coordinates of $\rho(\frac{d\tilde{C}}{ds})$ in $\mathbb{R}_{\mathcal{M}}$:

$$\rho(\frac{d\tilde{C}}{ds}) = b^1 \bar{\xi}_1 + \cdots + b^n \bar{n}_p \,, \tag{2.9}$$

$$\sum_{i=1}^{n}(b^i)^2 = 1\,. \tag{2.10}$$

Definition 2.1. *We call* $\bar{\eta}_a$ *the geodesic* $\tilde{\pi}$*-versor field of rank* a *of* $\bar{\xi}_1$ *in* $\mathcal{M}(\tilde{C}\,,\,\bar{\xi}_1\,,\,\bar{T}^m)$, *span*$\{\bar{\eta}_1,\cdots,\bar{\eta}_a\}$ *- the geodesic space of rank* a, $\bar{n}_\alpha$ *- the* $\tilde{\pi}$*- normal versor field of rank* α, *span*$\{\bar{n}_1,\cdots,\bar{n}_\alpha\}$ *- the normal space of rank* α.

The invariants of the Myller configurations are called: G_a*- the geodesic curvature of rank a,* N_{11}*-the normal curvature of the* $\tilde{\pi}$*- versor field* $\bar{\xi}_1$ *in* $\mathcal{M}(\tilde{C}\,,\,\bar{\xi}_1\,,\,\bar{T}^m)$.

Theorem 2.2. *(fundamental) Let*

$$G_a, N_{a\beta}, M_{\alpha\beta}\,,\ a\in\overline{1,m}\,,\ \alpha,\beta\in\overline{1,p}\,,\ b^1,\cdots,b^n,$$

be some continuous functions of parameter $s\in I$, *satisfying the conditions* $\sum_{i=1}^{n}(b^i)^2 = 1$, *(2.7) or (2.8) and let*

$$\mathbb{R}_0 = \{\bar{\eta}_{01},\cdots,\bar{\eta}_{0m},\bar{n}_{01},\cdots,\bar{n}_{0p}\}$$

be an $\bar{g}$*-orthonormal, positively orientated frame in* $\pi^*_{\tilde{x}_0}TM$, $\tilde{x}_0\in\widetilde{TM}$. *Then, there is in a neighborhood of* $\tilde{x}_0$ *an unique curve* $C\,:\,x^i = x^i(s)$ *on* M*, such that* s *is its arc-length parameter, there is an unique horizontal curve* $\tilde{C}$ *on* $\widetilde{TM}$ *with* $\pi\circ\tilde{C} = C$*, there is an unique regular distribution* $\bar{T}^m$ *of dimension* m *restricted to* $\tilde{C}$ *and an unique* $\tilde{\pi}$*-versor field* $\bar{\xi}_1$ *from this distribution, such that the invariants of* $\bar{\xi}_1$ *in* $\mathcal{M}(\tilde{C},\bar{\xi}_1,\bar{T}^m)$ *are exactly the given functions* $G_a, N_{a\beta}, M_{\alpha\beta}$ *and the following initial conditions are satisfied:*

$$\tilde{C}(s_0) = \tilde{x}_0\,,\ \bar{\eta}_a(s_0) = \bar{\eta}_{0a}\,,\ a\in\overline{1,m}\,,\ \bar{n}_\alpha(s_0) = \bar{n}_{0\alpha}\,,\alpha\in\overline{1,p}\,.$$

Remark 2.1. *We imposed the condition of horizontality to obtain the uniqueness of* $\tilde{C}$.

3 Torse forming versor / vector fields in the sense of Myller from $\bar{T}^m$

In this section we study a new notion, introduced by Al. Myller for 3 dimensional Euclidian case and extended by R. Miron to Riemannian spaces.

Definition 3.2. [8] *1)* $\bar{X}\in\bar{T}^m$ *is a torse forming* $\tilde{\pi}$*- vector field in the sense of Myller if*

$$\frac{\nabla\bar{X}}{ds}(s) = \alpha(s)\rho(\frac{d\tilde{C}}{ds}) + \beta(s)\bar{X}(s) + \gamma(s)\bar{n}(s)\,,\ s\in I\,,$$

where $\alpha,\beta,\gamma\in\mathcal{F}(\widetilde{TM})$*, restricted to* $\tilde{C}$ *and* $\bar{n}\in\bar{T}^p$ *is a* $\tilde{\pi}$ *- vector field normal to the Myller configuration* $\mathcal{M}(\tilde{C},\bar{\xi}_1,\bar{T}^m)$.

2) A $\tilde{\pi}$*-versor field* $(\tilde{C},\bar{\xi}_1)$ *is called a torse forming versor field in the sense of Myller if there is a* $\tilde{\pi}$*- vector field* $\bar{X}(s) = \lambda(s)\bar{\xi}_1(s)$*, torse forming in the sense of Myller.* $(\tilde{C},\bar{\xi}_1)$ *is named concurrent in the sense of Myller if* $\alpha = cst. \neq 0$ *and* $\beta = 0$*, recurrent in the sense of Myller if* $\alpha = 0$ *and parallel in the sense of Myller if* $\alpha = 0$ *and* $\beta = 0$.

Theorem 3.3. *$(\tilde{C}, \bar{\xi}_1)$ with $G_1 > 0$ is a torse forming versor field in the sense of Myller ($\alpha \neq 0$) if and only if*

$$b^3 = \cdots = b^m = 0\,. \tag{3.1}$$

Proof: We suppose that $\alpha = 1$ and that there exists a vector field $\overline{X}(s) = \lambda(s)\overline{\xi}_1(s)$, some real, differentiable functions β, γ defined on $\tilde{C}$ and a versor field $\overline{n} \in \overline{T}^p$, such that

$$\frac{d\lambda}{ds}(s) + \lambda(s)\frac{\nabla\overline{\xi}_1}{ds}(s) = \rho(\frac{d\tilde{C}}{ds}) + \beta(s)\lambda(s)\overline{\xi}_1(s) + \gamma(s)\overline{n}(s). \tag{3.2}$$

Replacing $\frac{\nabla\bar{\xi}_1}{ds}$ and $\rho(\frac{d\tilde{C}}{ds})$ from the fundamental equations, we get the system:
$b^3 = b^4 = \cdots = b^m$, $\frac{d\lambda}{ds} = b^1 + \beta\lambda$, $G_1\lambda = b^2$.

So, $b^3 = b^4 = \cdots = b^m$ are necessary conditions for $\overline{\xi}_1$ to be a torse forming versor field in the sense of Myller. For the converse statement, $G_1 > 0$ implies $b^2 \neq 0$. We consider $\overline{X} = \frac{b^2}{G_1}\overline{\xi}_1$. Then, there exists the functions $\alpha = 1, \beta = \frac{G_1}{b^2}(\frac{d}{ds}(\frac{b^2}{G_1}) - b^1)$, such that the definition 3.2 is satisfied. We obtained the next result:

For concurrent $\tilde{\pi}$- versor fields we obtain a result similar with one of Professor's Miron [7]. The proof is similar with the former one.

Theorem 3.4. *$(\tilde{C}, \bar{\xi}_1)$ with $G_1 > 0$ is a concurrent versor field in the sense of Myller $\Leftrightarrow$*

$$\begin{cases} b^3 & = b^4 = \cdots = b^m = 0, \\ \frac{d}{ds}(\frac{b^2}{G_1}) & = b^1. \end{cases} \tag{3.3}$$

And for parallelism:

Theorem 3.5. *The $\tilde{\pi}$-versor field $(\tilde{C}, \bar{\xi}_1)$ is parallel in the sense of Myller $\Leftrightarrow$ $G_1 = 0$.*

Applying these characterizations and the fundamental theorem, we formulate theorems of existence and uniqueness. Next, we can study the case of vector fields:

Theorem 3.6. *A $\tilde{\pi}$- vector field*

$$\bar{X} = \sum_{a=1}^{m} X^a \bar{\eta}_a$$

is a torse forming vector field in the sense of Myller if and only if its components in $\mathcal{R}_{\mathcal{M}}$ verify the next system of differentiable equations:

$$\frac{dX^a}{ds} - G_a X^{a+1} + G_{a-1}X^{a-1} = \alpha b^a + \beta X^a\,, \tag{3.4}$$

with $a \in \overline{1, m}$, $G_0 = G_m = 0$ and α, β functions of class C^∞ on $\widetilde{TM}$, restricted to $\tilde{C}$.

The next theorem of existence and uniqueness is a consequence of the existence and uniqueness of the solutions of the system above:

Theorem 3.7. *Let $\mathcal{M}(\tilde{C},\bar{\xi}_1,\bar{T}^m)$ be a Myller configuration, $\alpha,\beta \in \mathcal{F}(\widetilde{TM})$ some differentiable functions restricted to $\tilde{C}$ and $\bar{X}_0 \in \bar{T}^m(\tilde{C}(s_0))$. Then, there is an unique torse forming $\tilde{\pi}$-vector field $\bar{X}$ from $\bar{T}^m$ which satisfies $\bar{X}(s_0) = \bar{X}_0$.*

All these results can be particularized for concurrent, recurrent and parallel vector fields. We also can introduce the parallel transport in the sense of Myller of vector fields from $\bar{T}^m$ and prove:

Theorem 3.8. *The parallel transport in the sense of Myller of $\tilde{\pi}$-vector fields from $\bar{T}^m$ preserves the length of vectors and the angle between them.*

Proof Multiplying the equations of the former system with X^a, respectively, we find that $\frac{d}{ds}\overline{g}(\overline{X},\overline{X}) = 0$. Here, by "the angle" of vectors we understand not the usual angle used in Finsler geometry but the formal generalisation $\cos\theta = \frac{\overline{g}(X,Y)}{\overline{g}(X,X)\overline{g}(Y,Y)}$.

A similar study can be made for $\tilde{\pi}$**- torse forming versor/vector fields from** $\bar{T}^p$. The entire theory of torse forming vector fields in the sense of Myller is presented in an article in preparation.

4 Applications to the study of vector fields tangent or normal to a Finsler submanifold

We consider an differentiable immersion of class C^∞ $j : \check{M} \to M$ of a m dimensional submanifold $\check{M}$ in M. Locally, j is an embedding. It is known [2] that the Finsler structure of M induces on the given submanifold a Finsler structure. Let $C\Gamma = (\nabla, HTM)$ be the Cartan Finsler connection of F^n. We remember that HTM is the Cartan nonlinear connection on TM. We denote by $C\check{F} = (\check{\nabla}, HT\check{M})$ the induced Finsler connection on the given submanifold, and by $CF^\perp = (\nabla^\perp, HT\check{M})$ the Finsler connection induced on the normal bundle. $HT\check{M}$ is the nonlinear connection induced on $T\check{M}$ by the Cartan nonlinear connection HTM.
Now, we'll associate to the Finsler submanifold $\check{M}$ a special Myller configuration. Let C be a regular curve of class C^∞ on $\check{M}$, locally given by $C : s \to u^a(s),\ s \in I$, with s the arc-length parameter. We consider the next family of linear spaces along C:$C(s) \to T_{C(s)}\check{M} := T^m(C(s))$. Let ξ_1 a vector field along along C, tangent to $\check{M}$, $\xi_1(s) \in T^m(C(s)), s \in I$. The canonic lift $\tilde{C}$ of the curve C to $\widetilde{T\check{M}}$ is: $\tilde{C} : s \to (x^i(s), y^i(s) = \frac{dx^i}{ds}(s)), s \in I$ and $\bar{\xi}_1$, the lift of ξ_1 to a section of $\pi^*T\check{M}$ is: $\bar{\xi}_1(\tilde{C}(s)) = (\tilde{C}(s), \xi_1(C(s)))$. We consider the case when $\bar{\xi}_1$ is a $\tilde{\pi}$- versor field along $\tilde{C} \Leftrightarrow \bar{g}(\bar{\xi}_1,\bar{\xi}_1) = 1$.

We also consider $\bar{T}^m : \tilde{C}(s) \to \{\tilde{C}(s)\} \times T_{C(s)}\check{M} = (\pi^*T\check{M})_{\tilde{C}(s)},\ s \in I$. So, we defined a Myller configuration $\mathcal{M}_t(\tilde{C}, \bar{\xi}_1, \bar{T}^m)$ geometrically associated to the Finsler subspaces $\check{F}^m = (\check{M}, \check{F})$. Its invariants will be called the invariants of the vector field ξ_1 along C in the Finsler subspace $\check{F}^m$.

Theorem 4.1. *([9, 2]) The next Gauss-Weingarten formulae hold good:*

$$\begin{aligned} \nabla_{\tilde{X}}\bar{Y} &= \check{\nabla}_{\tilde{X}}\bar{Y} + \tilde{H}(\tilde{X},\bar{Y})\,,\ \ \tilde{X}\in\chi(\widetilde{T\check{M}})\,,\ \bar{Y}\in\Gamma(\pi^*T\check{M})\,, \\ \nabla_{\tilde{n}}\bar{\xi} &= -\tilde{B}_{\bar{n}}\tilde{X} + \nabla^{\perp}_{\tilde{X}}\bar{n}\,,\ \ \tilde{X}\in\chi(\widetilde{T\check{M}})\,,\ \bar{n}\in\Gamma(\mathcal{N})\,. \end{aligned} \tag{4.1}$$

$\check{\nabla}$ is the induced connection on $\pi^*T\check{M}$ by the connection ∇ on π^*TM (we know that it is metrical but it is not the Cartan connection of the subspace $\check{F}^m = (\check{M},\check{F}(u,v))$,
$\tilde{H}$ - the second fundamental form of the Finsler subspace $\check{M}$,
$\tilde{B}_{\bar{n}}$ - the Weingarten operator associated to the normal $\tilde{\pi}$- vector field $\bar{n}$,
$\nabla^{\perp}$- the normal connection induced on the normal bundle $\mathcal{N}$.

Using the Gauss-Weingarten formulae and the results of the former section, the next theorem can be easily obtained:

Theorem 4.2. *A $\tilde{\pi}$-vector field $\bar{X}$ along a curve on $\widetilde{T\check{M}}$, tangent to $\check{M}$, is a torse forming (concurrent / parallel) $\tilde{\pi}$-vector field in the sense of Myller in $\mathcal{M}_t(\tilde{C}\ ,\ \bar{\xi}_1\ ,\ \bar{T}^m)$, with respect to the Cartan connection FC if and only if it is a torse forming (concurrent / parallel) $\tilde{\pi}$-vector field with respect to the induced Finsler connection $\check{F}C = (\check{\nabla}, HT\check{M})$.*

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