Eleven limit cycles in a Hamiltonian system under five-order perturbed terms

Gheorghe Tigan

Abstract. In this paper we study the limit cycles of a Hamiltonian system under five-order perturbed terms. The system was studied previously under three-order perturbed terms ([1]), seven-order perturbed terms ([2]) and nine-order perturbed terms ([3]). In the present work we show that the system along with five families of systems have the same bifurcation diagrams. By numerical explorations the distribution of the limit cycles is pointed out.

M.S.C. 2000: 34C07, 37G15.
Key words: limit cycles, Abelian integral.

1 Introduction

Studies about limit cycles have roots in the Hilbert's 16th problem posed by Hilbert in 1900. It concerns about the number of limit cycles of a polynomial differential system of a given degree. This problem is still unsolved even for the quadratic polynomial differential systems. In the last time, studies about properties of limit cycles are of interest not only for mathematicians but also for physicists, and more recently for chemists, biologists, economists, etc. ([4],[5],[9],[10])

One way to produce limit cycles is by perturbing a Hamiltonian system which has one or more centers, in such a way that limit cycles bifurcate in the perturbed system from some of the periodic orbits in the original system. From ([2]) we recall the following results.

THEOREM 1.1 *Consider the perturbed Hamiltonian system*

$$\dot{x} = -\frac{\partial H}{\partial y} + P(x,y,\alpha), \dot{y} = \frac{\partial H}{\partial x} + Q(x,y,\alpha). \tag{1.1}$$

Assume that $P(x,y,0) = Q(x,y,0) = 0$, *the curve* C^h *defined by Hamiltonian* $H(x,h) = h$ *of system (1) is a periodic orbit extending outside as* h *increases and* $C^h(D)$ *is the area inside* C^h. *If there exists* h_0 *such that the function*

DIFFERENTIAL GEOMETRY - DYNAMICAL SYSTEMS, Vol.8, 2006, pp. 268-277.

$$A(h) = \int\limits_{C^h(D)} [P''_{x\alpha}(x,y,0) + Q''_{y\alpha}(x,y,0)]dxdy \tag{1.2}$$

satisfies $A(h_0) = 0, A'(h_0) \neq 0, \alpha A'(h_0) < 0(> 0)$, *then system (1) has only one stable (unstable) limit cycle nearby* C^{h_0} *for* α *very small. If the* C^h *constricts inside as* h *increases, the stability of the limit cycle is opposite with above. If* $A(h) \neq 0$, *then system (1) has no limit cycle.*

Consider the following system introduced in ([6]):

$$\begin{cases} \dot{x} = -\frac{\partial H}{\partial y} + \varepsilon x(p(x,y) - \lambda), \\ \dot{y} = \frac{\partial H}{\partial x} + \varepsilon y(q(x,y) - \lambda) \end{cases}. \tag{1.3}$$

where $p(0,0) = q(0,0) = 0$.

Using the above Theorem 1.1, from $A(h) = 0$, we get:

$$\lambda = \lambda(h) = \frac{\int\limits_{C^h(D)} f(x,y)dxdy}{2\int\limits_{C^h(D)} dxdy} \tag{1.4}$$

where $f(x,y) = xp'_x + yq'_y + p + q$.

This function $\lambda(h)$ is called *the detection function* of the system (1.3).

Using this detection function $\lambda(h)$ we get the following result ([3]):

Proposition 1.1 *Given* λ_0 : *a) If* $(h_0, \lambda(h_0))$ *is an intersecting point of line* $\lambda = \lambda_0$ *and the detection curve* $\lambda = \lambda(h)$, *and* $\lambda'(h_0) > 0(< 0)$, *then system (1.3) has only one stable (unstable) limit cycle nearby* C^{h_0} *when* $\lambda = \lambda_0$; *b) If line* $\lambda = \lambda_0$ *and the detection curve* $\lambda = \lambda(h)$ *have no intersecting point, then the system (1.3) has no any limit cycle when* $\lambda = \lambda_0$. *If the* C^h *constricts inside as* h *increases , the stability of the limit cycle is opposite with above.*

In the following we consider the perturbed system ([1], [2], [3]):

$$\begin{cases} \dot{x} = y(1 + x^2 - ay^2) + \varepsilon x(ux^n + vy^n - \lambda) \\ \dot{y} = -x(1 - cx^2 + y^2) + \varepsilon y(ux^n + vy^n - \lambda) \end{cases} \tag{1.5}$$

where $ac > 1, a > c > 0, 0 < \varepsilon \ll 1$, u, v, λ are the real parameters and $n = 2k, k$ integer positive.

Our purpose in the present work is to deepen the case $n = 4$, that corresponds to perturbation of five order. Another system related to the system (1.5) is studied in ([7]) and ([8]).

2 The behavior of the unperturbed system

The unperturbed system (2.1) corresponding to system (1.5) is

$$\begin{cases} \dot{x} = y(1 + x^2 - ay^2) \\ \dot{y} = -x(1 - cx^2 + y^2) \end{cases} \tag{2.1}$$

that is, system (1.5) in the case $\varepsilon = 0$, fig.1.

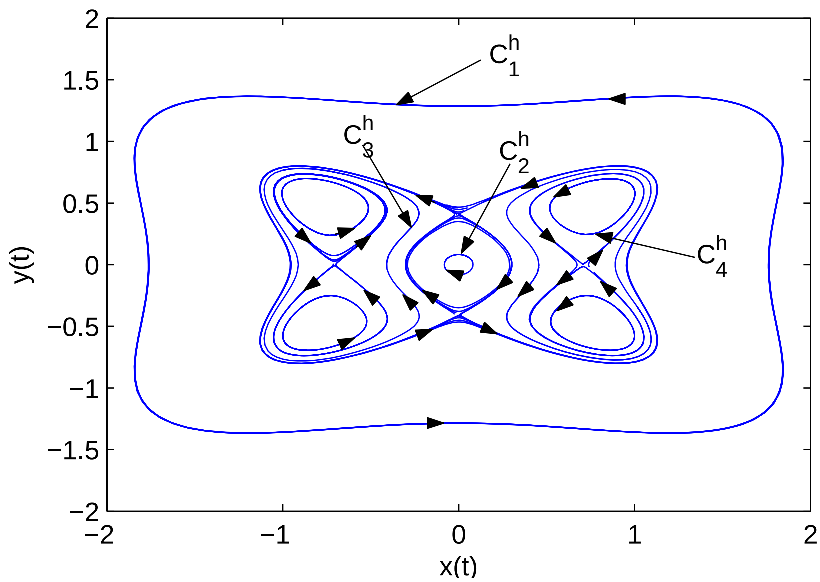

Figure 1: The phase portrait of the unperturbed system (2.1) when $a = 6, c = 2$

System (2.1) has nine finite singular points. By computing eigenvalues at each singular point we have that: $O(0,0), F_0^1(\sqrt{\frac{1+a}{ac-1}}, \sqrt{\frac{1+c}{ac-1}}), F_0^2(-\sqrt{\frac{1+a}{ac-1}}, \sqrt{\frac{1+c}{ac-1}}), F_0^3(-\sqrt{\frac{1+a}{ac-1}}, -\sqrt{\frac{1+c}{ac-1}})$, and

$F_0^1(\sqrt{\frac{1+a}{ac-1}}, -\sqrt{\frac{1+c}{ac-1}})$ are centers, while the other singular points $G_0^1(\sqrt{\frac{1}{c}}, 0), G_0^2(-\sqrt{\frac{1}{c}}, 0), G_0^3(0, \sqrt{\frac{1}{a}})$ and $G_0^4(0, -\sqrt{\frac{1}{a}})$ are hyperbolic saddle points. Further we have that the Hamiltonian of the system (2.1) is

$$H(x, y) = -(cx^4 + ay^4) + 2x^2y^2 + 2(x^2 + y^2) = h \tag{2.2}$$

and $H(0,0) = 0, H(F_0^1) = H(F_0^2) = H(F_0^3) = H(F_0^4) = \frac{a+c+2}{ac-1}$,
$H(G_0^1) = H(G_0^2) = \frac{1}{c}, H(G_0^3) = H(G_0^4) = \frac{1}{a}$, and $\frac{1}{a} < \frac{1}{c} < \frac{a+c+2}{ac-1}$.

In polar coordinates, $x = r\cos\theta, y = r\sin\theta$, system (2.1) becomes:

$$\dot{r} = -r^3 u'(\theta), \dot{\theta} = -1 + r^2 u(\theta) \tag{2.3}$$

and the Hamiltonian (2.2)

$$H(r,\theta) = -r^4 u(\theta) + 2r^2 = h, \tag{2.4}$$

where

$$u(\theta) = c\cos^4\theta + a\sin^4\theta - 2\cos^2\theta\sin^2\theta. \tag{2.5}$$

From (2.4), we get

$$r_{1,2} = r_{\pm}^2(\theta,h) = \frac{1 \pm \sqrt{1 - hu(\theta)}}{u(\theta)} \tag{2.6}$$

and from (2.6) and (2.3) we have

$$\dot{\theta} = \pm\frac{1}{2}[-(a+c+2)h\cos^2 2\theta + 2h(a-c)\cos 2\theta + 4 - h(a+c-2)]^{1/2}. \tag{2.7}$$

As h varies on the real line, the level curves defined by Hamiltonian (2.2) can be classified as follows ([2]):
$C_1^h : -\infty < h < \frac{1}{a}$, $C_2^h : \; 0 < h < \frac{1}{a}$,
$C_3^h : \; \frac{1}{a} < h < \frac{1}{c}$, and $C_4^h : \; \frac{1}{c} < h < \frac{a+c+2}{ac-1}$.

3 Detection functions and limit cycles of the perturbed system

Consider the following five families of perturbed systems:

$$\begin{cases} \dot{x} = y(1 + x^2 - ay^2) + p_{i,b} \\ \dot{y} = -x(1 - cx^2 + y^2) + q_{i,b}, \end{cases} \tag{3.1}$$

with b real, $i = 1-5$, $a, c, u, v, \varepsilon, \lambda, n$ as above, p, q positive integer and

$$\begin{cases} p_{1,b} = \varepsilon x(ux^n + vy^n - b\frac{q+1}{p+1}x^p y^q - \lambda),\; q_{1,b} = \varepsilon y(ux^n + vy^n + bx^p y^q - \lambda), \\ p_{2,b} = \varepsilon x(\frac{(n+2)u}{n+1}x^n - b\frac{q+1}{p+1}x^p y^q - \lambda),\; q_{2,b} = \varepsilon y(\frac{(n+2)v}{n+1}y^n + bx^p y^q - \lambda), \\ p_{3,b} = \varepsilon x((n+2)vy^n - b\frac{q+1}{p+1}x^p y^q - \lambda),\; q_{3,b} = \varepsilon y((n+2)ux^n + bx^p y^q - \lambda), \\ p_{4,b} = \varepsilon x \quad \frac{(n+2)u}{n+1}x^n + (n+2)vy^n - b\frac{q+1}{p+1}x^p y^q - 2\lambda \quad,\; q_{4,b} = \varepsilon bx^p y^{q+1}, \\ p_{5,b} = -\varepsilon b\frac{q+1}{p+1}x^{p+1}y^q,\; q_{5,b} = \varepsilon y((n+2)ux^n + \frac{(n+2)v}{n+1}y^n + bx^p y^q - 2\lambda) \end{cases} \tag{3.2}$$

For $b = 0$ we get the five systems related in ([3]).

We observe that $\frac{\partial^2 p_{i,b}}{\partial x\partial\varepsilon} + \frac{\partial^2 q_{i,b}}{\partial y\partial\varepsilon} = (2+n)\left(ux^n + vy^n\right) - 2\lambda = \frac{\partial^2 p_{i,0}}{\partial x\partial\varepsilon} + \frac{\partial^2 q_{i,0}}{\partial y\partial\varepsilon}$ for all $i = 1-5$.

Consequently, the four detection functions corresponding to the four closed curves C_j^h, $j = 1-4$, for the all above perturbations (3.2) are equal to the detection functions corresponding to system (1.5) and they are:

$$\lambda_j(h) = \frac{(n+2) \int\limits_{C_j^h(D)} (ux^n + vy^n)\, dxdy}{2 \int\limits_{C_j^h(D)} dxdy}, j = 1-4 \tag{3.3}$$

In polar coordinates ([3]) and for $a = 6, c = 2, n = 4$, (3.3) leads to:

(3.4)

$$\lambda_1(h) = \frac{\int\limits_0^{\pi/2} r_1^3(\theta, h)\, g(\theta)\, d\theta}{\int\limits_0^{\pi/2} r_1(\theta, h)\, d\theta}, -\infty < h < \frac{1}{6}, \quad \lambda_2(h) = \frac{\int\limits_0^{\pi/2} r_2^3(\theta, h)\, g(\theta)\, d\theta}{\int\limits_0^{\pi/2} r_2(\theta, h)\, d\theta}, 0 < h < \frac{1}{6},$$

$$\lambda_3(h) = \frac{\int\limits_0^{\theta_-(h)} \left(r_1^3(\theta, h) - r_2^3(\theta, h)\right) g(\theta)\, d\theta}{\int\limits_0^{\theta_-(h)} (r_1(\theta, h) - r_2(\theta, h))\, d\theta}, \ \frac{1}{6} < h < \frac{1}{2}, \tag{3.5}$$

$$\lambda_4(h) = \frac{\int\limits_{\theta_+(h)}^{\theta_-(h)} \left(r_1^3(\theta, h) - r_2^3(\theta, h)\right) g(\theta)\, d\theta}{\int\limits_{\theta_+(h)}^{\theta_-(h)} (r_1(\theta, h) - r_2(\theta, h))\, d\theta}, \ \frac{1}{2} < h < \frac{10}{11}, \tag{3.6}$$

where $\theta_{\pm}(h) = \frac{1}{2} arc\cos\left[\frac{2}{5} \pm \frac{1}{5}\sqrt{\frac{10}{h} - 11}\right], g(\theta) = u\cos^4\theta + v\sin^4\theta,$
and $r_{1,2}(\theta, h) = r_{\pm}^2(\theta, h)$.

Table 1

The values of the detection functions $\lambda_{1,2}(h)$ when $a = 6, c = 2, n = 4$.

h	$\lambda_1(h)$	h	$\lambda_1(h)$	h	$\lambda_2(h)$
-3.2	2.45452u+0.652181v	-0.7	1.32385u+0.308979v	0.001	$9.37998 \times 10^{-8} u + 9.39881 \times 10^{-8}$v
-3.1	2.41213u+0.63921v	-0.6	1.27386u+0.293878v	0.006	$3.38575 \times 10^{-6} u + 3.42734 \times 10^{-6}$v
-3	2.36958u+0.626197v	-0.5	1.22327u+0.27858v	0.011	0.0000114099u+0.000116721v
-2.9	2.32686u+0.613141v	-0.4	1.17208u+0.263061v	0.016	0.0000242034u+0.0000250289v
-2.8	2.28397u+0.60004v	-0.3	1.12027u+0.247295v	0.021	0.0000418029u+0.000043714v
-2.7	2.24089u+0.586892v	-0.2	1.06789u+0.23125v	0.026	0.0000642448u+0.0000679605v
-2.6	2.19762u+0.573695v	-0.1	1.01508u+0.214895v	0.031	0.0000915651u+0.0000980204v
-2.5	2.15416u+0.560446v	0.0	0.962323u+0.198202v	0.036	0.000123799u+0.000134167v
-2.4	2.11049u+0.547143v	0.01	0.957092u+0.196515v	0.041	0.000160981u+0.000176698v
-2.3	2.0666u+0.533785v	0.02	0.951878u+0.194824v	0.046	0.000203144u+0.000225938v
-2.2	2.02249u+0.520367v	0.03	0.946683u+0.19313v	0.051	0.00025032u+0.000282243v
-2.1	1.97814u+0.506886v	0.04	0.941512u+0.191434v	0.056	0.000302541u+0.000346001v
-2.0	1.93354u+0.493341v	0.05	0.936369u+0.189734v	0.061	0.000359834u+0.000417646v
-1.9	1.88868u+0.479726v	0.06	0.93126u+0.188033v	0.066	0.000422227u+0.000497653v
-1.8	1.84355u+0.466038v	0.07	0.926192u+0.18633v	0.071	0.000489743u+0.000586555v
-1.7	1.79813u+0.452273v	0.08	0.926192u+0.18633v	0.076	0.000562403u+0.000684946v
-1.6	1.75241u+0.438426v	0.09	0.916212u+0.18292v	0.081	0.000640223u+0.000793497v
-1.5	1.70637u+0.424491v	0.1	0.911325u+0.181215v	0.086	0.000723216u+0.000912965v
-1.4	1.65999u+0.410463v	0.11	0.906529u+0.179512v	0.096	0.000904735u+0.00118825v
-1.3	1.61325u+0.396334v	0.12	0.90185u+0.177813v	0.106	0.00110691u+0.00151946v
-1.2	1.56612u+0.382099v	0.13	0.897325u+0.17612v	0.116	0.00132952u+0.00191857v
-1.1	1.51859u+0.367748v	0.14	0.893012u+0.174439v	0.136	0.00183342u+0.00300013v
-1.	1.47063u+0.353272v	0.15	0.88902u+0.17278v	0.146	0.00211157u+0.00375836v
-0.9	1.42221u+0.33866v	0.16	0.885608u+0.171165v	0.156	0.00240174u+0.00478404v
-0.8	1.37329u+0.323901v	0.167	0.884367u+0.170139v	0.166	0.00268476u+0.00653876v

Table 2

The values of the detection function $\lambda_3(h)$ when $a = 6, c = 2, n = 4$.

h	$\lambda_3(h)$	h	$\lambda_3(h)$	h	$\lambda_3(h)$
0.167	0.996651u+0.190922v	0.287	1.04745u+0.188066v	0.407	1.07028u+0.184172v
0.177	1.00626u+0.191266v	0.297	1.04967u+0.187651v	0.417	1.07223u+0.18411v
0.187	1.01283u+0.191273v	0.307	1.05178u+0.187238v	0.427	1.07429u+0.184134v
0.197	1.01821u+0.191149v	0.317	1.05379u+0.18683v	0.437	1.0765u+0.184263v
0.207	1.02286u+0.190943v	0.327	1.05573u+0.186432v	0.447	1.07891u+0.184522v
0.217	1.02697u+0.19068v	0.337	1.05761u+0.186047v	0.457	1.08158u+0.184946v
0.227	1.03066u+0.190373v	0.347	1.05944u+0.18568v	0.467	1.0846u+0.185586v
0.237	1.03402u+0.190034v	0.357	1.06124u+0.185336v	0.477	1.08814u+0.186522v
0.247	1.03711u+0.189669v	0.367	1.06302u+0.18502v	0.487	1.09247u+0.187908v
0.257	1.03997u+0.189286v	0.377	1.06479u+0.184737v	0.497	1.09839u+0.190177v
0.267	1.04262u+0.188888v	0.387	1.06658u+0.184496v	0.4999	1.10103u+0.191341v
0.277	1.04511u+0.18848v	0.397	1.0684u+0.184304v		

Table 3

The values of the detection function $\lambda_4(h)$ when $a = 6, c = 2, n = 4$.

h	$\lambda_4(h)$	h	$\lambda_4(h)$	h	$\lambda_4(h)$
0.5	1.10118u+0.191414v	0.61	1.14928u+0.207886v	0.8	1.19565u+0.21932v
0.505	1.10555u+0.193321v	0.62	1.15235u+0.208744v	0.82	1.19948u+0.220125v
0.515	1.11179u+0.195783v	0.63	1.15532u+0.209559v	0.84	1.20316u+0.22088v
0.525	1.11703u+0.197708v	0.64	1.1582u+0.210337v	0.86	1.20671u+0.221587v
0.535	1.12173u+0.199354v	0.67	1.16637u+0.21247v	0.87	1.20843u+0.221923v
0.54	1.12394u+0.200104v	0.69	1.17147u+0.21375v	0.88	1.21012u+0.222249v
0.55	1.12813u+0.201489v	0.71	1.17632u+0.214934v	0.89	1.21179u+0.222565v
0.57	1.13581u+0.203921v	0.72	1.17866u+0.215493v	0.895	1.21261u+0.222719v
0.58	1.13939u+0.205007v	0.74	1.18319u+0.216552v	0.9	1.21342u+0.222871v
0.59	1.14281u+0.206024v	0.76	1.18752u+0.217539v	0.905	1.21422u+0.22302v
0.6	1.1461u+0.206981v	0.78	1.19167u+0.218459v	0.909	1.21486u+0.223138v

Consider $u = \frac{k}{2}$, $v = \frac{s}{2}$ and $-200 < k < 200$, $-200 < s < 200$, k, s integers. In order to get a maximum number of limit cycles we study the graphs of the detection functions $\lambda_j(h), j = 1-4$. First we pay attention to the last detection function $\lambda_4(h)$ because it produces most limit cycles. We want to see how many local extremum points can we get. After some tedious computations we observe three cases:
4a) $\lambda_4(h)$ has no local extremum,
4b) $\lambda_4(h)$ has a single local maximum for some values of $(u, v) \in (-44, 0) \times [0, 100)$, fig.2a)
4c) $\lambda_4(h)$ has a single local minimum for some values of $(u, v) \in (0, 44) \times (-100, 0]$, fig.2b).

Then we study $\lambda_3(h)$. We remark three cases again:
3a) $\lambda_3(h)$ has no local extremum,
3b) $\lambda_3(h)$ has a single local minimum or maximum for some values of (u, v), fig.3a),
3c) $\lambda_3(h)$ has a local maximum and a local minimum for some values of (u, v), fig.3b),

From Proposition 1.1, a line $\lambda = \lambda_0$ has to intersect the two graphs in a maximum number of points. It is clear that the cases 4a) and 3a) provide a minimum number

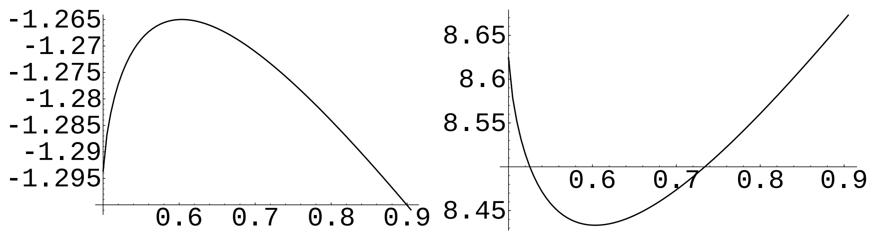

Figure 2: The detection function λ_4 for a) $u = -3, v = 10.5$, (left) and, respectively, b) $u = 20, v = -70$, (right)

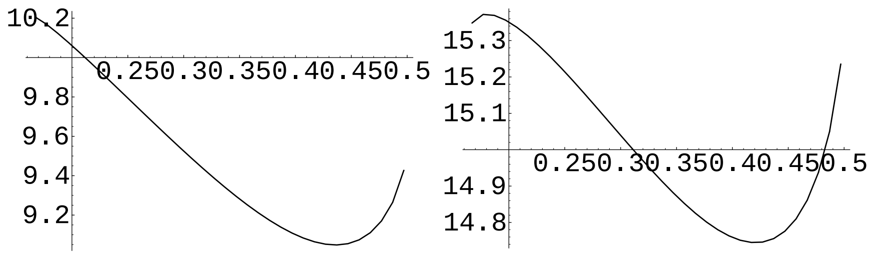

Figure 3: The detection functions λ_3 for a) $u = -7, v = 90$ (left) and, respectively, b) $u = -0.5, v = 83$ (right)

of limit cycles, so are not important. Further for any $(u, v) \in (-44, 0) \times [0, 100)$, in the case 3b) and 4b), by effectively testing, we obtained that $localmin(\lambda_3(h)) < \lambda_3(0.499) < \lambda_4(0.5) < localmax(\lambda_4(h)) < max((\lambda_3(h)))$ such that we can get three intersecting points in this case ($localmin(\lambda_3(h))$ means local minimum of $\lambda_3(h)$ for h on the corresponding interval,etc). From 3c) and 4b) we could get a maximum of four intersecting points, but after testing, we have that $localmin(\lambda_3(h)) < \lambda_3(0.499) < \lambda_4(0.5) < localmax(\lambda_4(h)) < \lambda_3(0.1667) < localmax(\lambda_3(h))$, that is, the maximum intersecting points can not be four but only three, that corresponds to ten limit cycles, two of them in the neighborhood of each closed orbit of the form C_4^h and one in the neighborhood of each closed orbit of the form C_3^h.

Because the values of $\lambda_2(h)$ are very small, this detection function does not help us to produce more limit cycles.

For the first detection function $\lambda_1(h)$ we have obtained again three possibilities:
1a) $\lambda_1(h)$ has no local extremum,
1b) $\lambda_1(h)$ has a single local minimum or maximum for some values of (u, v), fig.4a),
1c) $\lambda_1(h)$ has a local minimum and a local maximum for some values of (u, v), fig.4b).

Comparing $\lambda_1(h)$ with $\lambda_4(h)$ we have that, in the cases 1b) and 4b) $max(\lambda_4(h)) < \lambda_1(0.1667) < max(\lambda_1(h))$ and in the case 1c) and 4b) $max(\lambda_4(h)) < localmin(\lambda_1(h))$ that is, $\lambda_1(h)$ can contribute with only one limit cycle.

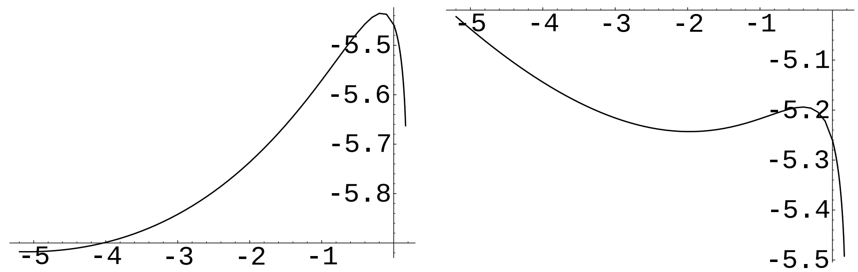

Figure 4: The detection functions λ_1 for a) $u = -17, v = 55$ (left) and, respectively, b) $u = -17, v = 56$ (right)

The case 4c), (fig.2b), for $(u, v) \in (0, 44) \times (-100, 0]$ with the corresponding cases for the other three detection functions is similar with 4b), providing again the same scenario of the distribution of the limit cycles, (fig.5, fig.6).

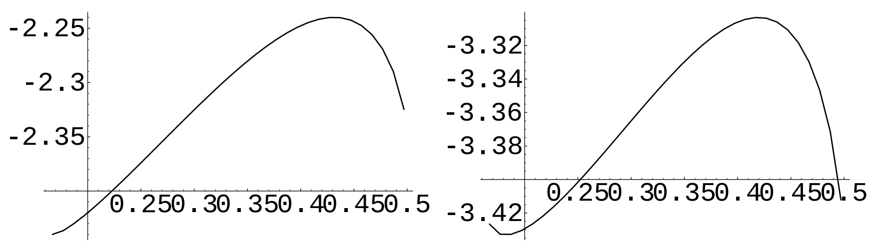

Figure 5: The detection function $\lambda_3(h)$ for a) $u = 1, v = -18$, (left) and, respectively, b) $u = 0.01, v = -18$, (right)

Therefore, we have the following conclusion:

Theorem 3.1 *For* $a = 6, c = 2, u = \frac{k}{2}, v = \frac{s}{2}, -200 < k < 200, -200 < s < 200$ *, k, s integer numbers,* $0 < \epsilon \ll 1$*, the system (2.1) along with the five systems (3.1) can have at least 11 limit cycles. In particular, for* $u = 8, v = -24.2$ *we have:*
a) If $4.17779 < \lambda < 4.19235$*, then the system (1.5) along with the five systems (3.1) have at least 9 limit cycles (fig.8a),*
b) If $4.14885 < \lambda < 4.17723$*, then the system (1.5) along with the five systems (3.1) have at least 11 limit cycles (fig.8b).*

For the values $u = 8$ and $v = -24.2$ we have that, local minimum of $\lambda_4(h)$ is $min_4 = 4.14885$, $\lambda_4(0.5) = 4.17723$, $\lambda_3(0.499) = 4.17779$, local maximum of $\lambda_3(h), max_3 = 4.19235$, (fig.7).

The distribution of the limit cycles is drawn in fig.8a,b. From fig.7 we could extract more results but we register only these two more important cases.

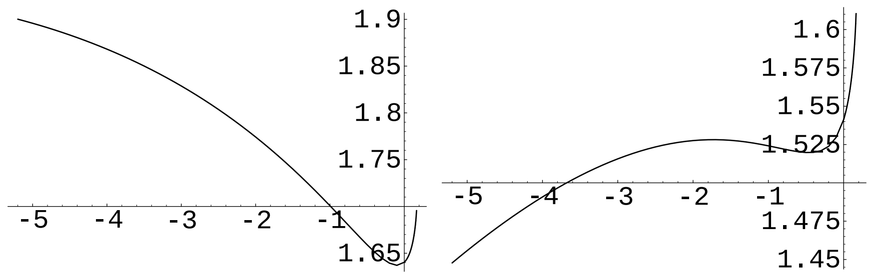

Figure 6: The detection function $\lambda_1(h)$ for a) $u = 5, v = -16$, (left) and, respectively, b) $u = 5, v = -16.5$, (right)

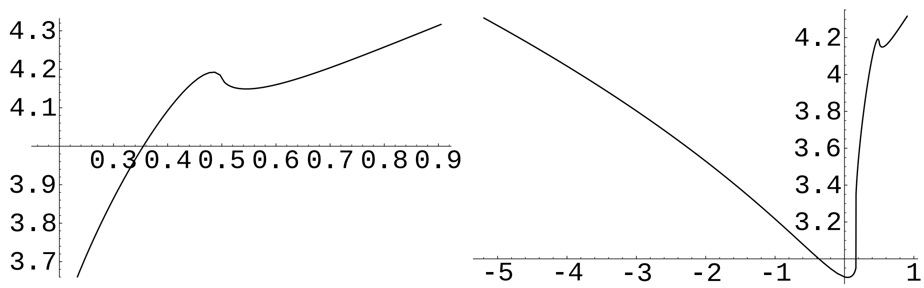

Figure 7: The detection function a) $\lambda_3(h), \lambda_4(h)$ (left) and, respectively, b) $\lambda_1(h), \lambda_2(h), \lambda_3(h), \lambda_4(h)$, (right) for $u = 8, v = -24.2$

4 Conclusions

The system (2.1) together with the five families of systems (3.1) when $a = 6, c = 2, n = 4, 0 < \epsilon \ll 1, u = \frac{k}{2}, v = \frac{s}{2}$ and $-200 < k < 200, -200 < s < 200$, k, s integer numbers, can have at least eleven limit cycles. The detection function method was employed. By numerical explorations we have obtained the shape of the graphs of the detection functions from which we have drawn the distribution of the limit cycles.

5 Acknowledgements

This work was supported through a European Community Marie Curie Fellowship and in the framework of the CTS, contract number HPMT-CT-2001-00278.

References

[1] JB. Li, ZR. Liu , *Bifurcation set and limit cycles forming compound eyes in a perturbed Hamiltonian system*, Publ.Math. 35 (1991), 487-506.

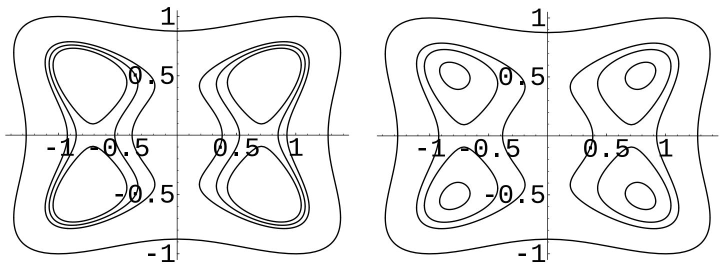

Figure 8: Distribution diagram corresponding to: a) nine (left), b) eleven (right) limit cycles of the system (2.1) when $a = 6, c = 2, u = 8, v = -24.2, n = 4$

[2] H. Cao, Z. Liu , Z. Jing, *Bifurcation set and distribution of limit cycles for a class of cubic Hamiltonian system with higher-order perturbed terms*, Chaos, Solitons and Fractals 11 (2000), 2293–2304.

[3] M. Tang, X. Hong, *Fourteen limit cycles in a cubic Hamiltonian system with nine-order perturbed term*, Chaos, Solitons and Fractals 14 (2002), 1361-1369.

[4] SN. Chow, C. Li, D. Wang, *Normal Forms and Bifurcation of Planar Vector Fields*, Cambridge University Press, Cambridge, 1994.

[5] Y. Yanqian, *Theory of Limit Cycles*, Translations of Math. Monographs, vol. 66, Amer. Math. Soc.,Providence, RI, 1986.

[6] CF. Li, JB. Li, *Distribution of limit cycles for planar cubic Hamiltonian systems*, Acta Math Sinica 28 (1985), 509-521.

[7] Gh. Tigan, *Existence and distribution of limit cycles in a Hamiltonian system*, Applied Mathematics E–Notes, Taiwan (accepted), to appear.

[8] Gh. Tigan, *Analysis of a Hamiltonian system under seven-order perturbed terms*, in review.

[9] M. Viano, J. Llibre, H. Giacomini, *Arbitrary order bifurcations for perturbed Hamiltonian planar systems via the reciprocal of an integrating factor*, Nonlinear Analysis, 48 (2002), 117-136.

[10] Z. Hong, W. Chen, Z. Tonghua, *Perturbation from a cubic Hamiltonian with three figure eight-loops*, Chaos, Solitons and Fractals, 22 (2004), 61-74.

Author's address:

Gheorghe Tigan
"Politehnica" University of Timisoara, Department of Mathematics,
P-ta R. Maria, Nr. 1, Timisoara, Timis, Romania.
email: gtigan73@yahoo.com