On decomposability of the curvature tensor in recurrent conformal Finsler spaces

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Abstract. The decomposability of curvature tensor in a Finsler manifold was studied by P. N. Pandey [2]. The purpose of the present paper is to decompose the curvature tensor in recurrent conformal Finsler space and study the properties of conformal decomposition tensors.

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1 Introduction

Let the two distinct metric functions $F(x,\dot{x})$ and $\bar{F}(x,\dot{x})$ are defined over a n-dimensional Finsler space Fn. Then the two metrices resulting from these functions are called conformal, if the corresponding metric tensors $g_{ij}(x,\dot{x})$, $\bar{g}_{ij}(x,\dot{x})$ are proportional to each other. Knebelman [1] has proved that the factor of proportionality between them is at most point function. Thus we have

$$\bar{g}_{ij}(x,\dot{x}) = e^{2\sigma} g_{ij}(x,\dot{x}), \tag{1.1}$$

where

$$\sigma = \sigma(x). \tag{1.2}$$

Hence,

$$\bar{g}^{ij}(x,\dot{x}) = e^{-2\sigma} g^{ij}(x,\dot{x}), \tag{1.3}$$

and

$$\bar{F}(x,\dot{x}) = e^{\sigma} F(x,\dot{x}). \tag{1.4}$$

The space equiped with quantities $\bar{F}(x,\dot{x})$, $\bar{g}(x,\dot{x})$ etc is called a conformal Finsler space usually denoted by $\bar{F}n$.

In conformal Finsler space the Berwald's connection coefficient may be written as

$$\bar{G}^i_{jk}(x,\dot{x}) = G^i_{jk}(x,\dot{x}) - \dot{\partial}_k \dot{\partial}_j B^{im}(x,\dot{x})\sigma_m, \tag{1.5}$$

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where,

$$B^{im}(x,\dot{x}) = \frac{1}{2}F^2 g^{im} - \dot{x}^i\dot{x}^m. \tag{1.6}$$

The functins B^{im} are homogeneous of the second degree in its directional arguments.

The curvature tensor H^i_{jkh} under the conformal change (1.1) as

$$\begin{aligned}\bar{H}^i_{jkh}(x,\dot{x}) \;=\;& H^i_{jkh}(x,\dot{x}) - 2\sigma_m\dot{\partial}_j\{\dot{\partial}_{[k}B^{im}\}_{(h)]} + \\ & 2\sigma_{m[(k)}\dot{\partial}_{h]}\dot{\partial}_j B^{im} + 2\sigma_r(\dot{\partial}_{[k}B^{im})G^r_{h]mj} \\ & +2\sigma_m\sigma_r\dot{\partial}_j(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{ir}.\end{aligned} \tag{1.7}$$

The decomposition of curvature tensor H^i_{jkh} is defined by P.N.Pandey [2].

$$H^i_{jkh} = X^i_j A_{kh}, \tag{1.8}$$

where X^i_j is non zero tensor and A_{kh} is skew symmetric decomposition tensor.

The recurrent curvature tensor H^i_{jkh} is characterised by the condition

$$H^i_{jkh(l)} = V_l H^i_{jkh}, \tag{1.9}$$

where

$$H^i_{jkh} \neq 0. \tag{1.10}$$

The covariant vector V_l is called the recurrence vector. The space equipped with such curvature tensor is called recurrent Finsler space.

Transvecting (1.8) by y_i, we get

$$y_i H^i_{jkh} = \lambda_j A_{kh}, \tag{1.11}$$

where

$$\begin{cases} a) & y_i = \dot{x}^j g_{ij}\ , \\ b) & \lambda_j = y_i X^i_j. \end{cases} \tag{1.12}$$

A non zero vector λ_j and choose another vector V^j such that

$$V^j\lambda_j = 1. \tag{1.13}$$

Transvecting the Bianchi identity

$$H^i_{jkh} + H^i_{khj} + H^i_{hjk} = 0, \tag{1.14}$$

by y_i, and using (1.11), we have

$$\lambda_j A_{kh} + \lambda_k A_{hj} + \lambda_h A_{jk} = 0. \tag{1.15}$$

The conformal transformation of vector λ_j, V^j and tensor X^i_j and the directional argument $\dot{x}^j$ may be written as

$$(1.16) \qquad \begin{cases} a) & \bar{X}^i_j = e^{-\sigma} X^i_j, \\ b) & \bar{\lambda}_j = e^{\sigma} \lambda_j, \\ c) & \bar{V}^j = e^{-\sigma} V^j, \\ d) & \overline{\dot{x}^j} = \dot{x}^j. \end{cases}$$

The Berwald's covariant derivative of metric tensor is given by [3].

$$(1.17) \qquad g_{ij(l)} = -2A_{ijl|h} l^h.$$

Transvecting (1.17) by $\dot{x}^j$, we get

$$(1.18) \qquad \dot{x}^j g_{ij(l)} = 0.$$

The Berwald's covariant derivative of $\dot{x}^j$ and F vanish, i.e.

$$(1.19) \qquad \begin{cases} a) & \dot{x}^j_{(l)} = 0, \\ b) & F_{(l)} = 0. \end{cases}$$

Differentiating (1.12a) covariantly with respect to x^l in the sence of Berwald's, we get

$$(1.20) \qquad y_{i(l)} = \dot{x}^j_{(l)} g_{ij} + \dot{x}^j g_{ij(l)},$$

In the view of (1.18) and (1.19a), above equation reduces to

$$(1.21) \qquad y_{i(l)} = 0.$$

The equation (1.13) may be written as

$$(1.22) \qquad V^j y_i X^i_j = 1.$$

Differentiating (1.22) covariantly with respect to x^l, we get

$$(1.23) \qquad V^j_{(l)} y_i X^i_j + V^j y_{i(l)} X^i_j + V^j y_i X^i_{j(l)} = 0.$$

If the tensor X^i_j is covariant constant, then above equation reduces to

$$(1.24) \qquad V^j_{(l)} y_i X^i_j = 0,$$

in the light of equation (1.21).

Since y_i is non zero vector and X^i_j is non zero tensor, it implies

$$(1.25) \qquad V^j_{(l)} = 0,$$

which shows that $V^j_{(l)}$ is covariant constant.

2 Decomposition of the curvature tensor in $R - \bar{F}_n$.

We considered the decomposition of conformal curvature tensor $\bar{H}^i_{jkh}$ in the form

$$\bar{H}^i_{jkh} = \bar{X}^i_j \bar{A}_{kh}, \tag{2.1}$$

where $\bar{X}^i_j$ is non zero conformal tensor and $\bar{A}_{kh}$ is skew symmetric conformal decompositon tensor.

The recurrent conformal curvature tensor $\bar{H}^i_{jkh}$ is characterised by the condition

$$\bar{H}^i_{jkh(\bar{l})} = \bar{V}_l \bar{H}^i_{jkh}, \tag{2.2}$$

where

$$\bar{H}^i_{jkh} \neq 0. \tag{2.3}$$

The covariant vector $\bar{V}_l$ is called conformal recurrence vector. The space equipped with such recurrent conformal curvature tensor is called recurrent conformal Finsler space and we denote it by $R - \bar{F}_n$.

Transvecting (2.1) by $\bar{y}_i$, we have

$$\bar{y}_i \bar{H}^i_{jkh} = \bar{\lambda}_j \bar{A}_{kh}, \tag{2.4}$$

where

$$\begin{cases} a) & \bar{y}_i = \overline{\dot{x}^j} \bar{g}_{ij}, \\ b) & \bar{\lambda}_j = \bar{y}_i \bar{X}^i_j. \end{cases} \tag{2.5}$$

We may choose a vector $\bar{V}^j$ such that $\bar{V}^j \bar{\lambda}_j = 1$. Transvecting (2.4) by such a vector $\bar{V}^j$, we get

$$\bar{A}_{kh} = \bar{V}^j \bar{y}_i \bar{H}^i_{jkh}, \tag{2.6}$$

using equation (1.7) in (2.6), we get

$$\begin{aligned} \bar{A}_{kh} \quad = \quad & \bar{V}^j \bar{y}_i [\bar{H}^i_{jkh} - 2\{\sigma_m \dot{\partial}_j (\dot{\partial}_{[k} B^{im})_{(h)]} - \\ & \sigma_{m[(k)} \dot{\partial}_{h]} \dot{\partial}_j B^{im} - \ \sigma_r (\dot{\partial}_{[k} B^{im}) G^r_{h]mj} \\ & -\sigma_m \sigma_r \dot{\partial}_j (\dot{\partial}_{[k} B^{sm}) \dot{\partial}_{h]} \dot{\partial}_s B^{ir}\}]. \end{aligned} \tag{2.7}$$

Applying equations (1.16c), (1.16d), (2.5a) and (1.1) in (2.7), we obtain

$$\begin{aligned} \bar{A}_{kh} \quad = \quad & e^\sigma V^j y_i H^i_{jkh} - e^\sigma V^j y_i 2\{\sigma_m \dot{\partial}_j (\dot{\partial}_{[k} B^{im})_{(h)]} - \\ & \sigma_{m[(k)} (\dot{\partial}_{h]} \dot{\partial}_j B^{im}) - \ \sigma_m \sigma_r \dot{\partial}_j (\dot{\partial}_{[k} B^{sm}) \dot{\partial}_{h]} \dot{\partial}_s B^{ir}\}, \end{aligned} \tag{2.8}$$

using the relations (1.8), (1.12b) and (1.13) in (2.8), it yields

$$\begin{aligned} \bar{A}_{kh} \quad = \quad & e^\sigma A_{kh} - e^\sigma V^j y_i 2[\sigma_m \dot{\partial}_j \{\dot{\partial}_{[k} B^{im}\}_{(h)]} - \\ & \sigma_{m[(k)} \dot{\partial}_{h]} \dot{\partial}_j B^{im} - \sigma_m \sigma_r \dot{\partial}_j \{(\dot{\partial}_{[k} B^{sm}) \dot{\partial}_{h]} \dot{\partial}_s B^{ir}\}]. \end{aligned} \tag{2.9}$$

Hence we can state:

Theorem 1. *Under the decomposition (2.1), the skew-symmetric conformal decomposition tensor $\bar A_{kh}$ is expressed in the form (2.9).*

Differentiating (2.1) covariantly with respect to $\bar x^l$ in the sence of Berwald's, we get

$$\bar H^i_{jkh(\bar l)} = \bar X^i_{j(\bar l)} \bar A_{kh} + \bar A_{kh(\bar l)} \bar X^i_j. \tag{2.10}$$

Applying (2.1) and (2.2) in the above equation, we find

$$\bar V_l \bar X^i_j \bar A_{kh} = \bar X^i_{j(\bar l)} \bar A_{kh} + \bar X^i_j \bar A_{kh(\bar l)}. \tag{2.11}$$

Let us assume that the conformal vector $\bar X^i_j$ is covariant constant, then (2.11) reduces to

$$\bar A_{kh(\bar l)} = \bar V_l \bar A_{kh}. \tag{2.12}$$

Conversely, if (2.12) is true, the equation (2.11), yields

$$\bar X^i_{j(\bar l)} \bar A_{kh} = 0, \tag{2.13}$$

Since $\bar A_{kh}$ is non zero conformal tensor, it implies

$$\bar X^i_{j(\bar l)} = 0, \tag{2.14}$$

which shows that $\bar X^i_j$ is covariant constant.

Thus we can state:

Theorem 2. *In a $R-\ \bar F_n$, the necessary and sufficient condition for the skew symmetric conformal decomposition tensor $\bar A_{kh}$ to be recurrent is that the conformal tensor $\bar X^i_j$ is covariant constant in the sense of Berwald's.*

Differentiating (1.7) covariantly with respect to $\bar x^l$ in the sence of Berwald's, we get

$$\begin{aligned}(2.15)\,\bar H^i_{jkh(\bar l)} \quad = \quad & H^i_{jkh(l)} - 2[\sigma_{m(l)} \dot\partial_j (\dot\partial_{[k} B^{im})_{(h)]} + \sigma_m \{\dot\partial_j (\dot\partial_{[k} B^{im})_{(h)]}\}_{(l)} \\ & -\sigma_{m[(k)(<l>)} \dot\partial_{h]} \dot\partial_j B^{im} - \sigma_{m[(k)} (\dot\partial_{h]} \dot\partial_j B^{im})_{(l)} - \\ & \sigma_{r(l)} (\dot\partial_k B^{im}) G^i_{h]mj} - \sigma_r (\dot\partial_k B^{im})_{(l)} G^i_{h]mj} - \sigma_r (\dot\partial_k B^{im}) \\ & (G^i_{h]mj})_{(l)} - (\sigma_{m(l)} \sigma_r - \sigma_m \sigma_{r(l)}) \dot\partial_j (\dot\partial_{[k} B^{sm}) \dot\partial_{h]} \dot\partial_s B^{ir} - \\ & \sigma_m \sigma_r \{(\dot\partial_j \dot\partial_{[k} B^{sm})_{(l)} (\dot\partial_{h]} \dot\partial_s B^{ir}) - (\dot\partial_j \dot\partial_{[k} B^{sm}) (\dot\partial_{h]} \dot\partial_s B^{ir})_{(l)}\}].\end{aligned}$$

Adding the expressions obtained by the cyclic change of the indices k, h and l in (2.15), we have

$$\begin{aligned}(2.16) \quad 2Z \quad = \quad & \bar H^i_{j[kh(\bar l)]} - H^i_{j[kh(l)]} - 2[\sigma_{r(l)} (\dot\partial_{[k} B^{im}) G^r_{h]mj} + \sigma_{r(k)} \\ & (\dot\partial_{[h} B^{im}) G^r_{l]mj} + \sigma_{r(h)} (\dot\partial_{[l} B^{im}) G^r_{k]mj} + \sigma_r (\dot\partial_{[k} B^{im})_{(l)} G^r_{h]mj} \\ & +\sigma_r (\dot\partial_{[h} B^{im})_{(k)} G^r_{l]mj} + \sigma_r (\dot\partial_{[l} B^{im})_{(h)} G^r_{k]mj} + \sigma_r (\dot\partial_{[k} B^{im}) \\ & G^r_{h]mj(l)} + \sigma_r (\dot\partial_{[h} B^{im}) G^r_{l]mj(k)} + \sigma_r (\dot\partial_{[l} B^{im}) G^r_{k]mj(h)}],\end{aligned}$$

where

$$
\begin{aligned}
(2.17)\qquad Z \;=\;& [-\sigma_{m(l)}\{\dot{\partial}_j(\dot{\partial}_{[k}B^{im})_{(h)]}\} - \sigma_{m(k)}\{\dot{\partial}_j(\dot{\partial}_{[h}B^{im})_{(l)]}\} -\\
& \sigma_{m(h)}\{\dot{\partial}_j(\dot{\partial}_{[l}B^{im})_{(k)]}\} - \sigma_m\{\dot{\partial}_j(\dot{\partial}_{[k}\dot{B}^{im})_{(h)]}\}_{(l)} -\\
& \sigma_m\{\dot{\partial}_j(\dot{\partial}_{[h}B^{im})_{(l)]}\}_{(k)} - \sigma_m\{\dot{\partial}_j(\dot{\partial}_{[l}B^{im})_{(k)]}\}_{(h)} +\\
& \sigma_{m[(k)<(l)>}(\dot{\partial}_{h]}\dot{\partial}_jB^{im}) + (\sigma_{m[(h)<(k)>}(\dot{\partial}_{l]}\dot{\partial}_jB^{im}) +\\
& (\sigma_{m[(l)<(h)>}(\dot{\partial}_{k]}\dot{\partial}_jB^{im}) + \sigma_{m[(k)}(\dot{\partial}_{h]}\dot{\partial}_jB^{im})_{(l)} +\\
& \sigma_{m[(h)}(\dot{\partial}_{l]}\dot{\partial}_jB^{im})_{(k)} + \sigma_{m[(l)}(\dot{\partial}_{k]}\dot{\partial}_jB^{im})_{(h)} + \{(\sigma_{m(l)}\sigma_r\\
& +\sigma_m\sigma_{r(l)})(\dot{\partial}_j\dot{\partial}_{[k}B^{sm})(\dot{\partial}_{h]}\dot{\partial}_sB^{ir}) + \sigma_m\sigma_r\{(\dot{\partial}_j\dot{\partial}_{[k}B^{sm})_{(l)}\\
& (\dot{\partial}_{h]}\dot{\partial}_sB^{ir}) + (\dot{\partial}_j\dot{\partial}_{[k}B^{sm})(\dot{\partial}_{h]}\dot{\partial}_sB^{ir})_{(l)}\} + \{(\sigma_{m(k)}\sigma_r +\\
& \sigma_m\sigma_{r(k)})(\dot{\partial}_j\dot{\partial}_{[h}B^{sm})(\dot{\partial}_{l]}\dot{\partial}_sB^{ir})\} + \sigma_m\sigma_r\{(\dot{\partial}_j\dot{\partial}_{[h}B^{sm})_{(k)}\\
& (\dot{\partial}_{l]}\dot{\partial}_sB^{ir}) + (\dot{\partial}_j\dot{\partial}_{[h}B^{sm})(\dot{\partial}_{l]}\dot{\partial}_sB^{ir})_{(k)}\} + \{(\sigma_{m(h)}\sigma_r +\\
& \sigma_m\sigma_{r(h)})(\dot{\partial}_j\dot{\partial}_{[l}B^{sm})(\dot{\partial}_{k]}\dot{\partial}_sB^{ir})\} + \sigma_m\sigma_r\{(\dot{\partial}_j\dot{\partial}_{[l}B^{sm})_{(h)}\\
& (\dot{\partial}_{k]}\dot{\partial}_sB^{ir}) + (\dot{\partial}_j\dot{\partial}_{[l}B^{sm})(\dot{\partial}_{k]}\dot{\partial}_sB^{ir})_{(h)}.
\end{aligned}
$$

The Berwald's covariant derivative of (2.9), gives

$$
\begin{aligned}
(2.18)\; \bar{A}_{kh(\bar{l})} \;=\;& e^\sigma A_{kh(l)} + e^\sigma\sigma_{(l)}A_{kh} - 2e^\sigma V^jy_i[\sigma_l\{\sigma_m\dot{\partial}_j(\partial_{[k}B^{im})_{(h)]}\\
& -\sigma_{m[(k)}\dot{\partial}_{h]}\dot{\partial}_jB^{im} - \sigma_m\sigma_r\dot{\partial}_j(\dot{\partial}_kB^{sm})\dot{\partial}_{h]}\dot{\partial}_sB^{ir}\} + \sigma_{m(l)}\\
& \dot{\partial}_j(\partial_{[k}B^{im})_{(h)]} + \sigma_m\{\dot{\partial}_j(\partial_{[k}B^{im})_{(h)]}\}_{(l)} - \sigma_{m[(k)<(l)>}\dot{\partial}_{h]}\\
& \dot{\partial}_jB^{im} - \sigma_{m[(k)}(\dot{\partial}_{h]}\dot{\partial}_jB^{im})_{(l)} - \sigma_{m(l)}\sigma_r\dot{\partial}_j(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_sB^{ir}\\
& -\sigma_m\sigma_{r(l)}\dot{\partial}_j(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_sB^{ir} - \sigma_m\sigma_r(\dot{\partial}_j\dot{\partial}_{[k}B^{sm})_{(l)}\dot{\partial}_{h]}\dot{\partial}_sB^{ir}\\
& \sigma_m\sigma_r(\dot{\partial}_j\dot{\partial}_{[h}B^{sm})_{(k)}\dot{\partial}_{l]}\dot{\partial}_sB^{ir}\}].
\end{aligned}
$$

by virtue of equations (1.21) and (1.25).

Adding the expressions obtained by the cyclic change of (2.18) with respect to indices k, h and l, we have

$$
\begin{aligned}
(2.19)\qquad \bar{A}_{[kh(\bar{l})]} \;=\;& e^\sigma\{(A_{[kh(l)]}) + \sigma_lA_{kh} + \sigma_kA_{hl} + \sigma_hA_{lk})\} + e^\sigma V^jy_i\\
& 2[-\sigma_m\{\sigma_l\dot{\partial}_j(\partial_{[k}B^{im})_{(h)]} + \sigma_k\dot{\partial}_j(\dot{\partial}_{[h}B^{im})_{(l)]} +\\
& \sigma_h\dot{\partial}_j(\dot{\partial}_{[l}B^{im})_{(k)]}\} + \{\sigma_l\sigma_{m[(k)}(\dot{\partial}_{h]}\dot{\partial}_jB^{im}) +\\
& \sigma_k\sigma_{m[(h)}(\dot{\partial}_{l]}\dot{\partial}_jB^{im}) + \sigma_h\sigma_{m[(l)}(\dot{\partial}_{k]}\dot{\partial}_jB^{im})\} +\\
& \sigma_m\sigma_r\{\sigma_l(\dot{\partial}_j\dot{\partial}_kB^{sm})\dot{\partial}_{h]}\dot{\partial}_sB^{ir} + \sigma_k(\dot{\partial}_j\dot{\partial}_hB^{sm})\dot{\partial}_{l]}\dot{\partial}_sB^{ir}\\
& +\sigma_h(\dot{\partial}_j\dot{\partial}_lB^{sm})\dot{\partial}_{k]}\dot{\partial}_sB^{ir}\} + Z],
\end{aligned}
$$

In view of (2.16), equation (2.19) becomes

$$\begin{aligned} \bar{A}_{[kh(\bar{l})]} \quad = \quad & e^{\sigma}\{(A_{[kh(l)]}) + \sigma_l A_{kh} + \sigma_k A_{hl} + \sigma_h A_{lk})\} + e^{\sigma} V^j y_i \\ & 2[-\sigma_m\{\sigma_l \dot{\partial}_j(\dot{\partial}_{[k} B^{im})_{(h)]} + \sigma_k \dot{\partial}_j(\dot{\partial}_{[h} B^{im})_{(l)]} + \\ & \sigma_h \dot{\partial}_j(\dot{\partial}_{[l} B^{im})_{(k)]}\} - \sigma_l \sigma_{m[(k)}(\dot{\partial}_{h]} \dot{\partial}_j B^{im}) - \\ & \sigma_k \sigma_{m[(h)}(\dot{\partial}_{l]} \dot{\partial}_j B^{im}) + \sigma_h \sigma_{m[(l)}(\dot{\partial}_{k]} \dot{\partial}_j B^{im})\} + \\ & \sigma_m \sigma_r\{\sigma_l(\dot{\partial}_j \dot{\partial}_k B^{sm})\dot{\partial}_{h]} \dot{\partial}_s B^{ir} + \sigma_k(\dot{\partial}_j \dot{\partial}_h B^{sm})\dot{\partial}_{l]} \dot{\partial}_s B^{ir} \\ & + \sigma_h(\dot{\partial}_j \dot{\partial}_l B^{sm})\dot{\partial}_{k]} \dot{\partial}_s B^{ir}\}]. \end{aligned} \tag{2.20}$$

If we suppose σ is constant, then the equation (2.20) reduces to

$$\bar{A}_{[kh(\bar{l})]} = 0. \tag{2.21}$$

Accordingly we can state:

Theorem 3. *Under the decomposition (2.1), If the mapping is homothetic, then the conformal decomposition tensor $\bar{A}_{kh}$ satisfies the Bianchi identitiy (2.21).*

Theorem 4. *The conformal curvature tensor of a conformal Finsler space is decomposable, if and only if there exists a skew symmetric conformal decomposition tensor $\bar{A}_{kh}$ satisfying.*

$$\bar{A}_{lm}\bar{H}^i_{jkh} + \bar{A}_{lh}\bar{H}^i_{jmk} + \bar{A}_{lk}\bar{H}^i_{jhm} = 0. \tag{2.22}$$

Proof. If there exits skew symmetric conformal decomposition tensor $\bar{A}_{kh}$ satisfying (2.22), then the conformal curvature tensor $\bar{H}^i_{jkh}$ satisfies

$$\bar{H}^i_{jkh} = \bar{X}^i_j \bar{A}_{kh}, \tag{2.23}$$

where $\bar{X}^i_j$ is non zero tensor and hence the curvature tensor is decomposable.

Conversely, if the curvature tensor of a Finsler space is decomposable, it has the form (2.23). Since $\bar{H}^i_{jkh}$ is skew symmetric in k and h, the tensor $\bar{A}_{kh}$ is skew symmetric in k and h. Since the space is non flat, so $\bar{y}_i \bar{H}^i_{jkh} \neq 0$. Hence

$$\bar{y}_i \bar{H}^i_{jkh} = \bar{\lambda}_j \bar{A}_{kh}. \tag{2.24}$$

Transvecting the Bianchi Identity

$$\bar{H}^i_{jkh} + \bar{H}^i_{hjk} + \bar{H}^i_{khj} = 0, \tag{2.25}$$

by $\bar{y}_i$ and using (2.4), we have

$$\bar{\lambda}_j \bar{A}_{kh} + \bar{\lambda}_h \bar{A}_{jk} + \bar{\lambda}_k \bar{A}_{hj} = 0, \tag{2.26}$$

using equation (1.13) in the above equation and transvecting (2.26) by such vector $\bar{V}^j$, we get

$$\bar{A}_{kh} = -\bar{\lambda}_k \bar{\mu}_h - \bar{\lambda}_h \bar{\mu}_k = 0, \tag{2.27}$$

where $\bar{\mu}_h = \bar{A}_{jh} \bar{V}^j$, obviously $\bar{\mu}_k$ satisfies.

$$\bar{\mu}_j \bar{A}_{kh} + \bar{\mu}_h \bar{A}_{jk} + \bar{\mu}_k \bar{A}_{hj} = 0. \tag{2.28}$$

Transvecting (2.28) and (2.26) by $\bar{\lambda}_l$ and $\bar{\mu}_l$ respectively, and then subtracting, we have

$$\bar{A}_{ij} \bar{A}_{kh} + \bar{A}_{lh} \bar{A}_{jk} + \bar{A}_{lk} \bar{A}_{hj} = 0. \tag{2.29}$$

Transvecting (2.23) by $\bar{A}_{lm}$, taking skew-symmetric part with respect to the indices m, k and h and using (2.29), we get (2.22). □

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