A note on CR-integral curves of Kaehler manifolds

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Abstract. In this note we define and study CR-integral curves on manifolds. Moreover some properties and results of these curves are studied on Kaehler manifolds.

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1 Introduction

CR-submanifolds of a Kaehler manifold were defined and studied by A. Bejancu (see [1],[2]), and since then they were investigated by several authors e.g. ([3],[4]). The main purpose of this paper is to define what we call CR-integral curves of a Kaehlerian manifold and to study their properties. In Section 2, we first state some preliminary definitions of the integral curves on a Kaehlerian manifold for our subsequent use. In Section 3, we define and study some basic properties and results of these curves.

2 Preliminaries

Let $\overline{M}$ be an almost complex manifold with almost complex structure J, then $\overline{M}$ is said to be an Hermitian manifold if there exists a positive definite Riemannian metric g, such that:

$$g(JX, JY) = g(X, Y)$$

for any vector fields X and Y on $\overline{M}$.

A Hermitian manifold $\overline{M}$ is said to be a Kaehlerian manifold if for any tangent vector X on $\overline{M}$, we have $(\overline{\nabla}_X J) = 0$.

If $\overline{M}$ be a Kaehler manifold of complex dimension n (that is of real dimension $2n$), and M be submanifold of $\overline{M}$ with real dimension m. Then the submanifold M is called CR-submanifold of $\overline{M}$, if there exists two orthogonal differentiable distributions D and $D^{\perp}$, complimentary to each other such that the horizontal distribution D is invariant, while that of vertical distribution $D^{\perp}$, is anti-invariant by almost complex structure J.

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In this note we introduce the concept of CR-integral curves by generalizing the notion of integral curves.

Definition 2.1. *A curve $\alpha(t)$ on a differentiable manifold M is said to be an integral curve of a vector field X if the vector $X_{\alpha(t)}$ is tangent to $\alpha(t)$ for every t.*

Remark 2.1. For any given point p of manifold M, one can find a unique integral curve $\alpha(t)$ of a vector field X, defined for $|t| < \epsilon$, for some $\epsilon > 0$, such that $p = \alpha(0)$.

3 Definitions and some results

In this section we give some basic definitions and results.

Defintion 3.1. *A curve $\alpha(t)$ on a submanifold M is said to be $D^{\perp}$-integral curve of a vector field $X \in D$, if vector $X_{\alpha(t)}$ is tangent to $\alpha(t)$.*

Defintion 3.2. *A curve $\alpha(t)$ on a submanifold M is said to be $D^{\perp}$-integral curve of a vector field $X \in D^{\perp}$, if vector $X_{\alpha(t)}$ is tangent to $\alpha(t)$.*

Remark 3.1. It is manifest that D-integral curves (resp. $D^{\perp}$-integral curves) are general cases of integral curves.

Lemma 3.1. *Every geodesic curve on M is at least either one of the following.*
(a) D-integral curve
(b) $D^{\perp}$-integral curve
(c) Integral curve.

Definition 3.3. *A curve on M is said to be a real curve if and only if, $dim\, D = 0$.*

Theorem 3.1. *Let M be a 2-dimensional submanifold of a 2-dimensional Kaehler manifold $\overline{M}$, then every curve in M is D-integral curve.*

Proof. Since $dim\, D = 0$ is trivial. Taking $dim\, D = 2$, and $dim\, D^{\perp} = 0$, the result follows immediately.

References

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