New commutation formulas for δ-differentiation in a generalized Finsler space

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Abstract. In this work we consider a generalized Finsler space GF_N as an N-dimensional differentiable manifold on which a non-symmetric basic tensor $g_{ij}(x, \dot{x})$ is defined by virtue (1.3). Some basic properties of GF_N are given (§1). In the work [4] we defined two kinds of covariant derivative of a tensor in the Rund's sense and obtained 10 Ricci type idetities in a GF_N. Non-symmetric basic tensor $g_{ij}(x, \dot{x})$ gives possibility to define two more kinds of covariant derivative in the Rund's sense too. In this work we consider new 10 Ricci type identities, and we get 3 curvature tensors and 15 curvature pseudotensors, defined in [4] and also a new curvature tensor $\underset{4}{\widetilde{K}}$.

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1 Introduction

The Finsler space and it's generalizations were investigated by many authors, for example H. D., Pande and K. K., Gupta [7, 8], M. K. Gupta [3], C., Nitescu [6], A. C., Shamihoke [10]-[13], U. P., Singh [14, 15], M., Verma [16] and many others.

The generalized Finsler space (GF_N) is a differentiable manifold with non-symmetric basic tensor $g_{ij}(x^1, \dots, x^N, \dot{x}^1, \dots, \dot{x}^N) \equiv g_{ij}(x, \dot{x})$, where

$$g_{ij}(x, \dot{x}) \neq g_{ji}(x, \dot{x}), \quad (g = det(g_{ij}) \neq 0, \ \dot{x} = dx/dt). \tag{1.1}$$

Based on (1.1), one can defined the symmetric respectively anti-symmetric part of g_{ij}

$$g_{\underline{ij}} = \frac{1}{2}(g_{ij} + g_{ji}), \qquad g_{\underset{\vee}{ij}} = \frac{1}{2}(g_{ij} - g_{ji}), \tag{1.2}$$

where, following [10]-[13], is

$$a)\ g_{\underline{ij}}(x, \dot{x}) = \frac{1}{2}\frac{\partial^2 F^2(x, \dot{x})}{\partial \dot{x}^i \partial \dot{x}^j}, \qquad b)\ \frac{\partial g_{\underset{\vee}{ij}}}{\partial \dot{x}^k} = 0, \tag{1.3}$$

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where $F(x, \dot{x})$ is a metric function in GF_N, having the properties known from the theory of usual Finsler space (F_N) (see e.g. [1, 9]).

The lowering and the raising of indices one defines by the tensors $g_{\underline{ij}}$ and h^{ij} respectively, where h^{ij} is defined as follows

$$g_{\underline{ij}} h^{jk} = \delta_i^k, \quad (\underline{g} = det(g_{\underline{ij}}) \neq 0). \tag{1.4}$$

We can define the *generalized Cristoffel symbols* of the 1^{st} and the 2^{nd} kind:

$$\gamma_{i.jk} = \frac{1}{2}(g_{ji,k} - g_{jk,i} + g_{ik,j}) \neq \gamma_{i.kj}, \tag{1.5}$$

$$\gamma^i_{jk} = h^{ip}\gamma_{p.jk} = \frac{1}{2}h^{ip}(g_{jp,k} - g_{jk,p} + g_{pk,j}) \neq \gamma^i_{kj}, \tag{1.6}$$

where, e.g., $g_{ji,k} = \partial g_{ji}/\partial x^k$.

Then we have

$$\gamma^p_{jk} g_{\underline{ip}} = \gamma_{s.jk} h^{ps} g_{\underline{ip}} = \gamma_{s.jk}\delta^s_i = \gamma_{i.jk}. \tag{1.7}$$

Theorem 1.1. *In GF_N the following relations are valid*

$$\gamma_{i.jk} + \gamma_{j.ik} = g_{\underline{ij},k}, \quad \gamma_{i.jk} + \gamma_{k.ji} = g_{\underline{ik},j}, \tag{1.8}$$

$$\gamma_{i.jk}\dot{x}^i\dot{x}^j = \frac{1}{2}g_{\underline{ij},k}\dot{x}^i\dot{x}^j = \frac{1}{2}g_{ij,k}\dot{x}^i\dot{x}^j = \frac{1}{4}F^2_{\dot{x}^i\dot{x}^j x^k}\dot{x}^i\dot{x}^j, \tag{1.9}$$

$$\gamma_{i.jk}\dot{x}^i\dot{x}^k = \frac{1}{2}g_{\underline{ik},j}\dot{x}^i\dot{x}^k = \frac{1}{2}g_{ik,j}\dot{x}^i\dot{x}^k = \frac{1}{4}F^2_{\dot{x}^i x^j \dot{x}^k}\dot{x}^i\dot{x}^k, \tag{1.10}$$

$$\gamma_{i.jk}\dot{x}^j\dot{x}^k = (g_{\underline{ij},k} - \frac{1}{2}g_{\underline{jk},i})\dot{x}^j\dot{x}^k, \tag{1.11}$$

where, for example, $F^2_{\dot{x}^i\dot{x}^j x^k} = \frac{\partial^3 F^2}{\partial\dot{x}^i\partial\dot{x}^j\partial x^k}$.

Proof. The equations (1.8) follow from (1.5, 1.2) and the equations (1.9-1.11) from (1.5, 1.2, 1.3). □

Introducing a tensor C_{ijk} like as at F_N, we have

$$C_{ijk}(x, \dot{x}) \overset{def}{=} \frac{1}{2}g_{ij,\dot{x}^k} \underset{(1.3b)}{=} \frac{1}{2}g_{\underline{ij},\dot{x}^k} = \frac{1}{4}F^2_{\dot{x}^i\dot{x}^j\dot{x}^k}, \tag{1.12}$$

where " $\underset{(1.3b)}{=}$ " signifies "equal based on (1.3b)". We see that C_{ijk} is symmetric in relation to each pair of indices. Also, we have

$$C^i_{jk} \overset{def}{=} h^{ip}C_{pjk} \underset{(1.12)}{=} h^{ip}C_{jpk} = h^{ip}C_{jkp}. \tag{1.13}$$

With help of coefficients

$$P^i_{jk} = \gamma^i_{jk} - C^i_{jp}\gamma^p_{sk}\dot{x}^s \neq P^i_{kj} \tag{1.14}$$

one obtains coefficients of non-symmetric affine connections in the Rund's sense [9, 12]:

$$P^{*i}_{jk} = \gamma^i_{jk} - h^{il}(C_{jlp}P^p_{ks} + C_{klp}P^p_{js} - C_{jkp}P^p_{ls})\dot{x}^s \underset{(1.6)}{\neq} P^{*i}_{kj}, \tag{1.15}$$

$$P^*_{i.jk} = P^{*r}_{jk}g_{\underline{ir}} = \gamma_{i.jk} - (C_{ijp}P^p_{ks} + C_{ikp}P^p_{js} - C_{jkp}P^p_{is})\dot{x}^s \neq P^*_{i.kj}. \tag{1.16}$$

In the work [4], we defined two kinds of covariant derivative of a tensor in the GF_N. For example, for a tensor $a^{r_1\dots r_u}_{t_1\dots t_v}(x,\xi)$ we have *covariant derivative of the* 1^{st} *and* 2^{nd} *kind*

(1.17)

$$a^{r_1\dots r_u}_{t_1\dots t_v\underset{\frac{1}{2}}{|}m} = a^{\cdots}_{\dots,m} + a^{\cdots}_{\dots,\dot{p}}\xi^p_{,m} + \sum_{\alpha=1}^{n} P^{*r_\alpha}_{\underset{mp}{pm}} a^{r_1\dots,r_{\alpha-1}pr_{\alpha+1}\dots r_u}_{t_1\dots t_v} - \sum_{\beta=1}^{v} P^{*p}_{\underset{mt_\beta}{t_\beta m}} a^{r_1\dots r_u}_{r_1\dots r_{\beta-1}pr_{\beta+1}\dots r_v}.$$

In a space GF_N it is possible to define two new kinds of covariant derivative: *covariant derivative of the* 3^{rd} *kind* and *covariant derivative of the* 4^{th} *kind.* Generally is:

(1.18)

$$a^{r_1\dots r_u}_{t_1\dots t_v\underset{\frac{3}{4}}{|}m} = a^{\cdots}_{\dots,m} + a^{\cdots}_{\dots,\dot{p}}\xi^p_{,m} + \sum_{\alpha=1}^{n} P^{*r_\alpha}_{\underset{mp}{pm}} a^{r_1\dots,r_{\alpha-1}pr_{\alpha+1}\dots r_u}_{t_1\dots t_v} - \sum_{\beta=1}^{v} P^{*p}_{\underset{t_\beta m}{mt_\beta}} a^{r_1\dots r_u}_{r_1\dots r_{\beta-1}pr_{\beta+1}\dots r_v},$$

where $\xi(x)$ is an arbitrary tangent vector in the tangent space $T_N(x)$, and $a^{\cdots}_{\dots,\dot{p}} = \frac{\partial a^{\cdots}_{\cdots}}{\partial \dot{x}^p}$.

By the procedure that is similar to that one in a Finsler space, it can be proved that covariant derivative (1.17, 1.18) of a tensor also is a tensor.

Theorem 1.2. *For the tensor $g_{\underline{ij}}(x,\dot{x})$ based on both kinds of derivative (1.18) is valid*

$$g_{\underline{ij}\underset{\theta}{|}m}(x,\xi) = 2C_{ijp}(\xi^p_{,m} + P^{*p}_{ms}\dot{x}^s), \quad \theta = 3,4, \tag{1.19}$$

$$g_{\underline{ij}\underset{\theta}{|}m}(x,\dot{x}) = 2C_{ijp}\dot{x}^p_{\underset{4}{|}m} = 2C_{ijp}\dot{x}^p_{\underset{2}{|}m}, \quad \theta = 3,4. \tag{1.20}$$

Proof. Starting from (1.18), we get

$$\begin{aligned} g_{\underline{ij}\underset{3}{|}m}(x,\xi) &= g_{\underline{ij},m} + g_{\underline{ij},\dot{x}^p}\xi^p_{,m} - P^{*p}_{mi}g_{\underline{pj}} - P^{*p}_{mj}g_{\underline{ip}} \\ &\underset{(1.12)}{=} g_{\underline{ij},m} + 2C_{ijp}\xi^p_{,m} - (P^*_{j.mi} + P^*_{i.mj}). \end{aligned} \tag{1.21}$$

Based on (1.16) and using (1.8) one obtains

$$P^*_{j.mi} + P^*_{i.mj} = g_{\underline{ij},m} - 2C_{ijp}P^p_{ms}\dot{x}^s. \tag{1.22}$$

By substituting from (1.22) into (1.21) we get

$$g_{\underline{ij}\underset{3}{|}m}(x,\xi) = 2C_{ijp}(\xi^p_{,m} + P^p_{ms}\dot{x}^s).$$

The same result one obtains for $g_{ij\underset{4}{|}m}$. As

$$P^p_{ms}\dot{x}^s = P^{*p}_{ms}\dot{x}^s. \tag{1.23}$$

we obtain (1.19). For $\xi = \dot{x}$ one obtains

$$g_{\underline{ij}\underset{3}{|}m}(x,\dot{x}) = 2C_{ijp}(\dot{x}^p_{,m} + P^{*p}_{ms}\dot{x}^s),$$

and the same one obtains for $\theta = 4$. Because $\dot{x}^p$ do not depends from ξ, the previous equation becomes (1.20). □

Theorem 1.3. *For the Kronecker symbol is in force*

$$\delta^i_{j\underset{\theta}{|}m} = 0, \quad \theta = 1,\dots,4.$$

The proof is obtained by using the definition of δ-symbols and covariant derivatives (1.17, 1.18).

Theorem 1.4. *For h^{ij} is force*

$$h^{ij}_{\underset{\theta}{|}m} = -h^{ip}h^{jq}g_{\underline{pq}\underset{\theta}{|}m}, \quad \theta = 1,\dots,4, \tag{1.24}$$

Proof. From $h^{ip}g_{\underline{pq}} = \delta^i_q$ one obtains

$$h^{ip}_{\underset{\theta}{|}m}g_{\underline{pq}} + h^{ip}g_{\underline{pq}\underset{\theta}{|}m} = 0.$$

Composing with h^{jq}, it results that

$$h^{ip}_{\underset{\theta}{|}m}\delta^j_p + h^{ip}h^{jq}g_{\underline{pq}\underset{\theta}{|}m} = 0, \quad \theta = 3,4 \Rightarrow (1.24). \quad \square$$

From (1.17, 1.18) is evidently that

$$a^i_{\underset{3}{|}m} = a^i_{\underset{1}{|}m}, \quad a_{k\underset{3}{|}m} = a_{k\underset{2}{|}m}, \quad a^i_{\underset{4}{|}m} = a^i_{\underset{2}{|}m}, \quad a_{k\underset{4}{|}m} = a_{k\underset{1}{|}m}, \tag{1.25}$$

2 New Ricci type identities in GF_N

If we examine the differences

$$a^{r_1\dots r_u}_{t_1\dots t_v\underset{\pi}{|}m\underset{\rho}{|}n} - a^{r_1\dots r_u}_{t_1\dots t_v\underset{\sigma}{|}m\underset{\tau}{|}n} \quad (\pi,\rho,\sigma,\tau = 3,4) \tag{2.1}$$

we get ten new Ricci type identities. In these identities the same magnitude $\underset{1}{\widetilde{K}}$, $\underset{2}{\widetilde{K}}$, $\underset{3}{\widetilde{K}}$; $\underset{1}{\widetilde{A}},\dots,\underset{15}{\widetilde{A}}$ appear, but in different arrangement than they appear in the work [4]. Only in the last case appears a new curvature tensor $\underset{4}{\widetilde{K}}$.

Let us consider a tensor a^{hij}_{kl} and its covariant derivative of the 3^{rd} and the 4^{th} kind, also their combinations, by virtue of (2.1).

1. Based on (1.18) we obtain

$$a^{hij}_{kl\underset{3}{|}mn} = (a^{hij}_{kl\underset{3}{|}m})_{\underset{3}{|}n} = (a^{hij}_{kl\underset{3}{|}m})_{,n} + a^{hij}_{kl\underset{3}{|}m,\dot d}\xi^d_{,n} + P^{*h}_{sn}a^{sij}_{kl\underset{3}{|}m} + P^{*i}_{sn}a^{hsj}_{kl\underset{3}{|}m}$$
$$+ P^{*j}_{sn}a^{his}_{kl\underset{3}{|}m} - P^{*s}_{nk}a^{hij}_{sl\underset{3}{|}m} - P^{*s}_{nl}a^{hij}_{ks\underset{3}{|}m} - P^{*s}_{nm}a^{hij}_{kl\underset{3}{|}s} \quad \text{i.e.}$$

$$a^{hij}_{kl\underset{3}{|}mn} = a^{hij}_{kl,mn} + P^{*h}_{pm,n}a^{pij}_{kl} + P^{*h}_{pm}a^{pij}_{kl,n} + P^{*i}_{pm,n}a^{hpj}_{kl} + P^{*i}_{pm}a^{hpj}_{kl,n} + P^{*j}_{pm,n}a^{hip}_{kl}$$
$$+ P^{*j}_{pm}a^{hip}_{kl,n} - P^{*p}_{mk,n}a^{hij}_{pl} - P^{*p}_{mk}a^{hij}_{pl,n} - P^{*p}_{ml,n}a^{hij}_{kp} - P^{*p}_{ml}a^{hij}_{kp,n} + a^{hij}_{kl,\dot s n}\xi^s_{,m}$$
$$+ a^{hij}_{kl,\dot s\dot t}\xi^s_{,m}\xi^t_{,n} + a^{hij}_{kl}\xi^s_{,nm} + a^{hij}_{kl,m\dot d}\xi^d_{,n} + P^{*h}_{pm,\dot d}a^{pij}_{kl}\xi^d_{,n} + P^{*h}_{pm}a^{pij}_{kl,\dot d}\xi^d_{,n} + P^{*i}_{pm,\dot d}a^{hpj}_{kl}\xi^d_{,n}$$
$$+ P^{*i}_{pm}a^{hpj}_{kl,\dot d}\xi^d_{,n} + P^{*j}_{pm,\dot d}a^{hip}_{kl}\xi^d_{,n} + P^{*j}_{pm}a^{hip}_{kl,\dot d}\xi^d_{,n} - P^{*p}_{km,\dot d}a^{hij}_{pl}\xi^d_{,n} - P^{*p}_{km}a^{hij}_{pl,\dot d}\xi^d_{,n}$$
$$- P^{*p}_{lm,\dot d}a^{hij}_{kp}\xi^d_{,n} - P^{*p}_{lm}a^{hij}_{kp,\dot d} + a^{hij}_{kl,\dot s\dot d}\xi^s_{,m}\xi^d_{,n} + a^{hij}_{kl,\dot s\dot t}\xi^s_{,m}\xi^t_{,\dot d}\xi^d_{,n} + a^{hij}_{kl}\xi^s_{,m\dot d} + P^{*h}_{sn}a^{sij}_{kl,m}$$
$$+ P^{*h}_{sn}a^{sij}_{kl,\dot d}\xi^d_{,m} + P^{*h}_{sn}P^{*s}_{pm}a^{pij}_{kl} + P^{*h}_{sn}P^{*i}_{pm}a^{spj}_{kl} + P^{*h}_{sn}P^{*j}_{pm}a^{sip}_{kl} - P^{*h}_{sn}P^{*p}_{mk}a^{sij}_{pl}$$
$$- P^{*h}_{sn}P^{*p}_{ml}a^{sij}_{kp} + P^{*i}_{sn}a^{hsj}_{kl,m} + P^{*i}_{sn}a^{hsj}_{kl,\dot d}\xi^d_{,m} + P^{*i}_{sn}P^{*h}_{pm}a^{psj}_{kl} + P^{*i}_{sn}P^{*s}_{pm}a^{hpj}_{kl} + P^{*i}_{sn}P^{*j}_{pm}a^{hsp}_{kl}$$
$$- P^{*i}_{sn}P^{*p}_{mk}a^{hsj}_{pl} - P^{*i}_{sn}P^{*p}_{ml}a^{hsj}_{kp} + P^{*j}_{sn}a^{his}_{kl,m} + P^{*j}_{sn}a^{his}_{kl,\dot d}\xi^d_{,m} + P^{*j}_{sn}P^{*h}_{pm}a^{pis}_{kl} + P^{*j}_{sn}P^{*i}_{pm}a^{hps}_{kl}$$
$$+ P^{*j}_{sn}P^{*s}_{pm}a^{hip}_{kl} - P^{*j}_{sn}P^{*p}_{mk}a^{his}_{pl} - P^{*j}_{sn}P^{*p}_{ml}a^{his}_{kp} - P^{*s}_{nk}a^{hij}_{sl,m} - P^{*s}_{nk}a^{hij}_{sl,\dot d}\xi^d_{,m} - P^{*s}_{nk}P^{*h}_{pm}a^{pij}_{sl}$$
$$- P^{*s}_{nk}P^{*i}_{pm}a^{hpj}_{sl} - P^{*s}_{nk}P^{*i}_{pm}a^{hip}_{sl} + P^{*s}_{nk}P^{*p}_{ms}a^{hij}_{pl} + P^{*s}_{nk}P^{*p}_{ml}a^{hij}_{sp} - P^{*s}_{nl}a^{hij}_{ks,m} - P^{*s}_{nl}a^{hij}_{ks,\dot d}\xi^d_{,m}$$
$$- P^{*s}_{nl}P^{*h}_{pm}a^{pij}_{ks} - P^{*s}_{nl}P^{*i}_{pm}a^{hpj}_{ks} - P^{*s}_{nl}P^{*i}_{pm}a^{hip}_{ks} + P^{*s}_{nl}P^{*p}_{mk}a^{hij}_{ps} + P^{*s}_{nl}P^{*p}_{ms}a^{hij}_{kp} - P^{*s}_{nm}a^{hij}_{kl\underset{3}{|}s},$$

and from here

$$a^{hij}_{kl\underset{3}{|}mn} - a^{hij}_{kl\underset{3}{|}nm} = \underset{1}{\widetilde{K}}{}^h{}_{pmn}a^{hij}_{kl} + \underset{1}{\widetilde{K}}{}^i{}_{pmn}a^{hpj}_{kl} + \underset{1}{\widetilde{K}}{}^j{}_{pmn}a^{hip}_{kl}$$
$$- \underset{2}{\widetilde{K}}{}^p{}_{kmn}a^{hij}_{pl} - \underset{2}{\widetilde{K}}{}^p{}_{lmn}a^{hij}_{kp} + T^{*p}_{mn}a^{hij}_{kl\underset{3}{|}p},$$

where

(2.2) $$\underset{1}{\widetilde{K}}{}^i{}_{jmn}(x,\xi) = P^{*i}_{jm,n} - P^{*i}_{jn,m} + P^{*p}_{jm}P^{*i}_{pn} - P^{*p}_{jn}P^{*i}_{pm} + P^{*p}_{jm,\dot s}\xi^s_{,n} - P^{*p}_{jn,\dot s}\xi^s_{,m},$$

(2.3) $$\underset{2}{\widetilde{K}}{}^i{}_{jmn}(x,\xi) = P^{*i}_{mj,n} - P^{*i}_{nj,m} + P^{*p}_{mj}P^{*i}_{np} - P^{*p}_{nj}P^{*i}_{mp} + P^{*p}_{mj,\dot s}\xi^s_{,n} - P^{*p}_{nj,\dot s}\xi^s_{,m},$$

are Rund's type *curvature tensors of the* 1^{st} *and the* 2^{nd} *kind* respectively. The torsion tensor and the double simmetric part of the connection are respectively

(2.4) $$a)\ T^{*i}_{jk} = P^{*i}_{[jk]} = P^{*i}_{jk} - P^{*i}_{kj}, \qquad b)\ P^{*i}_{(jk)} = P^{*i}_{jk} + P^{*i}_{kj}$$

2. Using (1.18) we get

$$a^{hij}_{kl\underset{4}{|}mn} = (a^{hij}_{kl\underset{4}{|}m})_{\underset{4}{|}n} = (a^{hij}_{kl\underset{4}{|}m})_{,n} + a^{hij}_{kl\underset{4}{|}m,\dot d}\xi^d_{,n} + P^{*h}_{ns}a^{sij}_{kl\underset{4}{|}m} + P^{*i}_{ns}a^{hsj}_{kl\underset{4}{|}m}$$
$$+ P^{*j}_{ns}a^{his}_{kl\underset{4}{|}m} - P^{*s}_{kn}a^{hij}_{sl\underset{4}{|}m} - P^{*s}_{ln}a^{hij}_{ks\underset{4}{|}m} - P^{*s}_{mn}a^{hij}_{kl\underset{4}{|}s}$$

$$
\begin{aligned}
a^{hij}_{kl\underset{4}{|}mn} &= a^{hij}_{kl,mn} + P^{*h}_{mp,n}a^{pij}_{kl} + P^{*h}_{mp}a^{pij}_{kl,n} + P^{*i}_{mp,n}a^{hpj}_{kl} + P^{*i}_{mp}a^{hpj}_{kl,n} + P^{*j}_{mp,n}a^{hip}_{kl}\\
&+ P^{*j}_{mp}a^{hip}_{kl,n} - P^{*p}_{km,n}a^{hij}_{pl} - P^{*p}_{km}a^{hij}_{pl,n} - P^{*p}_{lm,n}a^{hij}_{kp} - P^{*p}_{lm}a^{hij}_{kp,n} + a^{hij}_{kl,\dot{s}n}\xi^{s}_{,m}\\
&+ a^{hij}_{kl,\dot{s}\dot{t}}\xi^{s}_{,m}\xi^{t}_{,n} + a^{hij}_{kl}\xi^{s}_{,mn} + a^{hij}_{kl,m\dot{d}}\xi^{d}_{,n} + P^{*h}_{mp,\dot{d}}a^{pij}_{kl}\xi^{d}_{,n} + P^{*h}_{mp}a^{pij}_{kl,\dot{d}}\xi^{d}_{,n} + P^{*i}_{mp,\dot{d}}a^{hpj}_{kl}\xi^{d}_{,n}\\
&+ P^{*i}_{mp}a^{hpj}_{kl,\dot{d}}\xi^{d}_{,n} + P^{*j}_{mp,\dot{d}}a^{hip}_{kl}\xi^{d}_{,n} + P^{*j}_{mp}a^{hip}_{kl,\dot{d}}\xi^{d}_{,n} - P^{*p}_{km,\dot{d}}a^{hij}_{pl}\xi^{d}_{,n} - P^{*p}_{km}a^{hij}_{pl,\dot{d}}\xi^{d}_{,n}\\
&- P^{*p}_{lm,\dot{d}}a^{hij}_{kp}\xi^{d}_{,n} - P^{*p}_{lm}a^{hij}_{kp,\dot{d}} + a^{hij}_{kl,\dot{s}\dot{d}}\xi^{s}_{,m}\xi^{d}_{,n} + a^{hij}_{kl,\dot{s}\dot{t}}\xi^{s}_{,m}\xi^{t}_{,\dot{d}}\xi^{d}_{,n} + a^{hij}_{kl}\xi^{s}_{,m\dot{d}} + P^{*h}_{ns}a^{sij}_{kl,m}\\
&+ P^{*h}_{ns}a^{sij}_{kl,\dot{d}}\xi^{d}_{,m} + P^{*h}_{ns}P^{*s}_{mp}a^{pij}_{kl} + P^{*h}_{ns}P^{*i}_{mp}a^{spj}_{kl} + P^{*h}_{ns}P^{*j}_{mp}a^{sip}_{kl} - P^{*h}_{ns}P^{*p}_{km}a^{sij}_{pl}\\
&- P^{*h}_{ns}P^{*p}_{lm}a^{sij}_{kp} + P^{*i}_{ns}a^{hsj}_{kl,m} + P^{*i}_{ns}a^{hsj}_{kl,\dot{d}}\xi^{d}_{,m} + P^{*i}_{ns}P^{*h}_{mp}a^{psj}_{kl} + P^{*i}_{ns}P^{*s}_{mp}a^{hpj}_{kl} + P^{*i}_{ns}P^{*j}_{mp}a^{hsp}_{kl}\\
&- P^{*i}_{ns}P^{*p}_{km}a^{hsj}_{pl} - P^{*i}_{ns}P^{*p}_{lm}a^{hsj}_{kp} + P^{*j}_{ns}a^{his}_{kl,m} + P^{*j}_{ns}a^{his}_{kl,\dot{d}}\xi^{d}_{,m} + P^{*j}_{ns}P^{*h}_{mp}a^{pis}_{kl} + P^{*j}_{ns}P^{*i}_{mp}a^{hps}_{kl}\\
&+ P^{*j}_{ns}P^{*s}_{mp}a^{hip}_{kl} - P^{*j}_{ns}P^{*p}_{km}a^{his}_{pl} - P^{*j}_{ns}P^{*p}_{lm}a^{his}_{kp} - P^{*s}_{kn}a^{hij}_{sl,m} - P^{*s}_{kn}a^{hij}_{sl,\dot{d}}\xi^{d}_{,m} - P^{*s}_{kn}P^{*h}_{mp}a^{pij}_{sl}\\
&- P^{*s}_{kn}P^{*i}_{mp}a^{hpj}_{sl} - P^{*s}_{kn}P^{*i}_{mp}a^{hip}_{sl} + P^{*s}_{kn}P^{*p}_{ms}a^{hij}_{pl} + P^{*s}_{kn}P^{*p}_{lm}a^{hij}_{sp} - P^{*s}_{ln}a^{hij}_{ks,m} - P^{*s}_{nl}a^{hij}_{ks,\dot{d}}\xi^{d}_{,m}\\
&- P^{*s}_{ln}P^{*h}_{mp}a^{pij}_{ks} - P^{*s}_{ln}P^{*i}_{mp}a^{hpj}_{ks} - P^{*s}_{ln}P^{*i}_{mp}a^{hip}_{ks} + P^{*s}_{ln}P^{*p}_{km}a^{hij}_{ps} + P^{*s}_{ln}P^{*p}_{ms}a^{hij}_{kp} - P^{*s}_{mn}a^{hij}_{kl\underset{4}{|}s},
\end{aligned}
$$

from where one obtains

$$
\begin{aligned}
a^{hij}_{kl\underset{4}{|}mn} - a^{hij}_{kl\underset{4}{|}nm} &= \underset{2}{\widetilde{K}}{}^{h}{}_{pmn}a^{hij}_{kl} + \underset{2}{\widetilde{K}}{}^{i}{}_{pmn}a^{hpj}_{kl} + \underset{2}{\widetilde{K}}{}^{j}{}_{pmn}a^{hip}_{kl}\\
&- \underset{1}{\widetilde{K}}{}^{p}{}_{kmn}a^{hij}_{pl} - \underset{1}{\widetilde{K}}{}^{p}{}_{lmn}a^{hij}_{kp} - T^{*p}_{mn}a^{hij}_{kl\underset{4}{|}p},
\end{aligned}
$$

Generally, we can state by total induction method (as like in [4]):

Theorem 2.1. *In a generalized Finsler space GF_N the 1^{st} and 2^{nd} new Ricci type identity are*

$$
\begin{aligned}
a^{r_1\dots r_u}_{t_1\dots t_v\underset{\theta}{|}mn} - a^{r_1\dots r_u}_{t_1\dots t_v\underset{\theta}{|}nm} &= \sum_{\alpha=1}^{u}\underset{\substack{1\\2}}{\widetilde{K}}{}^{r_\alpha}{}_{pmn}\binom{p}{r_\alpha}a^{\dots}_{\dots} - \sum_{\beta=1}^{v}\underset{\substack{2\\1}}{\widetilde{K}}{}^{p}{}_{t_\beta mn}\binom{t_\beta}{p}a^{\dots}_{\dots}\\
&- (-1)^{\theta}T^{*p}_{mn}a^{r_1\dots r_u}_{t_1\dots t_v\underset{\theta}{|}p}\ ,\theta = 3,4,
\end{aligned}
\tag{2.5}
$$

where $\underset{\theta}{\widetilde{K}}$, $\theta = 1,2$ are given in (2.2, 2.3), and

$$
\binom{p}{r_\alpha}a^{\dots}_{\dots} = a^{r_1\dots r_{\alpha-1}pr_{\alpha+1}\dots r_u}_{t_1\dots t_v},\qquad \binom{t_\beta}{p}a^{\dots}_{\dots} = a^{r_1\dots r_u}_{t_1\dots t_{\beta-1}pr_{\beta+1}\dots t_v}.
\tag{2.6}
$$

3. Combining the 3^{rd} and 4^{th} kind of derivation on the base of (1.18) we have:

$$
\begin{aligned}
a^{hij}_{kl\underset{3}{|}m\underset{4}{|}n} = (a^{hij}_{kl\underset{3}{|}m})_{\underset{4}{|}n} &= (a^{hij}_{kl\underset{3}{|}m})_{,n} + a^{hij}_{kl\underset{3}{|}m,\dot{d}}\xi^{d}_{,n} + P^{*h}_{ns}a^{sij}_{kl\underset{3}{|}m} + P^{*i}_{ns}a^{hsj}_{kl\underset{3}{|}m}\\
&+ P^{*j}_{ns}a^{his}_{kl\underset{3}{|}m} - P^{*s}_{kn}a^{hij}_{sl\underset{3}{|}m} - P^{*s}_{ln}a^{hij}_{ks\underset{3}{|}m} - P^{*s}_{mn}a^{hij}_{kl\underset{3}{|}s}\quad \text{i.e.}
\end{aligned}
$$

$$\begin{aligned}
a^{hij}_{kl\underset{3}{|}m\underset{4}{|}n} &= a^{hij}_{kl,mn} + P^{*h}_{pm,n}a^{pij}_{kl} + P^{*h}_{pm}a^{pij}_{kl,n} + P^{*i}_{pm,n}a^{hpj}_{kl} + P^{*i}_{pm}a^{hpj}_{kl,n} + P^{*j}_{pm,n}a^{hip}_{kl} \\
&+ P^{*j}_{pm}a^{hip}_{kl,n} - P^{*p}_{mk,n}a^{hij}_{pl} - P^{*p}_{mk}a^{hij}_{pl,n} - P^{*p}_{ml,n}a^{hij}_{kp} - P^{*p}_{ml}a^{hij}_{kp,n} + a^{hij}_{kl,\dot s n}\xi^s_{,m} \\
&+ a^{hij}_{kl,\dot s\dot t}\xi^s_{,m}\xi^t_{,n} + a^{hij}_{kl}\xi^s_{,mn} + a^{hij}_{kl,m\dot d}\xi^d_{,n} + P^{*h}_{pm,\dot d}a^{pij}_{kl}\xi^d_{,n} + P^{*h}_{pm}a^{pij}_{kl,\dot d}\xi^d_{,n} + P^{*i}_{pm,\dot d}a^{hpj}_{kl}\xi^d_{,n} \\
&+ P^{*i}_{pm}a^{hpj}_{kl,\dot d}\xi^d_{,n} + P^{*j}_{pm,\dot d}a^{hip}_{kl}\xi^d_{,n} + P^{*j}_{pm}a^{hip}_{kl,\dot d}\xi^d_{,n} - P^{*p}_{km,\dot d}a^{hij}_{pl}\xi^d_{,n} - P^{*p}_{km}a^{hij}_{pl,\dot d}\xi^d_{,n} \\
&- P^{*p}_{lm,\dot d}a^{hij}_{kp}\xi^d_{,n} - P^{*p}_{lm}a^{hij}_{kp,\dot d} + a^{hij}_{kl,\dot s\dot d}\xi^s_{,m}\xi^d_{,n} + a^{hij}_{kl,\dot s\dot t}\xi^s_{,m}\xi^t_{,\dot d}\xi^d_{,n} + a^{hij}_{kl}\xi^s_{,m\dot d} + P^{*h}_{ns}a^{sij}_{kl,m} \\
&+ P^{*h}_{ns}a^{sij}_{kl,\dot d}\xi^d_{,m} + P^{*h}_{ns}P^{*s}_{pm}a^{pij}_{kl} + P^{*h}_{ns}P^{*i}_{pm}a^{spj}_{kl} + P^{*h}_{ns}P^{*j}_{pm}a^{sip}_{kl} - P^{*h}_{ns}P^{*p}_{mk}a^{sij}_{pl} \\
&- P^{*h}_{ns}P^{*p}_{ml}a^{sij}_{kp} + P^{*i}_{ns}a^{hsj}_{kl,m} + P^{*i}_{ns}a^{hsj}_{kl,\dot d}\xi^d_{,m} + P^{*i}_{ns}P^{*h}_{pm}a^{psj}_{kl} + P^{*i}_{ns}P^{*s}_{pm}a^{hpj}_{kl} + P^{*i}_{ns}P^{*j}_{pm}a^{hsp}_{kl} \\
&- P^{*i}_{ns}P^{*p}_{mk}a^{hsj}_{pl} - P^{*i}_{ns}P^{*p}_{ml}a^{hsj}_{kp} + P^{*j}_{ns}a^{his}_{kl,m} + P^{*j}_{ns}a^{his}_{kl,\dot d}\xi^d_{,m} + P^{*j}_{ns}P^{*h}_{pm}a^{pis}_{kl} + P^{*j}_{ns}P^{*i}_{pm}a^{hps}_{kl} \\
&+ P^{*j}_{ns}P^{*s}_{pm}a^{hip}_{kl} - P^{*j}_{ns}P^{*p}_{mk}a^{his}_{pl} - P^{*j}_{ns}P^{*p}_{ml}a^{his}_{kp} - P^{*s}_{kn}a^{hij}_{sl,m} - P^{*s}_{kn}a^{hij}_{sl,\dot d}\xi^d_{,m} - P^{*s}_{kn}P^{*h}_{pm}a^{pij}_{sl} \\
&- P^{*s}_{kn}P^{*i}_{pm}a^{hpj}_{sl} - P^{*s}_{kn}P^{*i}_{pm}a^{hip}_{sl} + P^{*s}_{kn}P^{*p}_{ms}a^{hij}_{pl} + P^{*s}_{kn}P^{*p}_{ml}a^{hij}_{sp} - P^{*s}_{ln}a^{hij}_{ks,m} - P^{*s}_{nl}a^{hij}_{ks,\dot d}\xi^d_{,m} \\
&- P^{*s}_{ln}P^{*h}_{pm}a^{pij}_{ks} - P^{*s}_{ln}P^{*i}_{pm}a^{hpj}_{ks} - P^{*s}_{ln}P^{*i}_{pm}a^{hip}_{ks} + P^{*s}_{ln}P^{*p}_{mk}a^{hij}_{ps} + P^{*s}_{ln}P^{*p}_{ms}a^{hij}_{kp} - P^{*s}_{mn}a^{hij}_{kl\underset{3}{|}s},
\end{aligned}$$

So, one gets

$$\begin{aligned}
a^{hij}_{kl\underset{3}{|}m\underset{4}{|}n} - a^{hij}_{kl\underset{3}{|}n\underset{4}{|}m} &= \underset{1}{\widetilde{A}}{}^h_{pmn}a^{pij}_{kl} + \underset{1}{\widetilde{A}}{}^i_{pmn}a^{hpj}_{kl} + \underset{1}{\widetilde{A}}{}^j_{pmn}a^{hip}_{kl} - \underset{4}{\widetilde{A}}{}^p_{knm}a^{hij}_{pl} \\
&- \underset{4}{\widetilde{A}}{}^p_{lnm}a^{hij}_{kp} + \overline{a}^{hij}_{<[kl]>} + \overline{a}^{hij}_{\leqslant[kl]\geqslant} - T^{*p}_{mn}a^{hij}_{kl\underset{3}{|}s}.
\end{aligned} \tag{2.7}$$

The denotations, introduced in (2.7) [4] are

$$\underset{1}{\widetilde{A}}{}^i_{jmn} = P^{*i}_{jm,n} - P^{*i}_{jn,m} + P^{*p}_{jm}P^{*i}_{np} - P^{*p}_{jn}P^{*i}_{mp} + P^{*p}_{jm,\dot s}\xi^s_{,n} - P^{*p}_{jn,\dot s}\xi^s_{,m}, \tag{2.8}$$

$$\underset{4}{\widetilde{A}}{}^i_{jmn} = P^{*i}_{mj,n} - P^{*i}_{nj,m} + P^{*p}_{jm}P^{*i}_{np} - P^{*p}_{jn}P^{*i}_{mp} + P^{*p}_{mj,\dot s}\xi^s_{,n} - P^{*p}_{nj,\dot s}\xi^s_{,m}, \tag{2.9}$$

$$\begin{aligned}
\overline{a}^{hij}_{kl\langle mn\rangle} &= T^{*h}_{pm}(a^{pij}_{kl,n} + a^{pij}_{kl,\dot s}\xi^s_{,n}) + T^{*i}_{pm}(a^{hpj}_{kl,n} + a^{hpj}_{kl,\dot s}\xi^s_{,n}) \\
&+ T^{*j}_{pm}(a^{hip}_{kl,n} + a^{hip}_{kl,\dot s}\xi^s_{,n}) - T^{*p}_{mk}(a^{hij}_{pl,n} + a^{hij}_{pl,\dot s}\xi^s_{,n}) \\
&- T^{*p}_{ml}(a^{hij}_{kp,n} + a^{hij}_{kp,\dot s}\xi^s_{,n}),
\end{aligned}$$

$$\begin{aligned}
\overline{a}^{hij}_{kl\leqslant mn\geqslant} &= a^{psj}_{kl}P^{*h}_{[\underset{\smile}{p}m}P^{*i}_{\underset{\smile}{n}s]} + a^{pis}_{kl}P^{*h}_{[\underset{\smile}{p}m}P^{*j}_{\underset{\smile}{n}s]} + a^{hps}_{kl}P^{*i}_{[\underset{\smile}{p}m}P^{*j}_{\underset{\smile}{n}s]} - a^{pij}_{sl}P^{*h}_{[\underset{\smile}{p}m}P^{*s}_{\underset{\smile}{k}n]} \\
&- a^{pij}_{ks}P^{*h}_{[\underset{\smile}{p}m}P^{*s}_{\underset{\smile}{l}n]} - a^{hpj}_{sl}P^{*i}_{[\underset{\smile}{p}m}P^{*s}_{\underset{\smile}{k}n]} - a^{hpj}_{ks}P^{*i}_{[\underset{\smile}{p}m}P^{*s}_{\underset{\smile}{l}n]} - a^{hip}_{sl}P^{*j}_{[\underset{\smile}{p}m}P^{*s}_{\underset{\smile}{k}n]} \\
&- a^{hip}_{ks}P^{*j}_{[\underset{\smile}{p}m}P^{*s}_{\underset{\smile}{l}n]} + a^{hij}_{ps}P^{*p}_{[\underset{\smile}{m}k}P^{*s}_{\underset{\smile}{l}n]},
\end{aligned}$$

$$P^{*h}_{[\underset{\smile}{p}m}P^{*i}_{\underset{\smile}{n}s]} = P^{*h}_{pm}P^{*i}_{ns} - P^{*h}_{mp}P^{*i}_{sn}.$$

4. Differentiating in inverse order then in preceding case, we obtain

$$\begin{aligned}
a^{hij}_{kl\underset{4}{|}m\underset{3}{|}n} = (a^{hij}_{kl\underset{4}{|}m})_{\underset{3}{|}n} &= (a^{hij}_{kl\underset{4}{|}m})_{,n} + a^{hij}_{kl\underset{4}{|}m,\dot d}\xi^d_{,n} + P^{*h}_{sn}a^{sij}_{kl\underset{4}{|}m} + P^{*i}_{sn}a^{hsj}_{kl\underset{4}{|}m} \\
&+ P^{*j}_{sn}a^{his}_{kl\underset{4}{|}m} - P^{*s}_{nk}a^{hij}_{sl\underset{4}{|}m} - P^{*s}_{nl}a^{hij}_{ks\underset{4}{|}m} - P^{*s}_{nm}a^{hij}_{kl\underset{4}{|}s} \quad \text{i.e.}
\end{aligned}$$

$$
\begin{aligned}
a^{hij}_{kl\underset{4}{|}m\underset{3}{|}n} &= a^{hij}_{kl,mn} + P^{*h}_{mp,n}a^{pij}_{kl} + P^{*h}_{mp}a^{pij}_{kl,n} + P^{*i}_{mp,n}a^{hpj}_{kl} + P^{*i}_{mp}a^{hpj}_{kl,n} + P^{*j}_{mp,n}a^{hip}_{kl}\\
&+ P^{*j}_{mp}a^{hip}_{kl,n} - P^{*p}_{km,n}a^{hij}_{pl} - P^{*p}_{km}a^{hij}_{pl,n} - P^{*p}_{lm,n}a^{hij}_{kp} - P^{*p}_{lm}a^{hij}_{kp,n} + a^{hij}_{kl,\dot{s}n}\xi^s_{,m}\\
&+ a^{hij}_{kl,\dot{s}\dot{t}}\xi^s_{,m}\xi^t_{,n} + a^{hij}_{kl}\xi^s_{,mn} + a^{hij}_{kl,m\dot{d}}\xi^d_{,n} + P^{*h}_{mp,\dot{d}}a^{pij}_{kl}\xi^d_{,n} + P^{*h}_{mp}a^{pij}_{kl,\dot{d}}\xi^d_{,n} + P^{*i}_{mp,\dot{d}}a^{hpj}_{kl}\xi^d_{,n}\\
&+ P^{*i}_{mp}a^{hpj}_{kl,\dot{d}}\xi^d_{,n} + P^{*j}_{mp,\dot{d}}a^{hip}_{kl}\xi^d_{,n} + P^{*j}_{mp}a^{hip}_{kl,\dot{d}}\xi^d_{,n} - P^{*p}_{mk,\dot{d}}a^{hij}_{pl}\xi^d_{,n} - P^{*p}_{mk}a^{hij}_{pl,\dot{d}}\xi^d_{,n}\\
&- P^{*p}_{ml,\dot{d}}a^{hij}_{kp}\xi^d_{,n} - P^{*p}_{ml}a^{hij}_{kp,\dot{d}} + a^{hij}_{kl,\dot{s}\dot{d}}\xi^s_{,m}\xi^d_{,n} + a^{hij}_{kl,\dot{s}\dot{t}}\xi^s_{,m}\xi^t_{,\dot{d}}\xi^d_{,n} + a^{hij}_{kl}\xi^s_{,m\dot{d}} + P^{*h}_{sn}a^{sij}_{kl,m}\\
&+ P^{*h}_{sn}a^{sij}_{kl,\dot{d}}\xi^d_{,m} + P^{*h}_{sn}P^{*s}_{mp}a^{pij}_{kl} + P^{*h}_{sn}P^{*i}_{mp}a^{spj}_{kl} + P^{*h}_{sn}P^{*j}_{mp}a^{sip}_{kl} - P^{*h}_{sn}P^{*p}_{km}a^{sij}_{pl}\\
&- P^{*h}_{sn}P^{*p}_{lm}a^{sij}_{kp} + P^{*i}_{sn}a^{hsj}_{kl,m} + P^{*i}_{sn}a^{hsj}_{kl,\dot{d}}\xi^d_{,m} + P^{*i}_{sn}P^{*h}_{mp}a^{psj}_{kl} + P^{*i}_{sn}P^{*s}_{mp}a^{hpj}_{kl} + P^{*i}_{sn}P^{*j}_{mp}a^{hsp}_{kl}\\
&- P^{*i}_{sn}P^{*p}_{km}a^{hsj}_{pl} - P^{*i}_{sn}P^{*p}_{lm}a^{hsj}_{kp} + P^{*j}_{sn}a^{his}_{kl,m} + P^{*j}_{sn}a^{his}_{kl,\dot{d}}\xi^d_{,m} + P^{*j}_{sn}P^{*h}_{mp}a^{pis}_{kl} + P^{*j}_{sn}P^{*i}_{mp}a^{hps}_{kl}\\
&+ P^{*j}_{sn}P^{*s}_{mp}a^{hip}_{kl} - P^{*j}_{sn}P^{*p}_{km}a^{his}_{pl} - P^{*j}_{sn}P^{*p}_{lm}a^{his}_{kp} - P^{*s}_{nk}a^{hij}_{sl,m} - P^{*s}_{nk}a^{hij}_{sl,\dot{d}}\xi^d_{,m} - P^{*s}_{nk}P^{*h}_{mp}a^{pij}_{sl}\\
&- P^{*s}_{nk}P^{*i}_{mp}a^{hpj}_{sl} - P^{*s}_{nk}P^{*i}_{mp}a^{hip}_{sl} + P^{*s}_{nk}P^{*p}_{sm}a^{hij}_{pl} + P^{*s}_{nk}P^{*p}_{lm}a^{hij}_{sp} - P^{*s}_{nl}a^{hij}_{ks,m} - P^{*s}_{nl}a^{hij}_{ks,\dot{d}}\xi^d_{,m}\\
&- P^{*s}_{nl}P^{*h}_{mp}a^{pij}_{ks} - P^{*s}_{nl}P^{*i}_{mp}a^{hpj}_{ks} - P^{*s}_{nl}P^{*i}_{mp}a^{hip}_{ks} + P^{*s}_{nl}P^{*p}_{km}a^{hij}_{ps} + P^{*s}_{nl}P^{*p}_{sm}a^{hij}_{kp} - P^{*s}_{nm}a^{hij}_{kl\underset{4}{|}s}
\end{aligned}
$$

from where

$$
\begin{aligned}
a^{hij}_{kl\underset{4}{|}m\underset{3}{|}n} - a^{hij}_{kl\underset{4}{|}n\underset{3}{|}m} &= \underset{3}{\widetilde{A}}{}^h_{pmn}a^{pij}_{kl} + \underset{3}{\widetilde{A}}{}^i_{pmn}a^{hpj}_{kl} + \underset{3}{\widetilde{A}}{}^j_{pmn}a^{hip}_{kl} - \underset{2}{\widetilde{A}}{}^p_{knm}a^{hij}_{pl}\\
&- \underset{2}{\widetilde{A}}{}^p_{lnm}a^{hij}_{kp} - \overline{a}^{hij}_{<[kl]>} - \overline{a}^{hij}_{\leqslant[kl]\geqslant} + T^{*p}_{mn}a^{hij}_{kl\underset{3}{|}s},
\end{aligned}
\tag{2.10}
$$

where we have denoted [4]

$$
\underset{3}{\widetilde{A}}{}^i_{jmn} = P^{*i}_{mj,n} - P^{*i}_{nj,m} + P^{*p}_{mj}P^{*i}_{pn} - P^{*p}_{nj}P^{*i}_{pm} + P^{*p}_{mj,\dot{s}}\xi^s_{,n} - P^{*p}_{nj,\dot{s}}\xi^s_{,m},
\tag{2.11}
$$

$$
\underset{2}{\widetilde{A}}{}^i_{jmn} = P^{*i}_{jm,n} - P^{*i}_{jn,m} + P^{*p}_{mj}P^{*i}_{pn} - P^{*p}_{nj}P^{*i}_{pm} + P^{*p}_{jm,\dot{s}}\xi^s_{,n} - P^{*p}_{jn,\dot{s}}\xi^s_{,m},
\tag{2.12}
$$

The next two theorems are valid.

Theorem 2.2. *In GF_N the 3^{rd} new Ricci type identity is in force:*

$$
\begin{aligned}
a^{r_1\ldots r_u}_{t_1\ldots t_v\underset{3}{|}m\underset{4}{|}n} - a^{r_1\ldots r_u}_{t_1\ldots t_v\underset{3}{|}n\underset{4}{|}m} &= \sum_{\alpha=1}^{u}\underset{1}{\widetilde{A}}{}^{r_\alpha}_{pmn}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}\underset{4}{\widetilde{A}}{}^{p}_{t_\beta mn}\binom{t_\beta}{p}a^{\cdots}_{\cdots}\\
&+ \overline{a}^{r_1\ldots r_u}_{t_1\ldots t_v<[mn]>} + \overline{a}^{r_1\ldots r_u}_{t_1\ldots t_v\leqslant[mn]\geqslant} - T^{*p}_{mn}a^{r_1\ldots r_u}_{t_1\ldots t_v\underset{3}{|}p}\ ,
\end{aligned}
\tag{2.13}
$$

where $\underset{1}{\widetilde{A}}$, $\underset{4}{\widetilde{A}}$ are given at (2.8, 2.9) and

$$
\begin{aligned}
\overline{a}^{r_1\ldots r_u}_{t_1\ldots t_v<mn>} &= \sum_{\alpha=1}^{u}T^{*r_\alpha}_{pm}\binom{p}{r_\alpha}(a^{\cdots}_{\cdots,n} + a^{\cdots}_{\cdots,\dot{s}}\xi^s_{,n})\\
&- \sum_{\beta=1}^{u}T^{*p}_{mt_\beta}\binom{t_\beta}{p}(a^{\cdots}_{\cdots,n} + a^{\cdots}_{\cdots,\dot{s}}\xi^s_{,n})\ ,
\end{aligned}
$$

$$
\begin{aligned}
\overline{a}^{r_1\dots r_u}_{t_1\dots t_v\leqslant mn\geqslant} = {} & \sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{u-1}\sum_{\beta=2}^{u} P^{*r_\alpha}_{[\underset{\smile}{p}m} P^{*r_\beta}_{n\underset{\smile}{s}]} \binom{p}{r_\alpha}\binom{s}{r_\beta} a^{\cdots}_{\cdots} \\
& - \sum_{\alpha=1}^{u}\sum_{\beta=1}^{v} P^{*r_\alpha}_{[\underset{\smile}{p}m} P^{*s}_{\underset{\smile}{t_\beta}n]} \binom{p}{r_\alpha}\binom{t_\beta}{s} a^{\cdots}_{\cdots} \\
& + \sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{v-1}\sum_{\beta=1}^{v} P^{*p}_{[\underset{\smile}{m}t_\alpha} P^{*s}_{\underset{\smile}{t_\beta}n]} \binom{t_\alpha}{p}\binom{t_\beta}{s} a^{\cdots}_{\cdots}\,,
\end{aligned}
$$

$$
\binom{p}{r_\alpha}\binom{t_\beta}{s} a^{\cdots}_{\cdots} = a^{r_1\dots r_{\alpha-1}pr_{\alpha+1}\dots r_u}_{t_1\dots t_{\beta-1}sr_{\beta+1}\dots t_v}.
$$

Theorem 2.3. *Applying the two kinds of covariant derivatives in reverse order than in the preceding theorem, we obtain the* 4^{th} *new Ricci type identity in* GF_N

$$
\begin{aligned}
a^{r_1\dots r_u}_{t_1\dots t_v\underset{4}{|}m\underset{3}{|}n} - a^{r_1\dots r_u}_{t_1\dots t_v\underset{4}{|}n\underset{3}{|}m} = {} & \sum_{\alpha=1}^{u} \underset{3}{\widetilde{A}}{}^{r_\alpha}_{pmn}\binom{p}{r_\alpha} a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} \underset{2}{\widetilde{A}}{}^{p}_{t_\beta mn}\binom{t_\beta}{p} a^{\cdots}_{\cdots} \\
& - \overline{a}^{\cdots}_{\cdots<[mn]>} - \overline{a}^{\cdots}_{\cdots\leqslant[mn]\geqslant} + T^{*p}_{mn} a^{\cdots}_{\cdots\underset{4}{|}p},
\end{aligned}
\tag{2.14}
$$

where $\underset{3}{\widetilde{A}}$, $\underset{2}{\widetilde{A}}$ *are given at (2.11, 2.12).*

Following this procedure, we can conclude that the following theorems are valid.

Theorem 2.4. *In* GF_N *is in force the* 5^{th} *new Ricci type identity*

$$
\begin{aligned}
a^{r_1\dots r_u}_{t_1\dots t_v\underset{3}{|}mn} - a^{r_1\dots r_u}_{t_1\dots t_v\underset{4}{|}nm} = {} & \sum_{\alpha=1}^{u} \underset{5}{\widetilde{A}}{}^{r_\alpha}_{pmn}\binom{p}{r_\alpha} a^{\cdots}_{\cdots} + \sum_{\beta=1}^{v} \underset{6}{\widetilde{A}}{}^{p}_{t_\beta nm}\binom{t_\beta}{p} a^{\cdots}_{\cdots} \\
& + \overline{a}^{\cdots}_{\cdots<(mn)>} + \overline{a}^{\cdots}_{\cdots\leqslant(mn)\geqslant} - P^{*p}_{mn}\big(a^{\cdots}_{\cdots\underset{3}{|}p} - a^{\cdots}_{\cdots\underset{4}{|}p}\big),
\end{aligned}
\tag{2.15}
$$

where

$$
\begin{aligned}
\overline{a}^{r_1\dots r_u}_{t_1\dots t_v\leqslant mn\geqslant} = {} & \sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{u-1}\sum_{\beta=2}^{u} P^{*r_\alpha}_{[\underset{\smile}{p}m} P^{*r_\beta}_{\underset{\smile}{s}n]} \binom{p}{r_\alpha}\binom{s}{t_\beta} a^{\cdots}_{\cdots} \\
& - \sum_{\alpha=1}^{u}\sum_{\beta=1}^{v} P^{*r_\alpha}_{[\underset{\smile}{p}m} P^{*s}_{n\underset{\smile}{t_\beta}]} \binom{p}{r_\alpha}\binom{t_\beta}{s} a^{\cdots}_{\cdots} \\
& + \sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{v-1}\sum_{\beta=2}^{v} P^{*p}_{[\underset{\smile}{m}t_\alpha} P^{*s}_{n\underset{\smile}{t_\beta}]} \binom{t_\alpha}{p}\binom{t_\beta}{s} a^{\cdots}_{\cdots}
\end{aligned}
\tag{2.16}
$$

$$
\underset{5}{\widetilde{A}}{}^{i}_{jmn} = P^{*i}_{jm,n} - P^{*i}_{nj,m} + P^{*p}_{jm}P^{*i}_{pn} - P^{*p}_{nj}P^{*i}_{mp} + P^{*p}_{jm,\dot{s}}\xi^{s}_{,n} - P^{*p}_{nj,\dot{s}}\xi^{s}_{,m},
\tag{2.17}
$$

$$
\underset{6}{\widetilde{A}}{}^{i}_{jmn} = P^{*i}_{jm,n} - P^{*i}_{nj,m} + P^{*p}_{mj}P^{*i}_{np} - P^{*p}_{jn}P^{*i}_{pm} + P^{*p}_{jm,\dot{s}}\xi^{s}_{,n} - P^{*p}_{nj,\dot{s}}\xi^{s}_{,m}.
\tag{2.18}
$$

Theorem 2.5. *In GF_N is in force the 6^{th} new Ricci type identity*

$$a^{r_1\dots r_u}_{t_1\dots t_v\underset{3}{|}mn} - a^{r_1\dots r_u}_{t_1\dots t_v\underset{3}{|}n\underset{4}{|}m} = \sum_{\alpha=1}^{u} \underset{7}{\widetilde{A}}{}^{r_\alpha}_{pmn} \binom{p}{r_\alpha} a^{\dots}_{\dots} - \sum_{\beta=1}^{v} \underset{14}{\widetilde{A}}{}^{p}_{t_\beta mn} \binom{t_\beta}{p} a^{\dots}_{\dots} + \overline{a}^{\dots}_{\dots<mn>} + \overline{a}^{\dots}_{\dots\leqslant mn\geqslant}\ , \tag{2.19}$$

where

$$\begin{aligned}\overline{a}^{r_1\dots r_u}_{t_1\dots t_v\leqslant mn\geqslant} = &\sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{u-1}\sum_{\beta=2}^{u}(P^{*r_\alpha}_{[pm]}P^{*r_\beta}_{sn} + P^{*r_\alpha}_{pn}P^{*r_\beta}_{[sm]})\binom{p}{r_\alpha}\binom{s}{r_\beta}a^{\dots}_{\dots}\\ &-\sum_{\alpha=1}^{u}\sum_{\beta=1}^{v}(P^{*r_\alpha}_{[pm]}P^{*s}_{nt_\beta} + P^{*r_\alpha}_{pn}P^{*s}_{[mt_\beta]})\binom{p}{r_\alpha}\binom{t_\beta}{s}a^{\dots}_{\dots}\\ &+\sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{v-1}\sum_{\beta=2}^{v}(P^{*p}_{[mt_\alpha]}P^{*s}_{nt_\beta} + P^{*p}_{nt_\alpha}P^{*s}_{[mt_\beta]})\binom{t_\alpha}{p}\binom{t_\beta}{s}a^{\dots}_{\dots}\ ,\end{aligned} \tag{2.20}$$

$$\underset{7}{\widetilde{A}}{}^{i}_{jmn} = P^{*i}_{jm,n} - P^{*i}_{jn,m} + P^{*p}_{jm}P^{*i}_{pn} - P^{*p}_{jn}P^{*i}_{mp} + P^{*p}_{jm,\dot{s}}\xi^s_{,n} - P^{*p}_{jn,\dot{s}}\xi^s_{,m}, \tag{2.21}$$

$$\underset{14}{\widetilde{A}}{}^{i}_{jmn} = P^{*i}_{mj,n} - P^{*i}_{nj,m} + P^{*p}_{jm}P^{*i}_{np} - P^{*p}_{nj}P^{*i}_{mp} + P^{*p}_{mj,\dot{s}}\xi^s_{,n} - P^{*p}_{nj,\dot{s}}\xi^s_{,m}. \tag{2.22}$$

Theorem 2.6. *In GF_N the 7^{th} new Ricci type identity is valid*

$$a^{r_1\dots r_u}_{t_1\dots t_v\underset{3}{|}mn} - a^{r_1\dots r_u}_{t_1\dots t_v\underset{4}{|}n\underset{3}{|}m} = \sum_{\alpha=1}^{u} \underset{9}{\widetilde{A}}{}^{r_\alpha}_{pmn} \binom{p}{r_\alpha} a^{\dots}_{\dots} - \sum_{\beta=1}^{v} \underset{12}{\widetilde{A}}{}^{p}_{t_\beta mn} \binom{t_\beta}{p} a^{\dots}_{\dots} + \overline{a}^{\dots}_{\dots<nm>} + \overline{a}^{\dots}_{\dots\leqslant nm\geqslant} - (P^{*p}_{nm}a^{\dots}_{\dots\underset{3}{|}p} - P^{*p}_{mn}a^{\dots}_{\dots\underset{4}{|}p}), \tag{2.23}$$

where

$$\underset{9}{\widetilde{A}}{}^{i}_{jmn} = P^{*i}_{jm,n} - P^{*i}_{nj,m} + P^{*p}_{jm}P^{*i}_{pn} - P^{*p}_{nj}P^{*i}_{pm} + P^{*p}_{jm,\dot{s}}\xi^s_{,n} - P^{*p}_{nj,\dot{s}}\xi^s_{,m}, \tag{2.24}$$

$$\underset{12}{\widetilde{A}}{}^{i}_{jmn} = P^{*i}_{mj,n} - P^{*i}_{jn,m} + P^{*p}_{mj}P^{*i}_{pn} - P^{*p}_{nj}P^{*i}_{mp} + P^{*p}_{mj,\dot{s}}\xi^s_{,n} - P^{*p}_{jn,\dot{s}}\xi^s_{,m}. \tag{2.25}$$

Theorem 2.7. *In GF_N the 8^{th} new Ricci type identity is valid*

$$a^{r_1\dots r_u}_{t_1\dots t_v\underset{4}{|}mn} - a^{r_1\dots r_u}_{t_1\dots t_v\underset{3}{|}n\underset{4}{|}m} = \sum_{\alpha=1}^{u} \underset{11}{\widetilde{A}}{}^{r_\alpha}_{pmn} \binom{p}{r_\alpha} a^{\dots}_{\dots} - \sum_{\beta=1}^{v} \underset{10}{\widetilde{A}}{}^{p}_{t_\beta mn} \binom{t_\beta}{p} a^{\dots}_{\dots} - \overline{a}^{\dots}_{\dots<nm>} + \overline{a}^{\dots}_{\dots\leqslant mn\geqslant} + P^{*p}_{nm}a^{\dots}_{\dots\underset{3}{|}p} - P^{*p}_{mn}a^{\dots}_{\dots\underset{4}{|}p}, \tag{2.26}$$

where

$$\overline{a}^{r_1\dots r_u}_{t_1\dots t_v \lessgtr mn \gtrless} = \sum_{\substack{\alpha=1 \\ (\alpha<\beta)}}^{u-1}\sum_{\beta=2}^{u}(P^{*r_\alpha}_{mp}P^{*r_\beta}_{[ns]} + P^{*r_\alpha}_{[np]}P^{*r_\beta}_{ms})\binom{p}{r_\alpha}\binom{s}{r_\beta}a^{\cdots}_{\cdots}$$
$$- \sum_{\alpha=1}^{u}\sum_{\beta=1}^{v}(P^{*r_\alpha}_{mp}P^{*s}_{[t_\beta n]} + P^{*r_\alpha}_{[np]}P^{*s}_{t_\beta m})\binom{p}{r_\alpha}\binom{t_\beta}{s}a^{\cdots}_{\cdots} \tag{2.27}$$
$$+ \sum_{\substack{\alpha=1 \\ (\alpha<\beta)}}^{v-1}\sum_{\beta=1}^{v}(P^{*p}_{t_\alpha m}P^{*s}_{[t_\beta n]} + P^{*p}_{[t_\alpha n]}P^{*s}_{t_\beta m})\binom{t_\alpha}{p}\binom{t_\beta}{s}a^{\cdots}_{\cdots}\ ,$$

$$\underset{10}{\widetilde{A}}{}^i{}_{jmn} = P^{*i}_{jm,n} - P^{*i}_{nj,m} + P^{*p}_{jm}P^{*i}_{np} - P^{*p}_{jn}P^{*i}_{pm} + P^{*p}_{jm,\dot{s}}\xi^s_{,n} - P^{*p}_{nj,\dot{s}}\xi^s_{,m}, \tag{2.28}$$

$$\underset{11}{\widetilde{A}}{}^i{}_{jmn} = P^{*i}_{mj,n} - P^{*i}_{jn,m} + P^{*p}_{mj}P^{*i}_{np} - P^{*p}_{jn}P^{*i}_{mp} + P^{*p}_{mj,\dot{s}}\xi^s_{,n} - P^{*p}_{jn,\dot{s}}\xi^s_{,m}. \tag{2.29}$$

Theorem 2.8. *In GF_N the 9^{th} new Ricci type identity is valid*

$$a^{r_1\dots r_u}_{t_1\dots t_v \underset{4}{|} mn} - a^{r_1\dots r_u}_{t_1\dots t_v \underset{4}{|} n \underset{3}{|} m} = \sum_{\alpha=1}^{u}\underset{13}{\widetilde{A}}{}^{r_\alpha}{}_{pmn}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}\underset{8}{\widetilde{A}}{}^{p}{}_{t_\beta mn}\binom{t_\beta}{p}a^{\cdots}_{\cdots}$$
$$- \overline{a}^{\cdots}_{\cdots <mn>} + \overline{a}^{\cdots}_{\cdots \lessgtr nm \gtrless}, \tag{2.30}$$

where

$$\underset{8}{\widetilde{A}}{}^i{}_{jmn} = P^{*i}_{jm,n} - P^{*i}_{jn,m} + P^{*p}_{mj}P^{*i}_{pn} - P^{*p}_{jn}P^{*i}_{pm} + P^{*p}_{jm,\dot{s}}\xi^s_{,n} - P^{*p}_{jn,\dot{s}}\xi^s_{,m}, \tag{2.31}$$

$$\underset{13}{\widetilde{A}}{}^i{}_{jmn} = P^{*i}_{mj,n} - P^{*i}_{nj,m} + P^{*p}_{mj}P^{*i}_{np} - P^{*p}_{nj}P^{*i}_{pm} + P^{*p}_{mj,\dot{s}}\xi^s_{,n} - P^{*p}_{nj,\dot{s}}\xi^s_{,m}. \tag{2.32}$$

Theorem 2.9. *In GF_N the 10^{th} new Ricci type identity is valid*

$$a^{r_1\dots r_u}_{t_1\dots t_v \underset{3}{|} m \underset{4}{|} n} - a^{r_1\dots r_u}_{t_1\dots t_v \underset{4}{|} n \underset{3}{|} m} = \sum_{\alpha=1}^{u}\underset{15}{\widetilde{A}}{}^{r_\alpha}{}_{pnm}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}\underset{15}{\widetilde{A}}{}^{p}{}_{t_\beta nm}\binom{t_\beta}{p}a^{\cdots}_{\cdots}$$
$$- P^{*p}_{mn}(a^{\cdots}_{\cdots \underset{3}{|} p} - a^{\cdots}_{\cdots \underset{4}{|} p}), \tag{2.33}$$

where

$$\underset{15}{\widetilde{A}}{}^i{}_{jmn} = P^{*i}_{jm,n} - P^{*i}_{nj,m} + P^{*p}_{jm}P^{*i}_{np} - P^{*p}_{nj}P^{*i}_{pm} + P^{*p}_{jm,\dot{s}}\xi^s_{,n} - P^{*p}_{nj,\dot{s}}\xi^s_{,m}. \tag{2.34}$$

The identity (2.33) can be written in another form. Namely, counting the difference in the last brackets in (2.33), one obtains

$$a^{r_1\dots r_u}_{t_1\dots t_v \underset{3}{|} m \underset{4}{|} n} - a^{r_1\dots r_u}_{t_1\dots t_v \underset{4}{|} n \underset{3}{|} m} = \sum_{\alpha=1}^{u}\underset{4}{\widetilde{K}}{}^{r_\alpha}{}_{pmn}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} + \sum_{\beta=1}^{v}\underset{3}{\widetilde{K}}{}^{p}{}_{t_\beta nm}\binom{t_\beta}{p}a^{\cdots}_{\cdots}\ ,$$

where

$$\underset{3}{\widetilde{K}}{}^{i}{}_{jmn} = \underset{15}{\widetilde{A}}{}^{i}_{jmn} + P^{*p}_{nm}T^{*i}_{pj} = P^{*i}_{jm,n} - P^{*i}_{nj,m} + P^{*p}_{jm}P^{*i}_{np} - P^{*p}_{nj}P^{*i}_{pm} + P^{*p}_{jm,\dot{s}}\xi^{s}_{,n} - P^{*p}_{nj,\dot{s}}\xi^{s}_{,m} + P^{*p}_{nm}(P^{*i}_{pj} - P^{*i}_{jp}),$$

$$\underset{4}{\widetilde{K}}{}^{i}{}_{jmn} = \underset{15}{\widetilde{A}}{}^{i}_{jmn} + P^{*p}_{mn}T^{*i}_{pj} = P^{*i}_{jm,n} - P^{*i}_{nj,m} + P^{*p}_{jm}P^{*i}_{np} - P^{*p}_{nj}P^{*i}_{pm} + P^{*p}_{jm,\dot{s}}\xi^{s}_{,n} - P^{*p}_{nj,\dot{s}}\xi^{s}_{,m}P^{*p}_{mn}(P^{*i}_{pj} - P^{*i}_{jp}),$$

In the work [18] are given some properties for the mentioned tensors (the antisymmetry with respect of two indices, the cyclic symmetry, the symmetry with respect of pairs of indices).

Remark 2.1. *The magnitudes* $\underset{\theta}{\widetilde{K}}$, $\theta = 1,2,3,4$ *are tensors and magnitudes* $\underset{t}{\widetilde{A}}$, $t = \overline{1,15}$, *have the form and the role of the curvature tensor, but they are not tensors. For example, from (2.19) one obtains*

$$a^{i}_{\underset{3}{|}mn} - a^{i}_{\underset{3}{|}n\underset{4}{|}m} = \underset{7}{\widetilde{A}}{}^{i}_{pmn}a^{p} + T^{*i}_{pm}(a^{p}_{,n} + a^{p}_{,\dot{s}}\xi^{s}_{,n}),$$

from where we see that $\underset{7}{\widetilde{A}}$ *is not a tensor, because the expression in the bracket is not a tensor. These magnitudes* $\underset{t}{\widetilde{A}}$, $t = \overline{1,15}$ *we call* curvature pseudotensors *of the* $1^{st}, \ldots, 15^{th}$ *kind in* GF_N.

Remark 2.2. *For* $g_{ij}(x,\dot{x}) = g_{ji}(x,\dot{x})$ *we obtain usual Finsler space* F_N. *If* $g_{ij}(x) \neq g_{ji}(x)$ *one obtains a generalized Riemannian space* GR_N *[2]. For* $g_{ij}(x) = g_{ji}(x)$ GF_N *reduces to the Riemannian space* R_N.

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