On a type of Kenmotsu manifolds

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Abstract. The object of the present paper is to study Kenmotsu manifolds admitting a W_2-curvature tensor.

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1 Introduction

In [12], S. Tanno classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold M, the sectional curvature of plane sections containing ξ is a constant, say c. If $c > 0$, M is homogeneous Sasakian manifold of constant sectional curvature . If $c = 0$, M is the product of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature. If $c < 0$, M is a warped product space $\mathbb{R} \times_f \mathbb{C}^n$. In 1971, Kenmotsu studied a class of contact Riemannian manifolds satisfying some special conditions [7]. We call it Kenmotsu manifold. Recently, Kenmotsu manifolds have been studied by many authors such as De and Pathak [2], Jun, De and Pathak [6], Ozgür and De [9] and many others.

On the other hand, Pokhariyal and Mishra [11] have introduced new tensor fields, called W_2 and E-tensor fields, in a Riemannian manifold, and studied their properties. Then, Pokhariyal [10] has studied some properties of this tensor fields in a Sasakian manifold. Recently, Matsumoto, Ianus and Mihai [8] have studied P-Sasakian manifolds admitting W_2 and E-tensor fields and De and Sarkar [3] have studied P-Sasakian manifolds admitting W_2 tensor field.

The curvature tensor W_2 is defined by

$$W_2(X,Y,U,V) = R(X,Y,U,V) + \frac{1}{n-1}[g(X,U)S(Y,V) - g(Y,U)S(X,V)], \tag{1.1}$$

where S is a Ricci tensor of type $(0,2)$.

In the present paper we have studied some curvature conditions on Kenmotsu manifolds. Firstly, we have studied their geometric and relativistic properties in

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Kenmotsu manifolds satisfying $W_2 = 0$. Then we have studied W_2-semisymmetric Kenmotsu manifolds. Also, we have classified Kenmotsu manifolds which satisfy $P \cdot W_2 = 0$, $\tilde{Z} \cdot W_2 = 0$, $C \cdot W_2 = 0$ and $\tilde{C} \cdot W_2 = 0$ where P is the projective curvature tensor, $\tilde{Z}$ is the concircular curvature tensor, $\tilde{C}$ is the quasi-conformal curvature tensor and C is the conformal curvature tensor.

2 Preliminaries

Let $(M^n, \phi, \xi, \eta, g)$ be an n-dimensional (where $n = 2m + 1$) almost contact metric manifold, where ϕ is a $(1,1)$-tensor field, ξ is the structure vector field, η is a 1-form and g is the Riemannian metric. It is well known that the (ϕ, ξ, η, g) structure satisfies the conditions [1]

$$\phi\xi = 0, \ \eta(\phi X) = 0, \ \eta(\xi) = 1, \tag{2.1}$$

$$\phi^2 X = -X + \eta(X)\xi, \ g(X, \xi) = \eta(X), \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.3}$$

for any vector fields X and Y on M^n.

If moreover

$$(\nabla_X \phi)Y = -g(X, \phi Y)\xi - \eta(Y)\phi X, \tag{2.4}$$

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.5}$$

where ∇ denotes the Riemannian connection of g hold, then $(M^n, \phi, \xi, \eta, g)$ is called a Kenmotsu manifold. In this case, it is well known that [7]

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.6}$$

$$S(X, \xi) = -(n-1)\eta(X), \tag{2.7}$$

where S denotes the Ricci tensor. From (2.6), it easily follows that

$$R(X, \xi)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.8}$$

$$R(X, \xi)\xi = \eta(X)\xi - X. \tag{2.9}$$

The notion of the *quasi-conformal curvature tensor* was given by Yano and Sawaki [13]. According to them a quasi-conformal curvature tensor $\tilde{C}$ is defined by

$$\begin{aligned} \tilde{C}(X, Y)Z =& aR(X, Y)Z \\ &+ b[S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY] \\ &- \frac{\tau}{n}[\frac{a}{n-1} + 2b][g(Y, Z)X - g(X, Z)Y], \end{aligned} \tag{2.10}$$

where a and b are constants and R, S, Q and τ are the Riemannian curvature tensor type of $(1,3)$, the Ricci tensor of type $(0,2)$, the Ricci operator defined by $g(QX, Y) = S(X, Y)$ and scalar curvature of the manifold respectively. If $a = 1$ and $b = -\frac{1}{n-2}$

then (2.10) takes the form

$$\begin{aligned}\tilde{C}(X,Y)Z =& R(X,Y)Z\\ &-\frac{1}{n-2}[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY]\\ &+\frac{\tau}{(n-1)(n-2)}[g(Y,Z)X-g(X,Z)Y]=C(X,Y)Z,\end{aligned}\tag{2.11}$$

where C is the *conformal curvature tensor* [5].

We next define endomorphisms $R(X,Y)$ and $X\wedge_A Y$ of $\chi(M)$ by

$$R(X,Y)W=\nabla_X\nabla_Y W-\nabla_Y\nabla_X W-\nabla_{[X,Y]}W,$$
$$(X\wedge_A Y)W=A(Y,W)X-A(X,W)Y,$$

respectively, where $X,Y,W\in\chi(M)$ and A is the symmetric $(0,2)-$tensor.

On the other hand, the *projective curvature tensor* P and the *concircular curvature tensor* $\tilde{Z}$ in a Riemannian manifold (M^n,g) are defined by

$$P(X,Y)W=R(X,Y)W-\frac{1}{n-1}(X\wedge_S Y)W,\tag{2.12}$$

$$\tilde{Z}(X,Y)W=R(X,Y)W-\frac{\tau}{n(n-1)}(X\wedge_g Y)W,\tag{2.13}$$

respectively.

An almost contact metric manifold is said to be an η-Einstein manifold if the Ricci tensor S satisfies the condition

$$S(X,Y)=\lambda_1 g(X,Y)+\lambda_2\eta(X)\eta(Y),\tag{2.14}$$

where λ_1, λ_2 are certain scalars. A Riemannian or a semi-Riemannian manifold is said to semisymmetric if $R(X,Y)\cdot R=0$, where $R(X,Y)$ is considered as a derivation of the tensor algebra at each point of the manifold for the tangent vectors X,Y.

In a Kenmotsu manifold, using (2.7) and (2.8), equations (2.12), (2.13), (2.11), and (2.10) reduce to

$$P(\xi,X)Y=-g(X,Y)\xi-\frac{1}{n-1}S(X,Y)\xi,\tag{2.15}$$

$$\tilde{Z}(\xi,X)Y=(1+\frac{\tau}{n(n-1)})(-g(X,Y)\xi+\eta(Y)X),\tag{2.16}$$

$$\begin{aligned}C(\xi,Y)W=&\frac{n-1+\tau}{(n-1)(n-2)}\{g(Y,W)\xi-\eta(W)Y\}\\ &-\frac{1}{n-2}\{S(Y,W)\xi-\eta(W)QY\},\end{aligned}\tag{2.17}$$

$$\tilde{C}(\xi,Y)W=K\{\eta(W)Y-g(Y,W)\xi\}+b\{S(Y,W)\xi-\eta(W)QY\},\tag{2.18}$$

respectively, where $K=a+(n-1)b+\frac{\tau}{n}(\frac{a}{n-1}+2b)$.
Also we have [1]

$$(\nabla_X\eta)Y=g(X,Y)-\eta(X)\eta(Y).\tag{2.19}$$

A Kenmotsu manifold M^n is said to be Einstein if its Ricci tensor S is of the form

$$S(X,Y) = \lambda_1 g(X,Y), \tag{2.20}$$

for any vector fields X, Y and λ_1 is a certain scalar.

3 Kenmotsu manifolds satisfying $W_2 = 0$

In this section we consider a Kenmotsu manifold satisfying $W_2 = 0$. Then we have from (1.1)

$$R(X,Y,U,V) = \frac{1}{n-1}[g(Y,U)S(X,V) - g(X,U)S(Y,V)]. \tag{3.1}$$

Using $X = U = \xi$ in (3.1), we have

$$R(\xi,Y,\xi,V) = \frac{1}{n-1}[g(Y,\xi)S(\xi,V) - g(\xi,\xi)S(Y,V)]. \tag{3.2}$$

From (2.1), (2.7) and (2.9), we get

$$S(Y,V) = -(n-1)g(Y,V). \tag{3.3}$$

Thus M is an Einstein manifold.

Theorem 1. *Let M be an $n-$dimensional $(n > 1)$ Kenmotsu manifold satisfying $W_2 = 0$. Then M is an Einstein manifold.*

Now using (3.3) in (3.1), we get

$$R(X,Y,U,V) = g(X,U)g(Y,V) - g(Y,U)g(X,V).$$

Hence M is of constant curvature -1. Then M is locally isometric to the hyperbolic space $H^n(-1)$.

Corollary 1. *Let M be an $n-$dimensional $(n > 1)$ Kenmotsu manifold satisfying $W_2 = 0$ and let M be an Einstein manifold. Then M^n is locally isometric to the hyperbolic space $H^n(-1)$.*

4 W_2-semisymmetric Kenmotsu manifolds

Definition 1. *An n-dimensional Kenmotsu manifold is called W_2-semisymmetric if it satisfies*

$$R(X,Y)\cdot W_2 = 0, \tag{4.1}$$

where $R(X,Y)$ is to be considered as a derivation of the tensor algebra at each point of the manifold for tangent vectors X, Y.

Proposition 1. *Let M be an n-dimensional Kenmotsu manifold. Then the W_2-curvature tensor on M satisfies the condition*

$$W_2(X,Y,U,\xi) = 0. \tag{4.2}$$

Theorem 2. *A W_2-semisymmetric Kenmotsu manifold is an Einstein manifold.*

Proof. From (4.1) we have

$$R(X,Y)W_2(Z,U)V - W_2(R(X,Y)Z,U)V \\ -W_2(Z,R(X,Y)U)V - W_2(Z,U)R(X,Y)V = 0. \tag{4.3}$$

This equation implies

$$g(R(X,Y)W_2(Z,U)V,\xi) - g(W_2(R(X,Y)Z,U)V,\xi) \\ -g(W_2(Z,R(X,Y)U)V,\xi) - g(W_2(Z,U)R(X,Y)V,\xi) = 0. \tag{4.4}$$

Putting $X = \xi$ in (4.4) we obtain

$$g(R(\xi,Y)W_2(Z,U)V,\xi) - g(W_2(R(\xi,Y)Z,U)V,\xi) \\ -g(W_2(Z,R(\xi,Y)U)V,\xi) - g(W_2(Z,U)R(\xi,Y)V,\xi) = 0. \tag{4.5}$$

Using (4.4) in (4.5), we get

$$-g(Y,W_2(Z,U)V)\xi + \eta(W_2(Z,U)V)Y + g(Y,Z)g(W_2(\xi,U)V,\xi) \\ -\eta(Z)g(W_2(Y,U)V,\xi) + g(Y,U)g(W_2(Z,\xi)V,\xi) - \eta(U)g(W_2(Z,Y)V,\xi) \\ +g(Y,V)g(W_2(Z,U)\xi,\xi) - \eta(V)g(W_2(Z,U)Y,\xi) = 0. \tag{4.6}$$

Taking the inner product with ξ and using (4.2) in (4.6), we obtain

$$W_2(Z,U,V,Y) = 0.$$

Then it follows from (1.1) that

$$R(Z,Y,U,V) = \frac{1}{n-1}[g(Y,U)S(Z,V) - g(Z,U)S(Y,V)]. \tag{4.7}$$

Contracting (4.7) we have

$$S(Y,U) = -(n-1)g(Y,U). \tag{4.8}$$

Thus M is an Einstein manifold. □

Now using (4.8) in (3.1), we get

$$R(Z,Y,U,V) = g(Z,U)g(Y,V) - g(Y,U)g(Z,V).$$

Hence M is of constant curvature -1. Then M is locally isometric to the hyperbolic space $H^n(-1)$.

Corollary 2. *A W_2-symmetric Kenmotsu manifold is locally isommetric to the hyperbolic space H^n (-1).*

5 Kenmotsu manifolds satisfying $P(X,Y)\cdot W_2 = 0$

In this section we consider a Kenmotsu manifold M^n satisfying the condition

$$P(X,Y)\cdot W_2 = 0.$$

This equation implies

$$\begin{aligned} &P(X,Y)W_2(Z,U)V - W_2(P(X,Y)Z,U)V \\ &-W_2(Z,P(X,Y)U)V - W_2(Z,U)P(X,Y)V = 0. \end{aligned} \tag{5.1}$$

Putting $X=\xi$ in (5.1), we obtain

$$\begin{aligned} &g(P(\xi,Y)W_2(Z,U)V,\xi) - g(W_2(P(\xi,Y)Z,U)V,\xi) \\ &-g(W_2(Z,P(\xi,Y)U)V,\xi) - g(W_2(Z,U)P(\xi,Y)V,\xi) = 0. \end{aligned} \tag{5.2}$$

Using (2.15) in (5.2), we have

$$\begin{aligned} &g(Y,W_2(Z,U)V)\xi - g(Y,Z)W_2(\xi,U)V - g(Y,U)W_2(Z,\xi)V \\ &-g(Y,V)W_2(Z,U)\xi + \frac{1}{n-1}\{S(Y,W_2(Z,U)V)\xi - S(Y,Z)W_2(\xi,U)V \\ &-S(Y,U)W_2(Z,\xi)V - S(Y,V)W_2(Z,U)\xi\} = 0. \end{aligned} \tag{5.3}$$

Taking the inner product with ξ and using (4.2) in (5.3), we get

$$g(Y,W_2(Z,U)V) + \frac{1}{n-1}S(Y,W_2(Z,U)V) = 0. \tag{5.4}$$

Now using (1.1) in (5.4) we get

$$\begin{aligned} &R(Z,U,V,Y) + \frac{1}{n-1}\{g(Z,V)S(U,Y) - g(U,V)S(Z,Y)\} \\ &+\frac{1}{n-1}R(Z,U,V,QY) + \frac{1}{(n-1)^2}\{g(Z,V)S(U,QY) - g(U,V)S(Z,QY)\} = 0, \end{aligned} \tag{5.5}$$

where $S(QY,Z) = S^2(Y,Z)$.

Again using $Z=V=\xi$ in (5.5) and from (2.9) we get

$$S(QY,U) = -2(n-1)S(Y,U) - (n-1)^2 g(Y,U). \tag{5.6}$$

Hence we have the following:

Theorem 3. *In an n-dimensional ($n>1$) Kenmotsu manifold M if the condition $P(X,Y)\cdot W_2 = 0$ holds on M, then the equation (5.6) is satisfied on M.*

Lemma 1. *[4] Let A be a symmetric $(0,2)$-tensor at point x of a semi-Riemannian manifold (M^n,g), $n>1$, and let $T = g\,\overline{\wedge}\,A$ be the Kulkarni-Nomizu product of g and A. Then, the relation*

$$T\cdot T = \alpha Q(g,T), \quad \alpha\in\mathbb{R}$$

is satisfied at x if and only if the condition

$$A^2 = \alpha A + \lambda g, \quad \lambda\in\mathbb{R}$$

holds at x.

From Theorem 3 and Lemma 1 we get the following:

Corollary 3. *Let M be an $n-$dimensional $(n>1)$ Kenmotsu manifold satisfying the condition $P(X,Y)\cdot W_2=0$, then $T\cdot T=\alpha Q(g,T)$, where $T=g\barwedge S$ and $\alpha=-2(n-1)$.*

6 Kenmotsu manifolds satisfying $\tilde{Z}(X,Y)\cdot W_2=0$

In this section we consider a Kenmotsu manifold M^n satisfying the condition

$$\tilde{Z}(X,Y)\cdot W_2=0.$$

This equation implies

$$\begin{aligned}\tilde{Z}(X,Y)W_2(Z,U)V-W_2(\tilde{Z}(X,Y)Z,U)V\\ -W_2(Z,\tilde{Z}(X,Y)U)V-W_2(Z,U)\tilde{Z}(X,Y)V=0.\end{aligned}\tag{6.1}$$

Now $X=\xi$ in (6.1), we have

$$\begin{aligned}\tilde{Z}(\xi,Y)W_2(Z,U)V-W_2(\tilde{Z}(\xi,Y)Z,U)V\\ -W_2(Z,\tilde{Z}(\xi,Y)U)V-W_2(Z,U)\tilde{Z}(\xi,Y)V=0.\end{aligned}\tag{6.2}$$

Using (2.16) in (6.2), we get

$$\begin{aligned}&\{1+\frac{\tau}{n(n-1)}\}\{g(Y,W_2(Z,U)V)\xi-g(W_2(Z,U)V,\xi)Y-g(Y,Z)W_2(\xi,U)V\\ &+\eta(Z)W_2(Y,U)V-g(Y,U)W_2(Z,\xi)V+\eta(U)W_2(Z,Y)V\\ &-g(Y,V)W_2(Z,U)\xi+\eta(V)W_2(Z,U)Y\}=0.\end{aligned}\tag{6.3}$$

Taking the inner product with ξ and using (4.2) in (6.3), we have

$$\{1+\frac{\tau}{n(n-1)}\}g(Y,W_2(Z,U)V)=0.$$

Again from (2.16) we have $1+\frac{\tau}{n(n-1)}\neq 0$. Hence we have

$$W_2(Z,U,V,Y)=0.$$

From the proof of Theorem 1, Theorem 2 and Corollary 1, we can say:

Theorem 4. *An $n-$dimensional $(n>1)$ Kenmotsu manifold M satisfying the condition $\tilde{Z}(\xi,Y)\cdot W_2=0$ is an Einstein manifold and locally isometric to the hyperbolic space $H^n(-1)$.*

7 Kenmotsu manifolds satisfying $C(X,Y)\cdot W_2=0$

In this section we consider a Kenmotsu manifold M^n satisfying the condition

$$C(X,Y)\cdot W_2=0.$$

This equation implies

$$C(X,Y)W_2(Z,U)V - W_2(C(X,Y)Z,U)V \\ -W_2(Z,C(X,Y)U)V - W_2(Z,U)C(X,Y)V = 0. \tag{7.1}$$

Putting $X = \xi$ in (7.1), we have

$$C(\xi,Y)W_2(Z,U)V - W_2(C(\xi,Y)Z,U)V \\ -W_2(Z,C(\xi,Y)U)V - W_2(Z,U)C(\xi,Y)V = 0. \tag{7.2}$$

Using (2.17) in (7.1), we obtain

$$\frac{n-1+\tau}{(n-1)(n-2)}\{g(Y,W_2(Z,U)V)\xi - g(W_2(Z,U)V,\xi)Y \\ -g(Y,Z)W_2(\xi,U)V + \eta(Z)W_2(Y,U)V - g(Y,U)W_2(Z,\xi)V \\ +\eta(U)W_2(Z,Y)V - g(Y,V)W_2(Z,U)\xi + \eta(V)W_2(Z,U)Y\} = 0. \tag{7.3}$$

Taking the inner product with ξ and using (4.2),

$$\{\frac{n-1+\tau}{(n-1)(n-2)}\}g(Y,W_2(Z,U)V) = 0.$$

Let U_1 and U_2 be a part of M satisfying $\tau + n - 1 = 0$ and

$$W_2(Z,U,V,Y) = 0. \tag{7.4}$$

This leads to the following:

Theorem 5. *In $n-$dimensional $(n > 1)$ Kenmotsu manifold M satisfying the condition $C(X,Y)\cdot W_2 = 0$. Then either $\tau + n - 1 = 0$, or M is locally isometric to the hyperbolic space $H^n(-1)$.*

8 Kenmotsu manifolds satisfying $\tilde{C}(X,Y)\cdot W_2 = 0$

In this section we consider a Kenmotsu manifold M^n satisfying the condition

$$\tilde{C}(X,Y)\cdot W_2 = 0.$$

This equation implies

$$\tilde{C}(X,Y)W_2(Z,U)V - W_2(\tilde{C}(X,Y)Z,U)V \\ -W_2(Z,\tilde{C}(X,Y)U)V - W_2(Z,U)\tilde{C}(X,Y)V = 0. \tag{8.1}$$

Putting $X = \xi$ in (8.1), we have

$$\tilde{C}(\xi,Y)W_2(Z,U)V - W_2(\tilde{C}(\xi,Y)Z,U)V \\ -W_2(Z,\tilde{C}(\xi,Y)U)V - W_2(Z,U)\tilde{C}(\xi,Y)V = 0 \tag{8.2}$$

Using (2.18) in (8.2), we get

$$
\begin{aligned}
&K\{g(W_2(Z,U)V,\xi)Y - g(Y,W_2(Z,U)V)\xi - \eta(Z)W_2(Y,U)V\\
&+g(Y,Z)W_2(\xi,U)V - \eta(U)W_2(Z,Y)V + g(Y,U)W_2(Z,\xi)V\\
&-\eta(V)W_2(Z,U)Y + g(Y,V)W_2(Z,U)\xi\}\\
&+b\{S(Y,W_2(Z,U)V)\xi - g(W_2(Z,U)V,\xi)QY + \eta(Z)W_2(QY,U)V\\
&-S(Y,Z)W_2(\xi,U)V + \eta(U)W_2(Z,QY)V - S(Y,U)W_2(Z,\xi)V\\
&+\eta(V)W_2(Z,U)QY - S(Y,V)W_2(Z,U)\xi\} = 0,
\end{aligned}
\tag{8.3}
$$

where $K = a + (n-1)b + \frac{\tau}{n}(\frac{a}{n-1} + 2b)$. Taking the inner product with ξ and using (4.2) in (8.3), we get

$$Kg(Y,W_2(Z,U)V) - bS(Y,W_2(Z,U)V) = 0. \tag{8.4}$$

Now using $Z = V = \xi$ in (8.4). Then from (1.1) and (2.9), we have

$$\frac{b}{n-1}S(QY,U) - (\frac{K}{n-1} - b)S(Y,U) - Kg(Y,U) = 0. \tag{8.5}$$

If $b = 0$, then we get

$$K\{\frac{1}{n-1}S(Y,U) - g(Y,U)\} = 0.$$

Let U_1 and U_2 be a part of M satisfying $K = 0$ and

$$S(Y,U) = -(n-1)g(Y,U),$$

respectively. If $b \neq 0$, then we get

$$S(QY,W) = (\frac{K}{b} - n + 1)S(Y,W) + (n-1)\frac{K}{b}g(Y,W). \tag{8.6}$$

Therefore we have the following:

Theorem 6. *Let M be an n-dimensional $(n > 1)$ Kenmotsu manifold satisfying the condition $\tilde{C}(X,Y)\cdot W_2 = 0$. Then we get*

1. *if $b = 0$, then either $K = 0$ on M, or M is an Einstein manifold.*
2. *if $b \neq 0$, then the equation (8.6) holds on M.*

Corollary 4. *Let M be an $n-$dimensional $(n > 1)$ Kenmotsu manifold satisfying the condition $\tilde{C}(\xi,Y)\cdot\tilde{C} = 0$, then $T\cdot T = \alpha Q(g,T)$, where $T = g\,\overline{\wedge}\,S$ and $\alpha = \frac{K}{b} - n + 1$.*

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