Generalized chaos synchronization of unidirectionally coupled Shimizu-Morioka dynamical systems

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Abstract. This paper proposes a generalized approach via linear transformations for constructing chaotic unidirectionally synchronized coupled Shimizu-Morioka dynamical systems. This method is very simple and useful for synchronization of hyper-chaotic system because it does not require calculation of Lyapunov exponents. This method can predict the responding system's behavior by knowing the driving system's behavior. To show the feasibility and effectiveness of the approach, numerical simulation results are presented.

M.S.C. 2010: 37N35, 37L15, 37B25, 34D20.
Key words: Shimizu-Morioka dynamical equations, Chaos synchronization, unidirectionally coupled dynamical systems

1 Introduction

In 1990, Pecora and Carrol [3] introduced the chaos synchronization idea. Since then much attention has been received because of it's fundamental importance in non-linear dynamics and applications in many fields such as, information processing, electronic circuits, secure communication, biological and chemical systems etc. Two or more coupled chaotic systems are called synchronized if their behaviors are closely related. The term identical synchronization between two dynamical systems was introduced by Pecora and Carrol, which states that as time tends to infinity the distance between the corresponding states converge to zero. In 1995, Rulkov *et al.* [5] generalized this concept for unidirectionally coupled dynamical systems. According to them, if there exists a functional relation between the states of the coupled dynamical systems, it is called the generalized synchronization. For a pair of coupled dynamical systems, the coupling is said to be unidirectional if one of the systems is independent of the other. Mathematically, unidirectionally coupled systems can be written as

$$\begin{cases} \dot{\vec{X}} = F(\vec{X}) \\ \dot{\vec{Y}} = G(\vec{X}, \vec{Y}). \end{cases}$$

Differential Geometry - Dynamical Systems, Vol.13, 2011, pp. 101-106.
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Physically, they show that in part of the phase space, the behavior of one of the two systems has no influence on the behavior of the other. To design a channel-independent chaotic secure communication Chua *et al.* [1] used generalized synchronization. In generalized synchronization, if the synchronizing manifold is linear it is called linear generalized synchronization. Poria [4] studied the linear generalized synchronization of chaotic Lorenz systems. In this paper, synchronization of chaotic trajectories of unidirectionally coupled Shimizu-Morioka dynamical systems is studied via linear transformations. To show the efficiency of this method numerical simulation results are presented.

2 Linear generalized Synchronization

Any dynamical system can be expressed in the form

$$\dot{\vec{X}} = A\vec{X} + \phi(\vec{X}). \tag{2.1}$$

where A is n×n constant matrix and $\phi : \mathbb{R}^n \to \mathbb{R}^n$. We call the system (2.1) as driving system. Assuming that the driving system transmitting the signal $\phi(\vec{X})$ to the response system, the unidirectional synchronization scheme can be written as

$$\begin{cases} \dot{\vec{X}} = A\vec{X} + \phi(\vec{X}) & : \textit{Driving System} \\ \dot{\vec{Y}} = A\vec{Y} + B\phi(\vec{X}) & : \textit{Response System} \end{cases} \tag{2.2}$$

where B is n × n matrix. If the matrix B commutes with A, the two dynamical systems in (2.2) are in a state of generalized synchronization via the linear transformation

$$\dot{\vec{Y}} = B\vec{X} \tag{2.3}$$

iff all the eigenvalues of the matrix A have negative real parts. For a given matrix A we can construct the matrix B in many ways so that AB = BA is satisfied. Thus we can construct several methods of linear generalized synchronization between two chaotic systems. In this paper we consider three types of B for which AB = BA:

(i) B is a scalar matrix of the same order as A
(ii) B = A
(iii) B = A^{-1}.

3 Generalized synchronization of Shimizu-Marioka system

In this section we study the linear generalized synchronization of two Shimizu-Morioka systems [2, 8]. The model is described by the dynamical system

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = x_1 - \lambda x_2 - x_1 x_2 \\ \dot{x_3} = \alpha x_3 + {x_1}^2 \end{cases} \tag{3.1}$$

where α and λ are positive parameters.
This system can be decomposed into two parts as

$$\dot{\vec{X}} = A\vec{X} + \phi(\vec{X}). \tag{3.2}$$

where

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -\alpha \end{pmatrix}, \ \vec{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \ \phi(\vec{X}) = \begin{pmatrix} x_1 \\ x_1 - x_1x_3 \\ {x_1}^2 \end{pmatrix}$$

Clearly, A is negative definite matrix as $\lambda > 0$, and $\alpha > 0$. Therefore all the eigenvalues of A are negative. Let us take the response system as

$$\dot{\vec{Y}} = A\vec{Y} + B\phi(\vec{X}). \tag{3.3}$$

If the matrix B commutes with A, then the driving Shimizu-Marioka system (3.2) and the response Shimizu-Marioka system (3.3) are in a state of generalized synchronization.

4 Results and Discussions

Here we consider three forms of B and the corresponding simulation results are presented. Following Shilnikov [6, 7] the values of the parameters of Shimizu-Marioka system are taken as λ= 0.799 and α=0.54. The fourth order Runge-Kutta method is used for solving the coupled driving and response dynamical systems. The initial conditions for the driving and response Shimizu-Marioka Systems are taken as $(-0.1, 0.1, 0)$ and $(0, 0.1, -0.1)$ respectively.

Case 1. In this case we take

$$B = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$$

where $k \neq 0$ is a real number. Clearly, $AB = BA$. Therefore all the conditions for synchronization are satisfied. Our driving Shimizu-Morioka system is (3.2) and the response Shimizu-Marioka system is calculated as

$$\begin{cases} \dot{y_1} = -y_1 + y_2 + kx_1 \\ \dot{y_2} = -\lambda y_2 + k(x_1 - x_1x_3) \\ \dot{y_3} = \alpha y_3 + k{x_1}^2 \end{cases} \tag{4.1}$$

where $\vec{Y} = (y_1, y_2, y_3)^t$.
The simulation results are shown in figures 1.1, 1.2 and 1.3. We have chosen $k = 2$. The state variables of the driving system and the response system are connected by the linear transformations as

$$\begin{cases} y_1 = kx_1 \\ y_2 = kx_2 \\ y_3 = kx_3 \end{cases} \tag{4.2}$$

Case 2. In this case we take B = A = $\begin{pmatrix} -1 & 1 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -\alpha \end{pmatrix}$.

Clearly, $BA = AB$ is satisfied. Thus the response Shimizu-Marioka system is given by

$$\left\{ \begin{array}{l} \dot{y_1} = -y_1 + y_2 + x_1x_3 \\ \dot{y_2} = -\lambda y_2 + \lambda(x_1 - x_1x_3) \\ \dot{y_3} = \ \alpha y_3 + \alpha {x_1}^2 \end{array} \right. \tag{4.3}$$

and the corresponding linear transformation is

$$\left\{ \begin{array}{l} y_1 = -x_1 + x_2 \\ y_2 = -\lambda x_2 \\ y_3 = -\alpha x_3 \end{array} \right. \tag{4.4}$$

The simulation results are shown in figures 2.1, 2.2 and 2.3.

Case 3. Satisfying the condition BA = AB we take B= $A^{-1} = \begin{pmatrix} -1 & -1/\lambda & 0 \\ 0 & -1/\lambda & 0 \\ 0 & 0 & -1/\alpha \end{pmatrix}$.

In this case response Shimizu-Marioka system is

$$\left\{ \begin{array}{l} \dot{y_1} = -y_1 + y_2 - x_1 - (1/\lambda)(x_1 - x_1x_3) \\ \dot{y_2} = -\lambda y_2 - (1/\lambda)(x_1 - x_1x_3) \\ \dot{y_3} = \ -\alpha y_3 - (1/\alpha) {x_1}^2 \end{array} \right. \tag{4.5}$$

and the corresponding linear transformation is

$$\left\{ \begin{array}{l} y_1 = -x_1 - (1/\lambda)x_2 \\ y_2 = -(1/\lambda)x_2 \\ y_3 = -(1/\alpha)x_3 \end{array} \right. \tag{4.6}$$

The simulation results are shown in figures 3.1, 3.2, 3.3.

In the proposed method we obtained the functional relationship between the states of the driving system and the response system. Thus the behavior of the response system can be predicted in advance by knowing the behavior of the driving system. Again the matrix associated with the linear transformation is invertible and hence the behavior of the driving system can be determined by knowing the behavior of the response system.

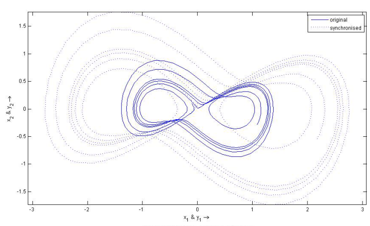

Fig 1.1 Simulation 1. x_1 vs x_2 with y_1 vs y_2

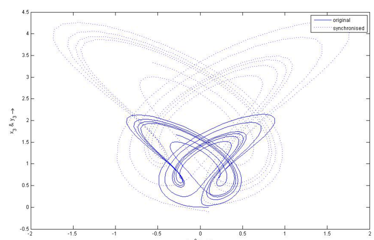

Fig 1.2 Simulation 1. x_2 vs x_3 with y_2 vs y_3

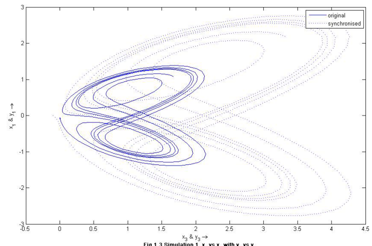

Fig 1.3 Simulation 1. x_3 vs x_1 with y_3 vs y_1

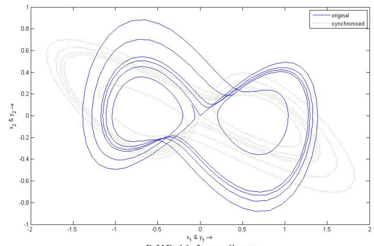

Fig 2.1 Simulation 2 x_1 vs x_2 with y_1 vs y_2

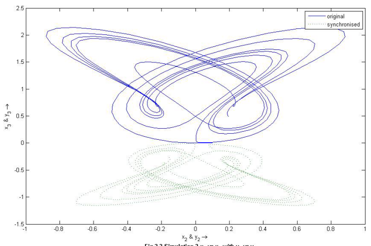

Fig 2.2 Simulation 2 x_2 vs x_3 with y_2 vs y_3

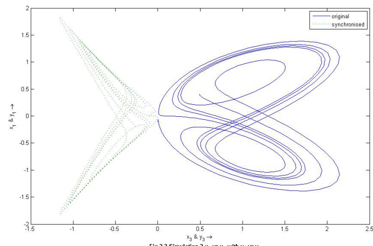

Fig 2.3 Simulation 2 x_1 vs x_3 with y_1 vs y_3

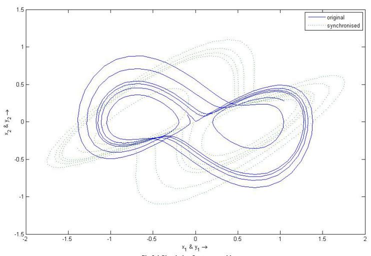

Fig 3.1 Simulation 3. x_2 vs x_1 with y_2 vs y_1

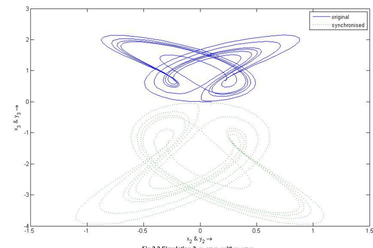

Fig 3.2 Simulation 3. x_3 vs x_2 with y_3 vs y_2

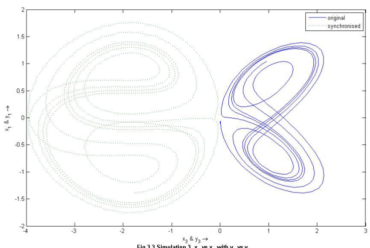

Fig 3.3 Simulation 3. x_1 vs x_3 with y_1 vs y_3

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