Synchronization of coupled hyper-chaotic systems

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Abstract. This paper presents the synchronization strategy for coupled hyperchaotic Chen systems via hybrid feedback control. The synchronization of coupled Chen systems via tracking control is also studied and the success of synchronization schemes is illustrated by numerical simulation results.

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Key words: synchronization; coupled hyperchaotic systems; tracking control.

1 Introduction

Since the pioneer works by Ott, Grebogi and Yorke [6] and Pecorra and Carroll [9], chaos control and synchronization has received increasing attention due to its theoretical challenges and its potential applications to various disciplines. Chaos synchronization is applied in many areas such as secure communication, information processing, biological systems, chemical reactions, neural networks and in engineering. The identical synchronization is a straightforward form of synchronization that may occur when two identical chaotic oscillators are mutually coupled, or when one of them drives the other. Let $(x_1, x_2, ..., x_n)$ and $(x'_1, x'_2, ..., x'_n)$ denote the set of dynamical variables that describe the states of the first and second oscillator, respectively. Identical synchronization occurs when there is a set of initial conditions $[x_1(0), x_2(0), ..., x_n(0)]$, $[x'_1(0), x'_2(0), ..., x'_n(0)]$ such that, (denoting the time by t), $\lim_{t\to\infty} |x'_i(t) - x_i(t)| = 0$, for $i = 1, 2, ..., n$. That means that for time large enough the state variables of the two oscillators satisfy $x'_i(t) = x_i(t)$, for $i = 1, 2, ..., n$. Many interesting investigations on chaos synchronization are done by scientists for the last two decades. Synchronization between two different noise perturbed chaotic system with fully unknown parameters was proposed by Sun Y.Cao [12]. Synchronization of unified chaotic system using adaptive feedback control was studied by Lu and Chen [3]. Poria et.al. [10] have investigated adaptive synchronization of two coupled chaotic neuronal systems. Various modern synchronization methods, such as adaptive control [12]-[17], backstepping design [18, 14], active control [1, 2, 16, 11], nonlinear feedback control [8] has been successfully applied to control chaos or obtain synchronized chaotic systems in recent years.

In this paper, we propose the synchronization scheme of coupled hyperchaotic Chen systems via hybrid feedback control. We also study the synchronization of

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coupled Chen systems via tracking control. We discuss the results both analytically and numerically.

2 Design of a hybrid controller

Any non-linear system of ordinary differential equation can be written as

$$\dot{x} = Ax + B\psi(x) \tag{2.1}$$

where $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$, $\psi(x) : \mathbb{R}^n \to \mathbb{R}^n$ is non-linear vector function. We construct a new system which is coupled with system (2.1) in the following way

$$\dot{y} = Ay + B[\psi(y) + u] \tag{2.2}$$

where u is the controller which control the motion of the system (2.2). The synchronization errors between the systems (2.1) and (2.2) are defined as: $e = (e_1, e_2, e_3)^T = (x_1 - y_1, x_2 - y_2, x_3 - y_3)^T$. Then the time evolution of the synchronization error will obey the following dynamical system

$$\dot{\mathbf{e}} = (A - BK)\mathbf{e} \tag{2.3}$$

In order to make identical synchronization of systems (2.1) and (2.2), the feedback controller u should be appropriately chosen. Assume that $u = u_1 + u_2$ with

$$u_1 = \psi(x) - \psi(y) \tag{2.4}$$

and

$$u_2 = K(x - y), \tag{2.5}$$

where $(x-y) = (x_1 - y_1, x_2 - y_2, x_3 - y_3)^T$ and $K = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{pmatrix}$ is the feedback matrix. Obviously u_1 is a non-linear controller, u_2 is a linear controller, so u is the hybrid controller. Yang.et.al [15] derived the sufficient condition for synchronization between (2.1) and (2.2). They found that if all the eigenvalues of the matrix $A - BK$ has negative real parts, then the error system (2.3) is asymptotically stable at origin. Hence synchronization between systems (2.1) and (2.2) will occur.

3 Synchronization of hyper-chaotic Chen system via hybrid feedback control

The hyper-chaotic Chen system is

$$\begin{cases} \dot{\mathbf{x_1}} = a(x_2 - x_1) + x_4 \\ \dot{\mathbf{x_2}} = dx_1 - x_1x_3 + cx_2 \\ \dot{\mathbf{x_3}} = x_1x_2 - bx_3 \\ \dot{\mathbf{x_4}} = x_2x_3 + rx_4, \end{cases} \tag{3.1}$$

where x_1, x_2, x_3 and x_4 are state variables and the parameters a, b, c, d and r are real constants. In the region of the parameter space where $a = 35, b = 3, c = 12, d = 7.0$ and $0. \leq r \leq .085$, the system (3.1) is chaotic, when $a = 35, b = 3, c = 12$ and $d = 7.0$ and $0.085 \leq r \leq .798$, system (3.1) is hyper-chaotic and when $a = 35, b = 3, c = 12, d = 7.0$ and $.798 \leq r \leq .9$, system shows periodic behavior. The system (3.1) can be written as

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{pmatrix} = \begin{pmatrix} -a & a & 0 & 1 \\ d & c & 0 & 0 \\ 0 & 0 & -b & 0 \\ 0 & 0 & 0 & r \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -x_1x_3 \\ x_1x_2 \\ x_2x_3 \end{pmatrix}$$

From this system we identify the matrix $A = \begin{pmatrix} -a & a & 0 & 1 \\ d & c & 0 & 0 \\ 0 & 0 & -b & 0 \\ 0 & 0 & 0 & r \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

and $\psi(x) = \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \\ x_2x_3 \end{pmatrix}$.

Then $A - BK = \begin{pmatrix} -a-k_1 & a & 0 & 1 \\ d & c-k_2 & 0 & 0 \\ 0 & 0 & -b-k_3 & 0 \\ 0 & 0 & 0 & r-k_4 \end{pmatrix}$.

By the theorem of Routh when $k_1 = 20$, $k_2 = 30$, $k_3 = 2$, $k_4 = 2$, the sufficient condition for synchronization is satisfied. Based on equations (2.4) and (2.5), the controllers for the Chen system are chosen as

$$u_1 = \begin{pmatrix} y_1y_3 - x_1x_3 \\ x_1x_2 - y_1y_2 \\ x_2x_3 - y_2y_3 \end{pmatrix} \text{ and } u_2 = \begin{pmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{pmatrix} \begin{pmatrix} x_1 - y_1 \\ x_2 - y_2 \\ x_3 - y_3 \\ x_4 - y_4 \end{pmatrix}.$$

Therefore the response system is the following

$$\begin{cases} \dot{y_1} = -ay_1 + ay_2 + y_4 + 20(x_1 - y_1) \\ \dot{y_2} = dy_1 + cy_2 - x_1x_3 + 30(x_2 - y_2) \\ \dot{y_3} = -by_3 + x_1x_2 + 2(x_3 - y_3) \\ \dot{y_4} = ry_4 + x_2x_3 + 2(x_4 - y_4). \end{cases} \tag{3.2}$$

4 Results

Numerical simulation are done for the parameter values $a = 35, b = 3, c = 12, d = 7, r = 1.09$ using fourth order Runge-Kutta method. The initial conditions of system (3.1) and (3.2) are selected as $x(0) = (1, -6, 0, -2)$ and $y(0) = (-7, 2, -10, 2)$. So the initial values of error system are $e(0) = (8, -8, 10, -4)$. The trajectories of the x_i, state of the drive system and y_i, state of the response system, for $i = 1, 2, 3, 4$ which are shown in fig.1. Time evolution of the synchronization errors is shown in Fig.2. It is clear from the figure that synchronization error goes to zero as We

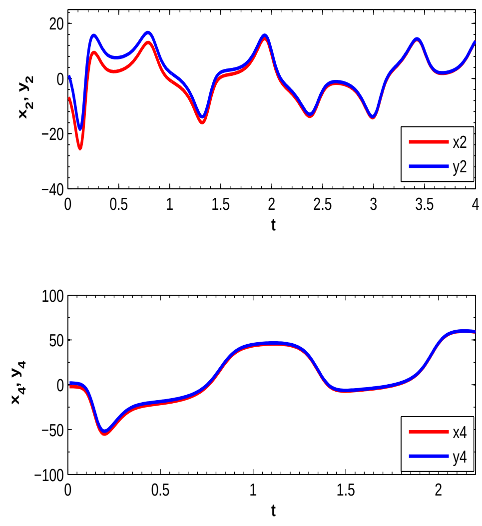

Figure 1: Time evolution of states of drive and the corresponding states of the response system are shown $x_i, y_i, i = 1, 2, 3, 4$.

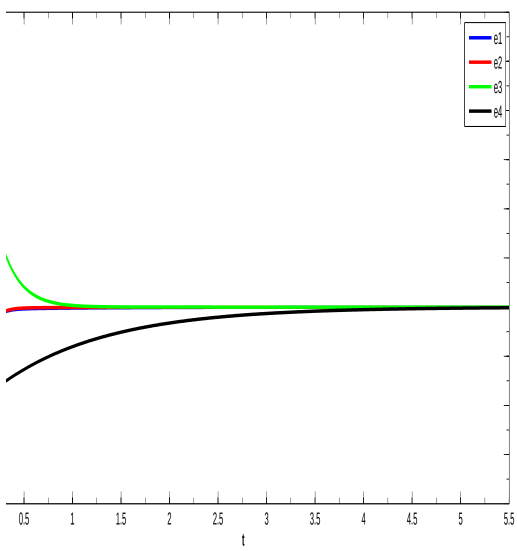

Figure 2: Time evolution of the synchronization error are shown.

observe that synchronization errors go to zero after some time. Therefore the identical synchronization between the coupled chaotic Chen systems is achieved.

5 Design of tracking controller

Any system of ordinary differential equations can be written as

$$\dot{x} = Ax + B\psi(x), \tag{5.1}$$

where $x \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$ are matrices and vectors of system parameters, $\psi(x) : \mathbb{R}^n \to \mathbb{R}^n$ is non-linear vector function. We assume that system (5.1) is the drive system. By introducing the control variable $\mathbf{U} \in \mathbb{R}^n$, the controlled response system is given by

$$\dot{y} = Ay + B\psi(y) + \mathbf{U}, \tag{5.2}$$

where $y \in \mathbb{R}^n$ denote the state vectors of the response system. Our aim is to design a controller $\mathbf{U}$ which synchronizes the state of both drive and response system. We substract (5.1) from (5.2) and we get

$$\dot{e} = Ae + B(\psi(y) - \psi(x)) + \mathbf{U},$$

where $e = y - x$. Now our aim is to make $\lim_{t\to\infty} ||e(t)|| = 0$. Then let the Lyapunov function be $V(e) = \frac{1}{2}e^T e$, where $V(e)$ is a positive definite function. Assuming that the parameters of the drive and response system are known and state of both systems are measurable. We may achieve the synchronization by selecting the controller $\mathbf{U}$ to make the first derivative $V(e)$ i.e $\dot{V}(e) < 0$. Then the states of the response and drive system synchronized globally asymptotically.

6 Synchronization of hyper-chaotic Chen systems via tracking control

Let the dynamical system (3.1) be the drive system; then the controlled response Chen hyper-chaotic system is given by

$$\begin{cases} \dot{y_1} = -ay_1 + ay_2 + y_4 + u_1 \\ \dot{y_2} = dy_1 + cy_2 - y_1y_3 + u_2 \\ \dot{y_3} = y_1y_2 - by_3 + u_3 \\ \dot{y_4} = ry_4 + y_2y_3 + u_4. \end{cases}$$

Let us define the error as

$$e_i = y_i - x_i \quad (i = 1, 2, 3, 4).$$

Therefore the error system is the following dynamical system

$$\begin{cases} \dot{e_1} = -ae_1 + ae_2 + e_4 + u_1 \\ \dot{e_2} = de_1 + ce_2 + x_1x_3 - y_1y_3 + u_2 \\ \dot{e_3} = y_1y_2 - x_1x_2 - be_3 + u_3 \\ \dot{e_4} = re_4 + y_2y_3 - x_2x_3 + u_4. \end{cases}$$

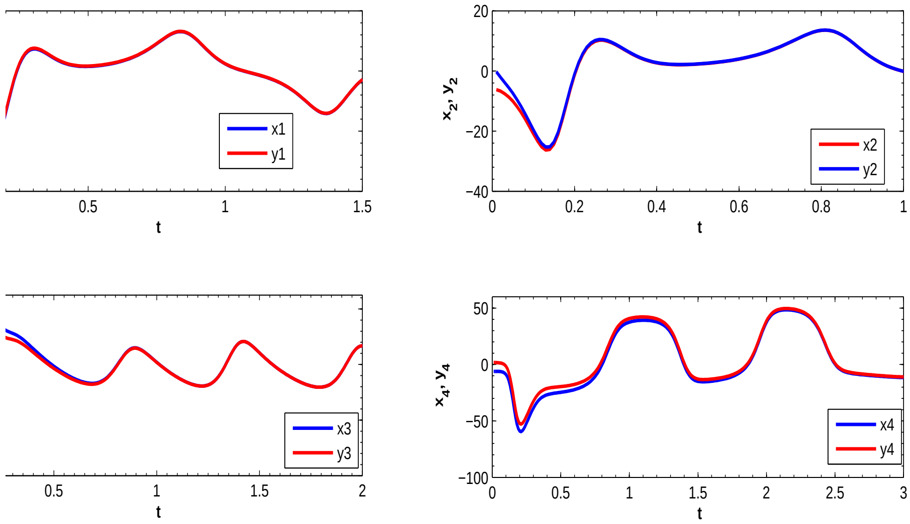

Figure 3: Time evolution of states of drive and the corresponding states of the response system are shown $x_i, y_i, i = 1, 2, 3, 4$.

If we choose the controller as

$$\begin{cases} u_1 = -ae_2 - e_4 \\ u_2 = -de_1 - (c+1)e_2 - x_1x_3 + y_1y_3 \\ u_3 = -y_1y_2 + x_1x_2 \\ u_4 = -(r+1)e_4 - y_2y_3 + x_2x_3. \end{cases}$$

Then the synchronization between the coupled hyper-chaotic Chen systems will occur via tracking control.

7 Results

Numerical simulations are done with the parameters $a = 35, b = 3, c = 12, d = 7, r = 1.09$. The initial conditions of the systems (3.1) and (3.2) are selected as $x(0) = (1, -6, 0, -2)$ and $y(0) = (-7, 2, -10, 2)$. So the initial values of error system are $e(0) = (8, -8, 10, -4)$. The trajectories of the x_i, states of the drive system and y_i, states of the response system, for $i = 1, 2, 3, 4$ are shown in fig.3. Time evolution of the synchronization errors is plotted in fig.4. All error goes to zero with time. Therefore, the synchronization between chaotic systems (3.1) and (3.2) is achieved.

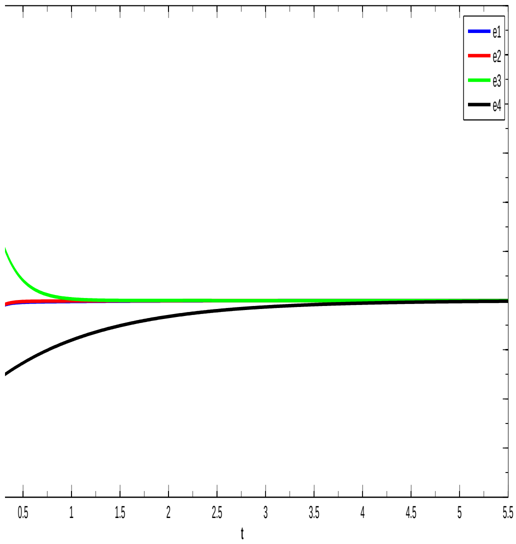

Figure 4: Time evolution of the synchronization error are shown.

8 Conclusions

We have successfully applied hybrid feedback control and tracking control techniques for synchronization of coupled hyper chaotic Chen systems. Both the above synchronization schemes are based on Lyapunov stability theorem. This results may be useful for electrical engineers and for sending secrete messages via chaotic encription.

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