

Dynamical connections on graded jet bundles

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Abstract. We introduce the concept of a dynamical connection on a graded jet bundle $J^{1,1}(\mathbb{R}^{1|1}, (M, \mathcal{A}))$ in terms of an almost tangent structure. We show that the dynamical connection and its associated semispray on $J_G^{1,1}(\pi)$ have the same paths. We study Lagrangian systems with constraints in the graded case. If L is a regular Lagrangian on $J_G^1\pi$, then in the presence of a constraint, a geometrical model will be introduced for the description of dynamical systems represented by graded Euler-Lagrange vector fields. At the end of the article we study the systems for which the constraints can be defined in terms of a connection.

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Key words: Semispray; dynamical connection; graded jet bundle; non-holonomic Lagrangian.

1 Introduction

The concept of connections on tangent bundles in terms of the almost tangent geometry was proposed by Grifone [2]. The situation on $\mathbb{R} \times TM$ was studied by Leon and Rodrigues. They have shown in [7] that for any semispray on $\mathbb{R} \times TM$ there is so-called dynamical connection with the same paths. The key point in the theory of connections is the use of a $(1, 1)$ - tensor field J , called almost tangent structure. It was showed that there is a one-to-one correspondence between the sprays and nonlinear connections, i.e. if we have a semispray on a manifold M (resp. $\mathbb{R} \times TM$) then we have a nonlinear connection on M (resp. $\mathbb{R} \times TM$) and vice versa.

In supergeometry, Vacaru introduced the nonlinear connection structures in vector superbundles but it was not studied on graded jet bundles [14]. The construction of graded jet bundles is introduced (see detailed study and basic references in [3]). Now, we want to transport the geometric structures defined on the tangent superbundle to graded jet bundles like the almost tangent structure. According to [8], some geometric structures, like Poincaré-Cartan form, lives on an intermediate graded bundle between $J^1(\mathbb{R}^{1|1}, (M, \mathcal{A}))$ and $J^2(\mathbb{R}^{1|1}, (M, \mathcal{A}))$, denoted by $J^{1,1}(\mathbb{R}^{1|1}, (M, \mathcal{A}))$, so we will define our structure on this bundle. We first define an almost tangent structure on $J^{1,1}(\mathbb{R}^{1|1}, (M, \mathcal{A}))$ and then by using it the dynamical connection will be defined such that its paths are just the paths of its associated semispray.

32 One of the important problems of theoretical mechanics is the study of constrained
 33 Lagrangian systems [1, 10, 11]. A geometric setting in terms of jet manifolds was
 34 developed for time-dependent non-holonomic Lagrangian systems (see [6], [7]). In
 35 this case, the Lagrangian function was supposed to be defined on the fibred manifold
 36 $J^1\pi$ over $E, \pi : E \mapsto \mathbb{R}$ and the constraints were obtained as the evaluation maps of
 37 a local cobasis of a distribution on E .

38 The aim of this paper is to study Lagrangian systems with constraints in the graded
 39 case: Let (M, \mathcal{A}) be a graded manifold and $J_G^1\pi$ the graded first-order jet bundle. If
 40 L is a regular Lagrangian on $J_G^1\pi$, then in the presence of a constraint, a geometrical
 41 model will be introduced for the description of dynamical systems represented by
 42 graded Euler-Lagrange vector fields. At the end of the article we study the systems
 43 for which the constraints can be defined in terms of a connection.

44 2 An almost tangent structure on $J^{1,1}(\mathbb{R}^{1|1}, (M, \mathcal{A}))$

45 We work in the category of C^∞ graded manifolds (see [5] and [15]) and definitions
 46 are taken from [5] and [9]. Let us recall some notations for graded coordinates. On
 47 a graded manifold (M, \mathcal{A}) of graded dimension (m, n) , positive indices are used for
 48 even coordinates: $x^i, i = 1, \dots, m$, and negative indices for odd coordinates: $x^i : i =$
 49 $-n, \dots, -1$. The coordinates of the graded manifold $\mathbb{R}^{1|1}$ are denoted by (t, s) , with
 50 $|t| = 0$ and $|s| = 1$.

51 The graded fibered coordinates on the graded first-order jet bundle $J^1(\mathbb{R}^{1|1}, (M, \mathcal{A}))$
 52 of local sections of $p_1 : \mathbb{R}^{1|1} \times (M, \mathcal{A}) \rightarrow \mathbb{R}^{1|1}$ are defined in [3] and [4] and denoted
 53 by $(t, s, x^a, x_t^a, x_s^a)$, $a = -n, \dots, -1, 1, \dots, m$. This coordinate system shows that the
 54 graded 1-jet bundle has dimension $(1 + 2m + n, 1 + m + 2n)$. Also the coordinates
 55 on $J^2(\mathbb{R}^{1|1}, (M, \mathcal{A}))$ are denoted by $(t, s, x^a, x_t^a, x_s^a, x_{tt}^a, x_{st}^a)$, $a = -n, \dots, -1, 1, \dots, m$,
 56 (compare with [12]). In [8], the authors produce a new definition of the Poincaré-
 57 Cartan form and demonstrate a bijection between the Berezinian critical sections of
 58 the corresponding action functional and the extremals of the Poincaré-Cartan form (in
 59 the regular case). The important point is that this Poincaré-Cartan form lives on an
 60 intermediate graded bundle between $J^1(\mathbb{R}^{1|1}, (M, \mathcal{A}))$ and $J^2(\mathbb{R}^{1|1}, (M, \mathcal{A}))$, denoted
 61 by $J^{1,1}(\mathbb{R}^{1|1}, (M, \mathcal{A})) \subset J^1(\mathbb{R}^{1|0}, J^1(\mathbb{R}^{0|1}, (M, \mathcal{A})))$ and defined by $s_t = 0$. Based on
 62 such statements, we try to introduce a dynamical connection on this bundle.

63 We notice that the role that $J^1(\mathbb{R}, M)$ plays in the classical case is played by
 64 $J^{1,1}(\mathbb{R}^{1|1}, (M, \mathcal{A}))$ in the graded case, but there is an outstanding difference: whereas
 65 $J^1(\mathbb{R}, M)$ is equal to $\mathbb{R} \times TM$, this is no longer true in the graded case. The sub-bundle
 66 $J^{1,1}(\mathbb{R}^{1|1}, (M, \mathcal{A}))$ is not the product of $\mathbb{R}^{1|1}$ times the supertangent.

67 Let $J^{1,1}(\mathbb{R}^{1|1}, (M, \mathcal{A}))$ be the graded subbundle of $J^2(\mathbb{R}^{1|1}, (M, \mathcal{A}))$ with local
 68 coordinates $(t, s, x^a, x_t^a, x_s^a, x_{st}^a)$. For the brevity we will write $J^{1,1}(\mathbb{R}^{1|1}, (M, \mathcal{A}))$ as
 69 $J_G^{1,1}(\pi)$. Consider the 1-form on $J_G^{1,1}(\pi)$ (called the contact 1-forms)

$$\Phi^a := dx^a - dt.x_t^a - ds.x_s^a, \quad \Psi^a := dx_s^a - dt.x_{st}^a.$$

70 A graded semispray is a projectable vector field $X \in \chi_G(J_G^{1,1}(\pi))$ such that its
 71 integral curves are 1|1-jet prolongations, i.e.

$$(2.1) \quad i_X(dt) = 1 = i_X(ds), \quad i_X(\Phi^a) = 0 = i_X(\Psi^a),$$

72 thus a graded semispray is given locally by

$$\xi = \frac{\partial}{\partial t} + \frac{\partial}{\partial s} + (x_t^a + x_s^a) \frac{\partial}{\partial x^a} + A^a \frac{\partial}{\partial x_t^a} + x_{st}^a \frac{\partial}{\partial x_s^a} + B^a \frac{\partial}{\partial x_{st}^a},$$

73 where A^a, B^a are superfunctions on $J_G^{1,1}(\pi)$ with degree $|x^a|$ and $|x^a| + 1$ respectively,
74 as follow [9],

$$\begin{aligned} A^a &= (-(-1)^{|x^b|}) \frac{d}{ds} \left(\frac{\partial L}{\partial x_s^b} + \frac{\partial L}{\partial x_t^b} \right) + \frac{\partial L}{\partial x^b} - x_t^a \frac{\partial^2 L}{\partial x^a \partial x_t^b} \\ &\quad + (-1)^{|x^b|} x_s^a \frac{\partial^2 L}{\partial x^a \partial x_t^b} - x_{st}^a \frac{\partial^2 L}{\partial x_s^a \partial x_t^b} \omega^{ab}, \\ B^a &= (-x_{st}^a \frac{\partial^2 L}{\partial x^a \partial x_t^b} + (-1)^{|x^b|}) \frac{d}{ds} \left(\frac{d}{ds} \left(\frac{\partial L}{\partial x_s^b} + \frac{\partial L}{\partial x_t^b} \right) \right) \omega^{ab}. \end{aligned}$$

75 We notice that ω^{ab} is the inverse matrix of the matrix $\omega_{ab} = \frac{\partial^2 L}{\partial x^a \partial x^b}$.

76 In the classical case, nonlinear connections on $E = \mathbb{R} \times TM$ are defined as distri-
77 butions complementary to the vertical distribution and the torsion and curvature of
78 such connections are defined using the almost tangent structure on E . This tensor
79 field defined as $\tilde{J} = J - C \otimes dt$, where J is the almost tangent structure and C is the
80 Liouville vector field on TM . The local form of it shows that it is the tensor product
81 of the vertical vector fields on E and the contact forms. It must be mention that the
82 paths of this connection and its associated semispray are the same. But in graded
83 case the situation is different.

84 We define two tensor fields \tilde{J}, J on $J_G^{1,1}(\pi)$ by

$$\begin{aligned} \tilde{J} &= (dx^a - dt.x_t^a - ds.x_s^a) \otimes \frac{\partial}{\partial x_t^a} + (dx_s^a + dt.x_{st}^a) \otimes \frac{\partial}{\partial x_s^a}, \\ (2.2) \quad J &= dx^a \cdot \frac{\partial}{\partial x_t^a} + dx_s^a \cdot \frac{\partial}{\partial x_s^a}, \end{aligned}$$

85 which are locally characterized by

$$\begin{aligned} \tilde{J}\left(\frac{\partial}{\partial t}\right) &= -x_t^a \frac{\partial}{\partial x_t^a} + x_{st}^a \frac{\partial}{\partial x_s^a}, & \tilde{J}\left(\frac{\partial}{\partial s}\right) &= -x_s^a \frac{\partial}{\partial x_t^a}, & \tilde{J}\left(\frac{\partial}{\partial x^a}\right) &= \frac{\partial}{\partial x_t^a}, \\ \tilde{J}\left(\frac{\partial}{\partial x_t^a}\right) &= 0, & \tilde{J}\left(\frac{\partial}{\partial x_s^a}\right) &= \frac{\partial}{\partial x_s^a}, & \tilde{J}\left(\frac{\partial}{\partial x_{st}^a}\right) &= 0, \end{aligned}$$

86 and

$$\begin{aligned} J\left(\frac{\partial}{\partial t}\right) &= 0, & J\left(\frac{\partial}{\partial s}\right) &= 0, & J\left(\frac{\partial}{\partial x^a}\right) &= \frac{\partial}{\partial x_t^a}, \\ J\left(\frac{\partial}{\partial x_t^a}\right) &= 0, & J\left(\frac{\partial}{\partial x_s^a}\right) &= \frac{\partial}{\partial x_s^a}, & J\left(\frac{\partial}{\partial x_{st}^a}\right) &= 0. \end{aligned}$$

87 Let $c : \mathbb{R}^{1|1} \rightarrow (M, \mathcal{A})$ be a graded curve and $c(t, s) = (x^a(t, s))$, then its 1|1-jet
88 prolongation is a section, denoted by $j^{1,1}c$, of $J_G^{1,1}(\pi)$. The graded curve $j^{1,1}c(t, s)$ is

an integral curve of a vector field denoted by $\widehat{j^{1,1}c}$ and has a local expression:

$$\widehat{j^{1,1}c} = \frac{\partial}{\partial t} + \frac{\partial}{\partial s} + (x_t^a + x_s^a) \frac{\partial}{\partial x^a} + (x_{tt}^a + x_{st}^a) \frac{\partial}{\partial x_t^a} + x_{st}^a \frac{\partial}{\partial x_s^a} + x_{tst}^a \frac{\partial}{\partial x_{st}^a}.$$

Let ξ be a semispray on $J_G^{1,1}(\pi)$. A curve c in (M, \mathcal{A}) is called a path of ξ if its canonical prolongation is an integral curve of ξ . So c is a path of ξ iff satisfies the following non-autonomous system of differential equations

$$\begin{cases} A^a(j^{1,1}c) = x_{tt}^a + x_{st}^a \\ B^a(j^{1,1}c) = x_{tst}^a \end{cases}$$

where ξ given by (2.1).

Consider the vector field $\frac{d}{ds}$ on $J^1(\mathbb{R}^{1|1}, (M, \mathcal{A}))$, which is the horizontal (or total) lifting of $\frac{\partial}{\partial s}$ as a vector field on (M, \mathcal{A}) and its local expression is $\frac{d}{ds} = \frac{\partial}{\partial s} + x_s^a \frac{\partial}{\partial x^a} + x_{st}^a \frac{\partial}{\partial x_t^a}$. We take $\mathcal{L}_{\frac{d}{ds}}^G$ and go over the graded bundle $J^2(\mathbb{R}^{1|1}, (M, \mathcal{A}))$. Define

$$\Theta^L := \mathcal{L}_{\frac{d}{ds}}^G \Theta_0^L$$

so that

$$\Theta^L = (dx_s^a - dt.x_{st}^a) \frac{\partial L}{\partial x_t^a} + (-1)^{|x^a|} (dx^a - dt.x_t^a - ds.x_s^a) \frac{d}{ds} \frac{\partial L}{\partial x_t^a} + dt. \frac{dL}{ds}.$$

Now, applying d to this expression, we will arrive at the Poincare-Cartan 2-form on $J^2(\mathbb{R}^{1|1}, (M, \mathcal{A}))$ (with the appropriate condition of regularity in L)

$$\Omega_L = d\Theta^L.$$

So the local expression of Ω_L is

$$\begin{aligned} \Omega_L = & dt dx_{st}^a \cdot \frac{\partial L}{\partial x_t^a} - (dx_s^a - dt.x_{st}^a) d\left(\frac{\partial L}{\partial x_t^a}\right) + (-1)^{|x^a|} (dt dx_t^a - ds dx_s^a) \frac{d}{ds} \frac{\partial L}{\partial x_t^a} \\ & - (-1)^{|x^a|} (dx^a - dt.x_t^a - ds.x_s^a) d\left(\frac{d}{ds} \frac{\partial L}{\partial x_t^a}\right) - dt d\left(\frac{dL}{ds}\right). \end{aligned}$$

Also the equations of motion can be written as follows

$$i_X \Omega_L = 0, \quad i_X dt = 1, \quad i_X ds = 1.$$

If L is regular, then the above equations have a unique solution, named the graded Euler-Lagrange vector field, which is a graded vector field on $J^{1|1}(\mathbb{R}^{1|1}, (M, \mathcal{A}))$.

In this paper we consider the case of a Lagrangian system subjected to a family of constraints. In this case the constrained equations of motion are considered by (5.1).

3 Dynamical connection on $J_G^{1,1}(\pi)$

In this section, we define a dynamical connection on the graded manifold $J_G^{1,1}(\pi)$. Another approach to the theory of nonlinear connection on the tangent superbundle has been studied by Vacaru (see [13] and [14]).

Definition 3.1. A dynamical connection on $J_G^{1,1}(\pi)$ is a tensor field Γ of type (1,1) on $J_G^{1,1}(\pi)$ which has the following conditions

- i) for vertical graded vector fields on $J_G^{1,1}(\pi)$, $\Gamma(X) = -X$,
- ii) for non-vertical graded vector fields on $J_G^{1,1}(\pi)$,

$$(3.1) \quad J\Gamma = \tilde{J}\Gamma = \tilde{J}, \quad \Gamma\tilde{J} = -\tilde{J}, \quad \Gamma J = -J,$$

A vertical graded vector field is a graded vector field V on $J_G^{1,1}(\pi)$ that is written locally as $V = A_1^a \frac{\partial}{\partial x_t^a} + A_2^a \frac{\partial}{\partial x_s^a} + A_3^a \frac{\partial}{\partial x_{st}^a}$.

By a straightforward computation from definition 3.1, we deduce that the local expressions of Γ are

$$(3.2) \quad \begin{aligned} \Gamma\left(\frac{\partial}{\partial t}\right) &= -x_t^a \frac{\partial}{\partial x^a} + \Gamma^a \frac{\partial}{\partial x_t^a} + x_{st}^a \frac{\partial}{\partial x_s^a} + \bar{\Gamma}^a \frac{\partial}{\partial x_{st}^a}, \\ \Gamma\left(\frac{\partial}{\partial s}\right) &= -x_s^a \frac{\partial}{\partial x^a} + \Gamma_1^a \frac{\partial}{\partial x_t^a} + \bar{\Gamma}_1^a \frac{\partial}{\partial x_{st}^a}, \\ \Gamma\left(\frac{\partial}{\partial x^a}\right) &= \frac{\partial}{\partial x^a} + \Gamma_a^b \frac{\partial}{\partial x_t^b} + \bar{\Gamma}_a^c \frac{\partial}{\partial x_{st}^c}, \\ \Gamma\left(\frac{\partial}{\partial x_t^a}\right) &= -\frac{\partial}{\partial x_t^a}, \quad \Gamma\left(\frac{\partial}{\partial x_s^a}\right) = -\frac{\partial}{\partial x_s^a}, \quad \Gamma\left(\frac{\partial}{\partial x_{st}^a}\right) = -\frac{\partial}{\partial x_{st}^a}, \end{aligned}$$

the functions $\Gamma^a, \bar{\Gamma}^a, \Gamma_1^a, \bar{\Gamma}_1^a, \Gamma_a^b, \bar{\Gamma}_a^c$ are superfunctions on $J_G^{1,1}(\pi)$ and will be called the component of the connection Γ . We notice that Γ is an even endomorphism and $|\Gamma^a| = |\bar{\Gamma}_1^a| = |x^a|$, $|\Gamma_1^a| = |\bar{\Gamma}^a| = 1 + |x^a|$, $|\Gamma_a^b| = 0$, $|\bar{\Gamma}_a^c| = 1$.

From (3.2) we easily deduce that $\Gamma^3 - \Gamma = 0$ and the eigenvalues corresponding to Γ are $\{0, 1, -1\}$. The eigenspace corresponding to the eigenvalue 1 will be denoted by H (called the strong-horizontal subspace) is as follow

$$H = \langle H^a \rangle = \text{span}\left\{ \frac{\partial}{\partial x^a} + \frac{1}{2}\Gamma_a^b \frac{\partial}{\partial x_t^b} + \frac{1}{2}\bar{\Gamma}_a^c \frac{\partial}{\partial x_{st}^c} \right\}.$$

The eigenspace corresponding to the eigenvalue -1 is the following vertical subspace

$$V = \langle V_1^a \rangle \oplus \langle V_2^a \rangle \oplus \langle V_3^a \rangle = \text{span}\left\{ \frac{\partial}{\partial x_t^a} \right\} \oplus \text{span}\left\{ \frac{\partial}{\partial x_s^a} \right\} \oplus \text{span}\left\{ \frac{\partial}{\partial x_{st}^a} \right\}.$$

Also we know that the eigenspace corresponding to 0 is $\ker\Gamma$ and will be denoted by K and is as follow

$$\begin{aligned}
K &= \langle \xi_1 \rangle \oplus \langle \xi_2 \rangle \\
&= \text{span}\left\{ \frac{\partial}{\partial t} + x_t^a \frac{\partial}{\partial x^a} + (\Gamma^a + x_t^b \Gamma_b^a) \frac{\partial}{\partial x_t^a} + x_{st}^a \frac{\partial}{\partial x_s^a} + (\bar{\Gamma}^a + x_t^c \bar{\Gamma}_c^a) \frac{\partial}{\partial x_{st}^a} \right\} \\
&\quad \oplus \text{span}\left\{ \frac{\partial}{\partial s} + x_s^a \frac{\partial}{\partial x^a} + (\Gamma_1^a + x_s^b \Gamma_b^a) \frac{\partial}{\partial x_t^a} + (\bar{\Gamma}_1^a + x_s^c \bar{\Gamma}_c^a) \frac{\partial}{\partial x_{st}^a} \right\}.
\end{aligned}$$

128 It is easy to see that the graded vector field $\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + (x_t^a + x_s^a) \frac{\partial}{\partial x^a} + (\Gamma^a + \Gamma_1^a +$
129 $x_t^b \Gamma_b^a + x_s^b \Gamma_b^a) \frac{\partial}{\partial x_t^a} + x_{st}^a \frac{\partial}{\partial x_s^a} + (\bar{\Gamma}^a + \bar{\Gamma}_1^a + x_t^c \bar{\Gamma}_c^a + x_s^c \bar{\Gamma}_c^a) \frac{\partial}{\partial x_{st}^a}$ of $K = \text{Ker}\Gamma$ is a semispray
130 which will be called the graded semispray associated to the dynamical connection.

131 Now, let $\chi(J_G^{1,1}(\pi))$ be the set of graded vector fields on $J_G^{1,1}(\pi)$ so we have a
132 canonical decomposition

$$(3.3) \quad \chi(J_G^{1,1}(\pi)) = K \oplus H \oplus V.$$

133 Let $K \oplus H = H'$ which H' will be called weak-horizontal subspace.

134 **Theorem 3.1.** *Let Γ be a dynamical connection on $J_G^{1,1}(\pi)$ then there exists a sub-*
135 *space $K \oplus H = H'$ such that the Whitney sum (3.3) holds.*

136 A graded vector field X on $J_G^{1,1}(\pi)$ belongs to H (resp. H') will be called a strong
137 (resp. weak) horizontal graded vector field.

138 From definition 3.1 and by a straightforward computation, we obtain

$$\begin{aligned}
\left(\frac{\partial}{\partial t}\right)^{H'} &= \frac{\partial}{\partial t} + (\Gamma^b + \frac{1}{2}x_t^a \Gamma_a^b) \frac{\partial}{\partial x_t^b} + x_{st}^a \frac{\partial}{\partial x_s^a} + (\bar{\Gamma}^c + \frac{1}{2}x_t^a \bar{\Gamma}_a^c) \frac{\partial}{\partial x_{st}^c}, \\
\left(\frac{\partial}{\partial s}\right)^{H'} &= \frac{\partial}{\partial s} + (\Gamma_1^b + \frac{1}{2}x_s^a \Gamma_a^b) \frac{\partial}{\partial x_t^b} + (\bar{\Gamma}_1^c + \frac{1}{2}x_s^a \bar{\Gamma}_a^c) \frac{\partial}{\partial x_{st}^c}, \\
\left(\frac{\partial}{\partial x^a}\right)^{H'} &= \frac{\partial}{\partial x^a} + \frac{1}{2}\Gamma_a^b \frac{\partial}{\partial x_t^b} + \frac{1}{2}\bar{\Gamma}_a^c \frac{\partial}{\partial x_{st}^c}.
\end{aligned}$$

139 It is easy to see that the dynamical connection, Γ , may be characterized by its hori-
140 zontal form

$$\begin{aligned}
h_\Gamma &= dt \otimes \left(\frac{\partial}{\partial t} + (\Gamma^b + \frac{1}{2}x_t^a \Gamma_a^b) \frac{\partial}{\partial x_t^b} + x_{st}^a \frac{\partial}{\partial x_s^a} + (\bar{\Gamma}^c + \frac{1}{2}x_t^a \bar{\Gamma}_a^c) \frac{\partial}{\partial x_{st}^c} \right) \\
&\quad + ds \otimes \left(\frac{\partial}{\partial s} + (\Gamma_1^b + \frac{1}{2}x_s^a \Gamma_a^b) \frac{\partial}{\partial x_t^b} + (\bar{\Gamma}_1^c + \frac{1}{2}x_s^a \bar{\Gamma}_a^c) \frac{\partial}{\partial x_{st}^c} \right) \\
&\quad + dx^a \otimes \left(\frac{\partial}{\partial x^a} + \frac{1}{2}\Gamma_a^b \frac{\partial}{\partial x_t^b} + \frac{1}{2}\bar{\Gamma}_a^c \frac{\partial}{\partial x_{st}^c} \right).
\end{aligned}$$

141 If we put $H^a = \left(\frac{\partial}{\partial x^a}\right)^{H'}$ and $V_1^a = \frac{\partial}{\partial x_t^a}, V_2^a = \frac{\partial}{\partial x_s^a}, V_3^a = \frac{\partial}{\partial x_{st}^a}$, one deduces that
142 $\{\xi_1, \xi_2, H^a, V_1^a, V_2^a, V_3^a\}$ is a local basis of vector fields on $J_G^{1,1}(\pi)$ and is called an
143 adapted basis to Γ .

144 We notice that the dual local basis of 1-forms of the adapted basis is given by
145 $\{\theta_1, \theta_2, \theta_3^a, \theta_4^a, \theta_5^a, \theta_6^a\}$ where $\theta_1 = dt, \theta_2 = ds, \theta_3^a = dx^a - dt.x_t^a - ds.x_s^a$, and

$$\begin{aligned}
 \theta_4^a &= -dt.(\Gamma^a + \frac{1}{2}x_t^b\Gamma_b^a) - ds(\Gamma_1^a + \frac{1}{2}x_s^b\Gamma_b^a) - dx^b.(\frac{1}{2}\Gamma_b^a) + dx_t^a, \\
 \theta_5^a &= -dt.x_{st}^a + dx_s^a, \\
 \theta_6^a &= -dt.(\bar{\Gamma}^a + \frac{1}{2}x_t^c\bar{\Gamma}_c^a) - ds(\bar{\Gamma}_1^a + \frac{1}{2}x_s^c\bar{\Gamma}_c^a) - dx^c.(\frac{1}{2}\bar{\Gamma}_c^a) + dx_{st}^a.
 \end{aligned}$$

146 With respect to these dual 1-forms, we can define a path of a dynamical connection
147 Γ on $J_G^{1,1}(\pi)$.

148 **Definition 3.2.** A graded curve $c : \mathbb{R}^{1|1} \mapsto (M, \mathcal{A})$ is called a path of Γ if the
149 1|1-prolongation is a weak-horizontal curve with respect to Γ , which means $j^{\widehat{1,1}}c(\theta_i) =$
150 0 for $i = 4, 5, 6$.

151 **Theorem 3.2.** A dynamical connection and its associated semispray on $J_G^{1,1}(\pi)$ have
152 the same paths.

153 *Proof.* In order to prove that the dynamical connection and its associated semispray
154 have the same paths it suffices to characterize the system of second-order differential
155 equations associated to path of the dynamical connection. Thus, if $c : \mathbb{R}^{1|1} \mapsto (M, \mathcal{A})$
156 is a path of Γ , then the following system of second-order differential equations is
157 satisfied along the section induced by c

$$(3.4) \quad \begin{cases} \Gamma^a + \Gamma_1^a + (x_t^b + x_s^b)\Gamma_b^a = x_{tt}^a + x_{st}^a, \\ \bar{\Gamma}^a + \bar{\Gamma}_1^a + (x_t^c + x_s^c)\bar{\Gamma}_c^a = x_{tst}^a. \end{cases}$$

158 On the other hand, let ξ be the associated semispray of Γ , i.e. $\xi \in \text{Ker}\Gamma$, then ξ is
159 locally given by

$$\xi = \frac{\partial}{\partial t} + \frac{\partial}{\partial s} + (x_t^a + x_s^a)\frac{\partial}{\partial x^a} + A^a\frac{\partial}{\partial x_t^a} + x_{st}^a\frac{\partial}{\partial x_s^a} + B^a\frac{\partial}{\partial x_{st}^a}$$

160 where $A^a = \Gamma^a + \Gamma_1^a + (x_t^b + x_s^b)\Gamma_b^a$, $B^a = \bar{\Gamma}^a + \bar{\Gamma}_1^a + (x_t^c + x_s^c)\bar{\Gamma}_c^a$. From (3.4) it is clear
161 that the paths of Γ and ξ satisfy the same system of differential equations. \square

162 4 Curvature of a dynamical connection

163 We can associate to Γ a horizontal operator h as follow:

$$h = \frac{1}{2}(I + \Gamma)\Gamma^2$$

164 which is locally given by

$$\begin{aligned}
 h\left(\frac{\partial}{\partial t}\right) &= -x_t^a\frac{\partial}{\partial x^a} - \frac{1}{2}x_t^b\Gamma_b^a\frac{\partial}{\partial x_t^a} - \frac{1}{2}x_t^c\bar{\Gamma}_c^a\frac{\partial}{\partial x_{st}^a}, \\
 h\left(\frac{\partial}{\partial s}\right) &= -x_s^a\frac{\partial}{\partial x^a} - \frac{1}{2}x_s^b\Gamma_b^a\frac{\partial}{\partial x_t^a} - \frac{1}{2}x_s^c\bar{\Gamma}_c^a\frac{\partial}{\partial x_{st}^a}, \\
 h\left(\frac{\partial}{\partial x^a}\right) &= \frac{\partial}{\partial x^a} + \frac{1}{2}\Gamma_b^a\frac{\partial}{\partial x_t^b} + \frac{1}{2}\bar{\Gamma}_c^a\frac{\partial}{\partial x_{st}^c}, \\
 h\left(\frac{\partial}{\partial x_s^a}\right) &= h\left(\frac{\partial}{\partial x_t^a}\right) = h\left(\frac{\partial}{\partial x_{st}^a}\right) = 0
 \end{aligned}$$

165 and in terms of adapted basis to Γ we have

$$h\xi_1 = h\xi_2 = 0, \quad hH^a = H^a, \quad hV_1^a = hV_2^a = hV_3^a = 0.$$

166 So the curvature of Γ is given by

$$R(X, Y) = [hX, hY] + h[X, Y] - h[hX, Y] - h[X, hY], \quad X, Y \in \chi(J_G^{1,1}(\pi)).$$

167 where h is the horizontal operator.

168 **Theorem 4.1.** *If h is the horizontal operator, then the horizontal, vertical and*
 169 *horizontal-vertical components of the curvature tensor are given as follows:*

$$\begin{aligned} R(\xi_1, \xi_2) &= (-\Gamma_1^a - x_s^b \Gamma_b^a + x_{st}^a) \frac{\partial}{\partial x^a} + \frac{1}{2} (-\Gamma_1^a - x_s^b \Gamma_b^a + x_{st}^a) \Gamma_a^b \frac{\partial}{\partial x_t^b} \\ &\quad + \frac{1}{2} (-\Gamma_1^a - x_s^b \Gamma_b^a + x_{st}^a) \bar{\Gamma}_a^c \frac{\partial}{\partial x_{st}^c}, \\ R(\xi_1, V_1^a) &= -H^a, \quad R(\xi_2, V_2^a) = (-1)^{|x^a|} H^a, \\ R(H^a, H^b) &= \frac{1}{2} \frac{\partial \Gamma_b^d}{\partial x^a} \frac{\partial}{\partial x_t^d} + \frac{1}{2} \frac{\partial \bar{\Gamma}_b^e}{\partial x^a} \frac{\partial}{\partial x_{st}^e} + (-1)^{|x^b||x^f|} \frac{1}{2} \frac{\partial \Gamma_a^f}{\partial x^b} \frac{\partial}{\partial x_t^f} + \frac{1}{4} \Gamma_a^f \frac{\partial \Gamma_b^d}{\partial x_t^f} \frac{\partial}{\partial x_t^d} \\ &\quad - (-1)^{|x^d||x^f|} \frac{1}{4} \Gamma_b^d \frac{\partial \Gamma_a^f}{\partial x_t^d} \frac{\partial}{\partial x_t^f} + \frac{1}{4} \Gamma_a^f \frac{\partial \bar{\Gamma}_b^e}{\partial x_t^f} \frac{\partial}{\partial x_{st}^e} - (-1)^{|x^e||x^f|} \frac{1}{4} \bar{\Gamma}_b^e \frac{\partial \Gamma_a^f}{\partial x_{st}^e} \frac{\partial}{\partial x_t^f} \\ &\quad - (-1)^{|x^b||x^g|} \frac{1}{2} \frac{\partial \bar{\Gamma}_a^g}{\partial x^b} \frac{\partial}{\partial x_{st}^g} + \frac{1}{4} \bar{\Gamma}_a^g \frac{\partial \Gamma_b^d}{\partial x_{st}^g} \frac{\partial}{\partial x_t^d} - (-1)^{|x^g||x^d|} \frac{1}{4} \Gamma_b^d \frac{\partial \bar{\Gamma}_a^g}{\partial x_t^d} \frac{\partial}{\partial x_{st}^g} \\ &\quad + \frac{1}{4} \bar{\Gamma}_a^g \frac{\partial \bar{\Gamma}_b^e}{\partial x_{st}^g} \frac{\partial}{\partial x_{st}^e} - (-1)^{|x^e||x^g|} \frac{1}{4} \bar{\Gamma}_b^e \frac{\partial \bar{\Gamma}_a^g}{\partial x_{st}^e} \frac{\partial}{\partial x_{st}^g} \end{aligned}$$

170 For the remaining set of the basic graded vector fields, the curvature tensor is zero.

171 *Proof.* A straightforward computation needs to prove the theorem. \square

172 5 Non-holonomic Lagrangian mechanics

173 Suppose that $L : J_G^1(\pi) \rightarrow \mathbb{R}^{1,1}$ is a regular Lagrangian subjected to a set of non-
 174 holonomic constraints given by a distribution D on $\mathbb{R}^{1,1} \times T(M, \mathcal{A})$. The constraint is
 175 used means that we have accepted only those curves that belong to D and $D \cap J_G^{1,1}\pi \neq$
 176 \emptyset .

177 If D^0 is the annihilator of D and $\{\mu_i, \gamma_i\}$ be a local basis of D^0 , i.e.

$$D^0 = \langle \mu_i, \gamma_i \rangle$$

178 such that

$$\begin{aligned} \mu_i &= dx^a \cdot \mu_a^i + dt \cdot h_\mu^i + ds \cdot k_\mu^i + dx_t^i, \\ \gamma_i &= dx^a \cdot \gamma_a^i + dt \cdot h_\gamma^i + ds \cdot k_\gamma^i + dx_s^i, \end{aligned}$$

179 then the complete lifts of μ_i and γ_i to $T(\mathbb{R}^{1|1} \times T(M, \mathcal{A}))$ are

$$\begin{aligned}\mu_i^c &= dx^a \cdot \bar{X}(\mu_a^i) + dx_t^a \cdot \mu_a^i + dx_s^a \cdot \mu_a^i + dt \cdot \bar{X}(h_\mu^i) + d\tau \cdot h_\mu^i + ds \cdot \bar{X}(k_\mu^i) \\ &\quad + d\varsigma \cdot k_\mu^i + dx_{tt}^i + dx_{ts}^i, \\ \gamma_i^c &= dx^a \cdot \bar{X}(\gamma_a^i) + dx_t^a \cdot \gamma_a^i + dx_s^a \cdot \gamma_a^i + dt \cdot \bar{X}(h_\gamma^i) + d\tau \cdot h_\gamma^i + ds \cdot \bar{X}(k_\gamma^i) \\ &\quad + d\varsigma \cdot k_\gamma^i + dx_{st}^i,\end{aligned}$$

180 where \bar{X} is as follow

$$\bar{X} = \tau \frac{\partial}{\partial t} + \varsigma \frac{\partial}{\partial s} + (x_t^a + x_s^a) \frac{\partial}{\partial x^a} + (x_{tt}^a + x_{ts}^a) \frac{\partial}{\partial x_t^a} + x_{st}^a \frac{\partial}{\partial x_s^a}.$$

181 Also, their restriction to $J_G^{1,1}(\pi)$ are given by

$$\begin{aligned}\mu_i^c / J_G^{1,1}\pi &= dx^a \cdot \bar{Y}(\mu_a^i) + dx_t^a \cdot \mu_a^i + dx_s^a \cdot \mu_a^i + dt \cdot \bar{Y}(h_\mu^i) + ds \cdot \bar{Y}(k_\mu^i) + dx_{tt}^i + dx_{ts}^i, \\ \gamma_i^c / J_G^{1,1}\pi &= dx^a \cdot \bar{Y}(\gamma_a^i) + dx_t^a \cdot \gamma_a^i + dx_s^a \cdot \gamma_a^i + dt \cdot \bar{Y}(h_\gamma^i) + ds \cdot \bar{Y}(k_\gamma^i) + dx_{st}^i,\end{aligned}$$

182 where

$$\bar{Y} = \frac{\partial}{\partial t} + \frac{\partial}{\partial s} + (x_t^a + x_s^a) \frac{\partial}{\partial x^a} + (x_{tt}^a + x_{ts}^a) \frac{\partial}{\partial x_t^a} + x_{st}^a \frac{\partial}{\partial x_s^a}.$$

183 *Remark.* As we mentioned before, $J_G^{1,1}(\pi)$ is an intermediate graded bundle between
184 $J_G^1(\pi)$ and $J_G^2(\pi)$, so we restricted the complete lift of μ_i and γ_i to $J_G^{1,1}(\pi)$. Thus we
185 are forced to consider the distribution D on $\mathbb{R}^{1|1} \times T(M, \mathcal{A})$. For this reason we did
186 not consider the distribution D on $\mathbb{R}^{1|1} \times (M, \mathcal{A})$.

187 We put $\bar{\mu}_i = \tilde{J}^*(\mu_i^c / J_G^{1,1}\pi)$ and $\bar{\gamma}_i = \tilde{J}^*(\gamma_i^c / J_G^{1,1}\pi)$. The following relations can be
188 used to rewrite $\bar{\mu}_i$ and $\bar{\gamma}_i$. So from

$$\begin{aligned}\tilde{J}^*(dt) &= \tilde{J}^*(ds) = \tilde{J}^*(dx^a) = 0, \\ \tilde{J}^*(dx_t^a) &= dx^a - dt \cdot x_t^a + ds \cdot x_s^a, \\ \tilde{J}^*(dx_s^a) &= dx_s^a - dt \cdot x_{st}^a,\end{aligned}$$

189 we get

$$\begin{aligned}\bar{\mu}_i &= (dx^a - dt \cdot x_t^a + ds \cdot x_s^a + dx_s^a - dt \cdot x_{st}^a) \mu_a^i, \\ \bar{\gamma}_i &= (dx^a - dt \cdot x_t^a + ds \cdot x_s^a + dx_s^a - dt \cdot x_{st}^a) \gamma_a^i.\end{aligned}$$

190 Now, let $\phi_i = (\hat{\mu}_i) / J_G^{1,1}\pi$ and $\psi_i = (\hat{\gamma}_i) / J_G^{1,1}\pi$ be the restriction of the functions $\hat{\mu}_i$
191 and $\hat{\gamma}_i$ to $J_G^{1,1}\pi$ respectively. We define evaluation functions $\hat{\mu}_i$ and $\hat{\gamma}_i$ on $T(\mathbb{R}^{1|1} \times$
192 $T(M, \mathcal{A}))$ by

$$\begin{aligned}\hat{\mu}_i(X) &= \langle \bar{X}, \mu_i \rangle = \tau \cdot h_\mu^i + \varsigma \cdot k_\mu^i + (x_t^a + x_s^a) \mu_a^i + x_{tt}^i + x_{ts}^i, \\ \hat{\gamma}_i(X) &= \langle \bar{X}, \gamma_i \rangle = \tau \cdot h_\gamma^i + \varsigma \cdot k_\gamma^i + (x_t^a + x_s^a) \gamma_a^i + x_{st}^i.\end{aligned}$$

193 So, we deduce that

$$\begin{aligned}\phi_i &= \hat{\mu}_i(X) / J_G^{1,1}\pi = h_\mu^i + k_\mu^i + (x_t^a + x_s^a) \mu_a^i + x_{tt}^i + x_{ts}^i, \\ \psi_i &= \hat{\gamma}_i(X) / J_G^{1,1}\pi = h_\gamma^i + k_\gamma^i + (x_t^a + x_s^a) \gamma_a^i + x_{st}^i,\end{aligned}$$

194 which are the usual form of the constraints functions.

195 Now, the constrained motion equations (along the points of $\tilde{D} = D \cap J_G^{1,1}\pi$) can
196 be written as follows

$$(5.1) \quad i_X \Omega_L = \lambda^i \bar{\mu}_i + \lambda'^i \bar{\gamma}_i, \quad i_X dt = 1, i_X ds = 1, \quad X(\phi_i) = 0, X(\psi_i) = 0.$$

197 For solving equation (5.1), first of all we get Z_i^1 and Z_i^2 from the following equation

$$(5.2) \quad i_{Z_i^1} \Omega_L = \bar{\mu}_i, \quad i_{Z_i^2} \Omega_L = \bar{\gamma}_i \quad i_{Z_i^1} dt = i_{Z_i^2} dt = 1, i_{Z_i^1} ds = i_{Z_i^2} ds = 1.$$

198 By solving the equation (5.2) we conclude that Z_i^1 and Z_i^2 are vertical graded vector
199 fields and in a local coordinate system, we have

$$Z_i^1 = a_i^1 \frac{\partial}{\partial x_t^i} + b_i^1 \frac{\partial}{\partial x_{st}^i}, \quad Z_i^2 = a_i^2 \frac{\partial}{\partial x_t^i} + b_i^2 \frac{\partial}{\partial x_{st}^i},$$

$$a_i^1 = (-1)^{(|L|+|L||x^a|)} \mu_i \omega^{ia},$$

$$b_i^1 = (-1)^{(|x^a|+|L||x^a|)} (\mu_i - (-1)^{(|x^i|+|L|+|L||x^a|)} (\mu_i) \omega^{ia} \frac{d}{ds} (\omega_{ia})) \omega^{ia},$$

$$a_i^2 = (-1)^{(|L|+|L||x^a|)} \gamma_i \omega^{ia},$$

$$b_i^2 = (-1)^{(|x^a|+|L||x^a|)} (\gamma_i - (-1)^{(|x^i|+|L|+|L||x^a|)} (\gamma_i) \omega^{ia} \frac{d}{ds} (\omega_{ia})) \omega^{ia}.$$

200 So, the graded vector field X satisfying equation (5.1) is

$$X = \xi - \alpha^i Z_i^1 - \beta^i Z_i^2,$$

201 with following conditions

$$X(\phi_i) = 0 \Rightarrow \xi(\phi_i) - \alpha^i Z_i^1(\phi_i) - \beta^i Z_i^2(\phi_i) = 0,$$

$$X(\psi_i) = 0 \Rightarrow \xi(\psi_i) - \alpha^i Z_i^1(\psi_i) - \beta^i Z_i^2(\psi_i) = 0,$$

202 where ξ is (2.1).

203 6 Constraints defined by dynamical connections

204 Let L be a Lagrangian function $L : J_G^1\pi \rightarrow \mathbb{R}^{1|1}$ subjected to non-holonomic
205 constraints given by the horizontal distribution H' of Γ . This means that the only
206 allowable motions are weak horizontal curves.

207 We will construct a suitable basis for H' . If we denote by $X^{H'}$ the weak horizontal
208 lift of a vector field X on $\mathbb{R}^{1|1} \times (M, \mathcal{A})$ to $J_G^{1,1}\pi$, we obtain
209

$$\begin{aligned} \left(\frac{\partial}{\partial t}\right)^{H'} &= \frac{\partial}{\partial t} + (\Gamma^b + \frac{1}{2}x_t^a \Gamma_a^b) \frac{\partial}{\partial x_t^b} + x_{st}^a \frac{\partial}{\partial x_s^a} + (\bar{\Gamma}^c + \frac{1}{2}x_t^a \bar{\Gamma}_a^c) \frac{\partial}{\partial x_{st}^c}, \\ \left(\frac{\partial}{\partial s}\right)^{H'} &= \frac{\partial}{\partial s} + (\Gamma_1^b + \frac{1}{2}x_s^a \Gamma_a^b) \frac{\partial}{\partial x_t^b} + (\bar{\Gamma}_1^c + \frac{1}{2}x_s^a \bar{\Gamma}_a^c) \frac{\partial}{\partial x_{st}^c}, \\ \left(\frac{\partial}{\partial x^a}\right)^{H'} &= \frac{\partial}{\partial x^a} + \frac{1}{2}\Gamma_a^b \frac{\partial}{\partial x_t^b} + \frac{1}{2}\bar{\Gamma}_a^c \frac{\partial}{\partial x_{st}^c}. \end{aligned}$$

210 Thus we have

$$H' = \text{span}\left\{\left(\frac{\partial}{\partial t}\right)^{H'}, \left(\frac{\partial}{\partial s}\right)^{H'}, \left(\frac{\partial}{\partial x^a}\right)^{H'}\right\}$$

211 and

$$\left\{\left(\frac{\partial}{\partial t}\right)^{H'}, \left(\frac{\partial}{\partial s}\right)^{H'}, \left(\frac{\partial}{\partial x^a}\right)^{H'}, \frac{\partial}{\partial x_t^a}, \frac{\partial}{\partial x_s^a}, \frac{\partial}{\partial x_{st}^a}\right\}$$

212 is a local basis of the set of vector fields on $J_G^{1,1}\pi$. A straightforward computation
213 shows that the dual local basis of 1-forms is given by $\{\theta_1, \theta_2, \theta_a, \mu_a, \gamma_a, \eta_a\}$ where
214 $\theta_1 = dt, \theta_2 = ds, \theta_a = dx^a - dt.x_t^a - ds.x_s^a$, and

$$\begin{aligned}\mu_a &= -dt.(\Gamma^a + \frac{1}{2}x_t^b\Gamma_b^a) - ds(\Gamma_1^a + \frac{1}{2}x_s^b\Gamma_b^a) - dx^b.(\frac{1}{2}\Gamma_b^a) + dx_t^a, \\ \gamma_a &= -dt.x_{st}^a + dx_s^a, \\ \eta_a &= -dt.(\bar{\Gamma}^a + \frac{1}{2}x_t^c\bar{\Gamma}_c^a) - ds(\bar{\Gamma}_1^a + \frac{1}{2}x_s^c\bar{\Gamma}_c^a) - dx^c.(\frac{1}{2}\bar{\Gamma}_c^a) + dx_{st}^a.\end{aligned}$$

215 Moreover, we have

$$(H')^0 = \text{span}\{\mu_a, \gamma_a, \eta_a\}.$$

216 Now, suppose that for $u \in J_G^{1,1}\pi$, X_u is as follow

$$X_u = \tau \frac{\partial}{\partial t} + \varsigma \frac{\partial}{\partial s} + (x_t^a + x_s^a) \frac{\partial}{\partial x^a} + (x_{tt}^a + x_{st}^a) \frac{\partial}{\partial x_t^a} + x_{st}^a \frac{\partial}{\partial x_s^a} + x_{tst}^a \frac{\partial}{\partial x_{st}^a},$$

217 and μ_a^c, η_a^c the complete lift of μ_a, η_a to $TJ_G^{1,1}\pi$ as follow

$$\begin{aligned}\mu_a^c &= -dt.X_u(\Gamma^a + \frac{1}{2}x_t^b\Gamma_b^a) - d\tau(\Gamma^a + \frac{1}{2}x_t^b\Gamma_b^a) - ds.X_u(\Gamma_1^a + \frac{1}{2}x_s^b\Gamma_b^a) \\ &\quad - d\varsigma(\Gamma_1^a + \frac{1}{2}x_s^b\Gamma_b^a) - dx^b.X_u(\frac{1}{2}\Gamma_b^a) - dx_t^b(\frac{1}{2}\Gamma_b^a) - dx_s^b(\frac{1}{2}\Gamma_b^a) + dx_{tt}^a + dx_{st}^a, \\ \eta_a^c &= -dt.X_u(\bar{\Gamma}^a + \frac{1}{2}x_t^c\bar{\Gamma}_c^a) - d\tau(\bar{\Gamma}^a + \frac{1}{2}x_t^c\bar{\Gamma}_c^a) - ds.X_u(\bar{\Gamma}_1^a + \frac{1}{2}x_s^c\bar{\Gamma}_c^a) \\ &\quad - d\varsigma(\bar{\Gamma}_1^a + \frac{1}{2}x_s^c\bar{\Gamma}_c^a) - dx^c.X_u(\frac{1}{2}\bar{\Gamma}_c^a) - dx_t^c(\frac{1}{2}\bar{\Gamma}_c^a) - dx_s^c(\frac{1}{2}\bar{\Gamma}_c^a) + dx_{tst}^a.\end{aligned}$$

218 Hence, their restriction to $J_G^{1,1}\pi$ are given by

$$\begin{aligned}\mu_c^a / J_G^{1,1}\pi &= -dt.Y_u(\Gamma^a + \frac{1}{2}x_t^b\Gamma_b^a) - ds.Y_u(\Gamma_1^a + \frac{1}{2}x_s^b\Gamma_b^a) - dx^b.Y_u(\frac{1}{2}\Gamma_b^a) \\ &\quad - dx_t^b(\frac{1}{2}\Gamma_b^a) - dx_s^b(\frac{1}{2}\Gamma_b^a) + dx_{tt}^a + dx_{st}^a, \\ \eta_c^a / J_G^{1,1}\pi &= -dt.Y_u(\bar{\Gamma}^a + \frac{1}{2}x_t^c\bar{\Gamma}_c^a) - ds.Y_u(\bar{\Gamma}_1^a + \frac{1}{2}x_s^c\bar{\Gamma}_c^a) - dx^c.Y_u(\frac{1}{2}\bar{\Gamma}_c^a) \\ &\quad - dx_t^c(\frac{1}{2}\bar{\Gamma}_c^a) - dx_s^c(\frac{1}{2}\bar{\Gamma}_c^a) + dx_{tst}^a,\end{aligned}$$

219 where

$$Y_u = \frac{\partial}{\partial t} + \frac{\partial}{\partial s} + (x_t^a + x_s^a) \frac{\partial}{\partial x^a} + (x_{tt}^a + x_{st}^a) \frac{\partial}{\partial x_t^a} + x_{st}^a \frac{\partial}{\partial x_s^a} + x_{tst}^a \frac{\partial}{\partial x_{st}^a}.$$

220 We put $\bar{\mu}_a = \tilde{J}^*(\mu_c^a/J^{1,1}\pi)$ and $\bar{\eta}_a = \tilde{J}^*(\eta_c^a/J^{1,1}\pi)$. Thus, we get

$$\begin{aligned}\bar{\mu}_a &= -\frac{1}{2}(dx^b - dt.x_t^b - ds.x_s^b + dx_s^b - dt.x_{st}^b)\Gamma_b^a, \\ \bar{\eta}_a &= -\frac{1}{2}(dx^c - dt.x_t^c - ds.x_s^c + dx_s^c - dt.x_{st}^c)\bar{\Gamma}_c^a.\end{aligned}$$

221 To obtain constraint functions we define the functions $\hat{\mu}_a : TJ_G^{1,1}\pi \rightarrow \mathbb{R}$ and $\hat{\eta}_a :$
222 $TJ_G^{1,1}\pi \rightarrow \mathbb{R}$ by

$$\hat{\mu}_a(X) = \langle \mu_a(u), X \rangle, \quad \hat{\eta}_a(X) = \langle \eta_a(u), X \rangle, \quad \forall X \in T_u J_G^{1,1}\pi,$$

223 So, we have

$$\begin{aligned}\hat{\mu}_a(X) &= -\tau(\Gamma^a + \frac{1}{2}x_t^b\Gamma_b^a) - \varsigma(\Gamma_1^a + \frac{1}{2}x_s^b\Gamma_b^a) - \frac{1}{2}(x_t^b + x_s^b)\Gamma_b^a + x_{tt}^a + x_{st}^a, \\ \hat{\eta}_a(X) &= -\tau(\bar{\Gamma}^a + \frac{1}{2}x_t^c\bar{\Gamma}_c^a) - \varsigma(\bar{\Gamma}_1^a + \frac{1}{2}x_s^c\bar{\Gamma}_c^a) - \frac{1}{2}(x_t^c + x_s^c)\bar{\Gamma}_c^a + x_{tst}^a.\end{aligned}$$

224 Thus, their restriction to $J_G^{1,1}\pi$ are just constraint functions ω_a and λ_a as follow

$$\begin{aligned}\omega_a &= -\Gamma^a - \Gamma_1^a - (x_t^b + x_s^b)\Gamma_b^a + x_{tt}^a + x_{st}^a, \\ \lambda_a &= -\bar{\Gamma}^a - \bar{\Gamma}_1^a - (x_t^c + x_s^c)\bar{\Gamma}_c^a + x_{tst}^a.\end{aligned}$$

225 Now, the constrained motion equations can be written as follows

$$i_X\Omega_L = \alpha^a\bar{\mu}_a + \beta^a\bar{\eta}_a, \quad i_X(dt) = 1 = i_X(ds), \quad X(\omega_a) = 0, X(\lambda_a) = 0$$

226 along the points of $\tilde{H}' = H' \cap J_G^{1,1}\pi$.

227 We deduce that the solutions of X satisfy the Euler-Lagrange superequations as

$$\frac{\partial L}{\partial x^a} - \frac{d}{dt} \frac{\partial L}{\partial x_t^a} - (-1)^{|x^a|} \frac{d}{ds} \frac{\partial L}{\partial x_s^a} = -\frac{1}{2}\alpha^a\Gamma_b^a - \frac{1}{2}\beta^a\bar{\Gamma}_c^a.$$

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