

On the conformal change of five-dimensional Finsler spaces

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1 **Abstract.** The purpose of the present paper is to deal with the theory of
2 conformal change in five-dimensional Finsler space. We have obtained the
3 conditions under which the h - and v - connection vectors are conformally
4 invariant in five-dimensional Finsler space.

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6 **Key words:** Finsler space; Conformal change; h and v -connection vectors; main
7 scalars.

8 1 Introduction

9 The conformal change and conformal transformation of n -dimensional Finsler spaces
10 have been studied in [6] and [1]. The conformal theory of two, three & four-dimensional
11 Finsler spaces have been discussed in [2, 8, 10] respectively. As far as author knows
12 there is no paper concerned with the conformal theory of five-dimensional Finsler
13 space. Recently, the present author has found that in a five-dimensional Finsler space
14 there are seventeen main scalars $H, I, J, K, H', I', J', K', H'', I'', J'', K'', M, M', M'',$
15 N, N' [11] in which the sum of H, I, K and M is LC , which is called unified main
16 scalar. It has been also shown by the present author that in a five-dimensional Finsler
17 space there exist six v -connection vectors $u_i, v_i, w_i, u'_i, v'_i, w'_i$ and six h -connection
18 vectors $h_i, J_i, k_i, h'_i, J'_i, k'_i$ [11]. The theory of five-dimensional Finsler space with constant
19 unified main scalars has been discussed in [12]. The orthonormal frame field
20 $(l^i, m^i, n^i, p^i, q^i)$, called the Miron frame plays an important role in five-dimensional
21 Finsler space. Here we discuss the theory of conformal change in five-dimensional
22 Finsler space.

23 2 Scalar components in Miron frame

24 Let F^5 be a five-dimensional Finsler space with fundamental function $L(x, y)$. The
25 metric tensor g_{ij} and Cartan C -tensor C_{ijk} of F^5 are defined by

26
$$g_{ij} = \frac{1}{2} \partial_i \partial_j L^2, \quad C_{ijk} = \frac{1}{2} \partial_k g_{ij} = \frac{1}{4} \partial_i \partial_j \partial_k L^2.$$

Throughout the paper, the symbols $\partial_i = \frac{\partial}{\partial y^i}$ and $\partial_i = \frac{\partial}{\partial x^i}$ have been used. The frame $\{e_{(\alpha)}^i\}$, $\alpha = 1, 2, 3, 4, 5$ is called the Miron frame of F^5 , where $e_{(1)}^i = l^i = \frac{y^i}{L}$ is called the normalized supporting element, $e_{(2)}^i = m^i = \frac{C^i}{C}$ is called the normalized torsion vector, $e_{(3)}^i = n^i, e_{(4)}^i = p^i, e_{(5)}^i = q^i$ are constructed from $g_{ij}e_{(\alpha)}^i e_{(\beta)}^j = \delta_{\alpha\beta}$. Here, C is the length of torsion vector $C_i = C_{ijk}g^{jk}$. The Greek letters $\alpha, \beta, \gamma, \delta$ vary from 1 to 5 throughout the paper. Summation convention is applied for both the Greek and Latin indices. In the Miron frame an arbitrary tensor can be expressed by scalar components along the unit vectors l^i, m^i, n^i, p^i, q^i . For instance; let $T = T_j^i$ be a tensor field of (1, 1) type, then the scalar components $T_{\alpha\beta}$ of T are defined by $T_{\alpha\beta} = T_j^i e_{(\alpha)i} e_{(\beta)}^j$ and the components T_j^i of the tensor T are expressed as $T_j^i = T_{\alpha\beta} e_{(\alpha)}^i e_{(\beta)j}$. From the equation $g_{ij}e_{(\alpha)}^i e_{(\beta)}^j = \delta_{\alpha\beta}$, we have

$$(2.1) \quad g_{ij} = l_i l_j + m_i m_j + n_i n_j + p_i p_j + q_i q_j$$

Next, the C -tensor $C_{ijk} = \frac{1}{2}\partial_k g_{ij}$ satisfies $C_{ijk}l^i = 0$ and is symmetric in i, j, k . Therefore, if $C_{\alpha\beta\gamma}$ are scalar components of LC_{ijk} i. e., if

$$(2.2) \quad LC_{ijk} = C_{\alpha\beta\gamma} e_{(\alpha)i} e_{(\beta)j} e_{(\gamma)k},$$

then, we have

$$(2.3) \quad \begin{aligned} LC_{ijk} = & C_{222}m_i m_j m_k + C_{223} \sum_{(ijk)} (m_i m_j n_k) + C_{233} \sum_{(ijk)} (m_i n_j n_k) \\ & + C_{333} (n_i n_j n_k) + C_{224} \sum_{(ijk)} (m_i m_j p_k) + C_{444} (p_i p_j p_k) \\ & + C_{244} \sum_{(ijk)} (m_i p_j p_k) + C_{225} \sum_{(ijk)} (m_i m_j q_k) + C_{255} \sum_{(ijk)} (m_i q_j q_k) \\ & + C_{555} (q_i q_j q_k) + C_{334} \sum_{(ijk)} (n_i n_j p_k) + C_{344} \sum_{(ijk)} (n_i p_j p_k) \\ & + C_{335} \sum_{(ijk)} (n_i n_j q_k) + C_{355} \sum_{(ijk)} (n_i q_j q_k) + C_{445} \sum_{(ijk)} (p_i p_j q_k) \\ & + C_{455} \sum_{(ijk)} (p_i q_j q_k) + C_{234} \sum_{(ijk)} \{m_i (n_j p_k + n_k p_j)\} \\ & + C_{235} \sum_{(ijk)} \{m_i (n_j q_k + n_k q_j)\} + C_{245} \sum_{(ijk)} \{m_i (p_j q_k + p_k q_j)\} \\ & + C_{345} \sum_{(ijk)} \{n_i (p_j q_k + p_k q_j)\}, \end{aligned}$$

where $\sum_{(ijk)} \{...\}$ denote the cyclic interchange of i, j, k and summation.

For instance; $\sum_{(ijk)} \{A_i B_j C_k\} = A_i B_j C_k + A_j B_k C_i + A_k B_i C_j$.

Contracting (2.2) with g^{jk} , we get $LCm_i = C_{\alpha\beta\beta}e_{(\alpha)i}$. Thus, if we put

$$(2.4) \quad \begin{aligned} C_{222} &= H, C_{233} = I, C_{244} = K, C_{333} = J, C_{344} = J', C_{444} = H', \\ C_{334} &= I', C_{234} = K', C_{255} = M, C_{355} = J'', C_{455} = M', C_{555} = H'', \\ C_{335} &= I'', C_{445} = K'', C_{235} = N, C_{245} = N', C_{345} = M'', \\ C_{224} &= -(H' + I' + M'), \quad C_{225} = -(H'' + I'' + K''). \end{aligned}$$

then, we have

$$(2.5) \quad H + I + K + M = LC, \quad C_{223} = -(J + J' + J''),$$

40 Hence, we have the following:

41 **Theorem 2.1.** *In a five-dimensional Finsler space there are seventeen main scalars*
 42 *$H, I, J, K, H', I', J', K', H'', I'', J'', K'', M, M', M'', N, N'$ in which the sum of H, I, K*
 43 *and M is LC which is called unified main scalar.*

Using (2.4) and (2.5), the equation (2.3) can be rewritten as [11]

$$(2.6) \quad \begin{aligned} LC_{ijk} &= Hm_i m_j m_k - (J + J' + J'') \sum_{(ijk)} (m_i m_j n_k) + I \sum_{(ijk)} (m_i n_j n_k) \\ &+ J (n_i n_j n_k) - (H' + I' + M') \sum_{(ijk)} (m_i m_j p_k) + H' (p_i p_j p_k) \\ &+ K \sum_{(ijk)} (m_i p_j p_k) - (H'' + I'' + K'') \sum_{(ijk)} (m_i m_j q_k) \\ &+ M \sum_{(ijk)} (m_i q_j q_k) + H'' (q_i q_j q_k) + I' \sum_{(ijk)} (n_i n_j p_k) + J' \sum_{(ijk)} (n_i p_j p_k) \\ &+ I'' \sum_{(ijk)} (n_i n_j q_k) + J'' \sum_{(ijk)} (n_i q_j q_k) + K'' \sum_{(ijk)} (p_i p_j q_k) \\ &+ M' \sum_{(ijk)} (p_i q_j q_k) + K' \sum_{(ijk)} \{m_i (n_j p_k + n_k p_j)\} + N \sum_{(ijk)} \{m_i (n_j q_k + n_k q_j)\} \\ &+ N' \sum_{(ijk)} \{m_i (p_j q_k + p_k q_j)\} + M'' \sum_{(ijk)} \{n_i (p_j q_k + p_k q_j)\}. \end{aligned}$$

The Cartan's connection $CT = (\Gamma_{jk}^i, G_j^i, C_{jk}^i)$ will be used in the following section of this paper. The h - and v -covariant derivatives of the frame field $e_{(\alpha)i}$ are given by [4]

$$(2.7) \quad e_{(\alpha)i;j} = H_{(\alpha)\beta\gamma} e_{(\beta)i} e_{(\gamma)j}, \quad Le_{(\alpha)i;j} = V_{(\alpha)\beta\gamma} e_{(\beta)i} e_{(\gamma)j},$$

where $H_{(\alpha)\beta\gamma}$ and $V_{(\alpha)\beta\gamma}$, γ being fixed, are given by

$$(2.8) \quad \begin{aligned} H_{(\alpha)\beta\gamma} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_\gamma & J_\gamma & k_\gamma \\ 0 & -h_\gamma & 0 & h'_\gamma & J'_\gamma \\ 0 & -J_\gamma & -h'_\gamma & 0 & k'_\gamma \\ 0 & -k_\gamma & -J'_\gamma & -k'_\gamma & 0 \end{pmatrix}, & V_{(\alpha)\beta\gamma} &= \begin{pmatrix} 0 & \delta_{2\gamma} & \delta_{3\gamma} & \delta_{4\gamma} & \delta_{5\gamma} \\ -\delta_{2\gamma} & 0 & u_\gamma & v_\gamma & w_\gamma \\ -\delta_{3\gamma} & -u_\gamma & 0 & u'_\gamma & v'_\gamma \\ -\delta_{4\gamma} & -v_\gamma & -u'_\gamma & 0 & w'_\gamma \\ -\delta_{5\gamma} & -w_\gamma & -v'_\gamma & -w'_\gamma & 0 \end{pmatrix} \end{aligned}$$

In (2.8), we have put

$$(2.9) \quad H_{2)3\gamma} = -H_{3)2\gamma} = h_\gamma, H_{2)4\gamma} = -H_{4)2\gamma} = J_\gamma, H_{2)5\gamma} = -H_{5)2\gamma} = k_\gamma,$$

$$\begin{aligned}
50 \quad H_{3)4\gamma} &= -H_{4)3\gamma} = h'_\gamma, H_{3)5\gamma} = -H_{5)3\gamma} = J'_\gamma, H_{4)5\gamma} = -H_{5)4\gamma} = k'_\gamma, \\
51 \quad V_{2)3\gamma} &= -V_{3)2\gamma} = u_\gamma, V_{2)4\gamma} = -V_{4)2\gamma} = v_\gamma, V_{2)5\gamma} = -V_{5)2\gamma} = w_\gamma, \\
52 \quad V_{3)4\gamma} &= -V_{4)3\gamma} = u'_\gamma, V_{3)5\gamma} = -V_{5)3\gamma} = v'_\gamma, V_{4)5\gamma} = -V_{5)4\gamma} = w'_\gamma.
\end{aligned}$$

53 Hence, we have the following:

54 **Theorem 2.2.** In a five-dimensional Finsler space there exist six h-connection vec-
55 tors $h_i, J_i, k_i, h'_i, J'_i, k'_i$ whose scalar components with respect to the frame $\{e^i_{(\alpha)}\}$ are
56 $h_\gamma, J_\gamma, k_\gamma, h'_\gamma, J'_\gamma, k'_\gamma$. i. e., $h_i = h_\gamma e_{(\gamma)i}, J_i = J_\gamma e_{(\gamma)i}, k_i = k_\gamma e_{(\gamma)i}, h'_i = h'_\gamma e_{(\gamma)i}, J'_i =$
57 $J'_\gamma e_{(\gamma)i}, k'_i = k'_\gamma e_{(\gamma)i}$.

58 **Theorem 2.3.** In a five-dimensional Finsler space there exist six v-connection vec-
59 tors $u_i, v_i, w_i, u'_i, v'_i, w'_i$ whose scalar components with respect to the frame $\{e^i_{(\alpha)}\}$ are
60 $u_\gamma, v_\gamma, w_\gamma, u'_\gamma, v'_\gamma, w'_\gamma$. i. e., $u_i = u_\gamma e_{(\gamma)i}, v_i = v_\gamma e_{(\gamma)i}, w_i = w_\gamma e_{(\gamma)i}, u'_i = u'_\gamma e_{(\gamma)i}, v'_i =$
61 $v'_\gamma e_{(\gamma)i}, w'_i = w'_\gamma e_{(\gamma)i}$.

In view of equations (2.8), (2.9) and using the theorems (2.2) and (2.3), the equations (2.7) may be explicitly written as [11]

$$\begin{aligned}
(2.10) \quad l_{ij} &= 0, m_{ij} = n_i h_j + p_i J_j + q_i k_j, n_{ij} = -m_i h_j + p_i h'_j + q_i J'_j, \\
p_{ij} &= -m_i J_j - n_i h'_j + q_i k'_j, q_{ij} = -m_i k_j - n_i J'_j - p_i k'_j
\end{aligned}$$

and

$$\begin{aligned}
(2.11) \quad Ll_{ij} &= m_i m_j + n_i n_j + p_i p_j + q_i q_j = g_{ij} - l_i l_j = h_{ij}, \\
Lm_{ij} &= -l_i m_j + n_i u_j + p_i v_j + q_i w_j, \\
Ln_{ij} &= -l_i n_j - m_i u_j + p_i u'_j + q_i v'_j, \\
Lp_{ij} &= -l_i p_j - m_i v_j - n_i u'_j + q_i w'_j, \\
Lq_{ij} &= -l_i q_j - m_i w_j - n_i v'_j - p_i w'_j.
\end{aligned}$$

62 Since m_i, n_i, p_i, q_i are homogeneous functions of degree zero in y^i , we have

$Lm_{i|j} l^j = Ln_{i|j} l^j = Lp_{i|j} l^j = Lq_{i|j} l^j = 0$ which in view of equation (2.11) and theorem (2.3) gives

$$(2.12) \quad u_1 = v_1 = w_1 = u'_1 = v'_1 = w'_1 = 0.$$

63 Thus we have the following:

64 **Theorem 2.4.** In a five-dimensional Finsler space, the first scalar components of
65 v-connection vectors $u_i, v_i, w_i, u'_i, v'_i, w'_i$ vanish identically.

The equations (2.11) and (2.6) lead to the following expressions for the partial derivatives with respect to y^j :

$$(2.13) \quad L\partial_j^i l_i = h_{ij} = m_i m_j + n_i n_j + p_i p_j + q_i q_j,$$

$$66 \quad L\partial_j^i m_i = -l_i m_j + n_i u_j + p_i v_j + q_i w_j + H m_i m_j + I n_i n_j + K p_i p_j + M q_i q_j$$

$$\begin{aligned}
& -(J + J' + J'')(m_i n_j + m_j n_i) - (H' + I' + M')(m_i p_j + m_j p_i) \\
& -(H'' + I'' + K'')(m_i q_j + m_j q_i) + K'(n_i p_j + n_j p_i) + N(n_i q_j + n_j q_i) \\
& + N'(p_i q_j + p_j q_i), \\
L\partial_j^i n_i &= -l_i n_j - m_i u_j + p_i u'_j + q_i v'_j - (J + J' + J'')m_i m_j + J n_i n_j + J' p_i p_j \\
& + J'' q_i q_j + I(m_i n_j + m_j n_i) + K'(m_i p_j + m_j p_i) + N(m_i q_j + m_j q_i) \\
& + I'(n_i p_j + n_j p_i) + I''(n_i q_j + n_j q_i) + M''(p_i q_j + p_j q_i), \\
L\partial_j^i p_i &= -l_i p_j - m_i v_j - n_i u'_j + q_i w'_j - (H' + I' + M') m_i m_j + I' n_i n_j + H' p_i p_j \\
& + M' q_i q_j + K'(m_i n_j + m_j n_i) + K(m_i p_j + m_j p_i) + N'(m_i q_j + m_j q_i) \\
& + J'(n_i p_j + n_j p_i) + M''(n_i q_j + n_j q_i) + K''(p_i q_j + p_j q_i), \\
L\partial_j^i q_i &= -l_i q_j - m_i w_j - n_i v'_j - p_i w'_j - (H'' + I'' + M'') m_i m_j + I'' n_i n_j + K'' p_i p_j \\
& + H'' q_i q_j + N(m_i n_j + m_j n_i) + N'(m_i p_j + m_j p_i) + M(m_i q_j + m_j q_i) \\
& + M''(n_i p_j + n_j p_i) + J''(n_i q_j + n_j q_i) + M'(p_i q_j + p_j q_i),
\end{aligned}$$

$$(2.14) \quad L\partial_j^i l^i = m^i m_j + n^i n_j + p^i p_j + q^i q_j,$$

$$\begin{aligned}
L\partial_j^i m^i &= -l^i m_j + n^i u_j + p^i v_j + q^i w_j - H m^i m_j - I n^i n_j - K p^i p_j - M q^i q_j \\
& + (J + J' + J'')(m^i n_j + m_j n^i) + (H' + I' + M')(m^i p_j + m_j p^i) \\
& + (H'' + I'' + K'')(m^i q_j + m_j q^i) - K'(n^i p_j + n_j p^i) - N(n^i q_j + n_j q^i) \\
& - N'(p^i q_j + p_j q^i), \\
L\partial_j^i n^i &= -l^i n_j - m^i u_j + p^i u'_j + q^i v'_j + (J + J' + J'')m^i m_j - J n^i n_j - J' p^i p_j \\
& - J'' q^i q_j - I(m^i n_j + m_j n^i) - K'(m^i p_j + m_j p^i) - N(m^i q_j + m_j q^i) \\
& - I'(n^i p_j + n_j p^i) - I''(n^i q_j + n_j q^i) - M''(p^i q_j + p_j q^i), \\
L\partial_j^i p^i &= -l^i p_j - m^i v_j - n^i u'_j + q^i w'_j + (H' + I' + M') m^i m_j - I' n^i n_j - H' p^i p_j \\
& - M' q^i q_j - K'(m^i n_j + m_j n^i) - K(m^i p_j + m_j p^i) - N'(m^i q_j + m_j q^i) \\
& - J'(n^i p_j + n_j p^i) - M''(n^i q_j + n_j q^i) - K''(p^i q_j + p_j q^i), \\
L\partial_j^i q^i &= -l^i q_j - m^i w_j - n^i v'_j - p^i w'_j + (H'' + I'' + M'') m^i m_j - I'' n^i n_j \\
& - K'' p^i p_j - H'' q^i q_j - N(m^i n_j + m_j n^i) + N'(m^i p_j + m_j p^i) \\
& - M(m^i q_j + m_j q^i) + M''(n^i p_j + n_j p^i) - J''(n^i q_j + n_j q^i) - M'(p^i q_j + p_j q^i),
\end{aligned}$$

The h -scalar derivative of the adapted components $T_{\alpha\beta}$ of the tensor T_j^i of (1, 1) type is defined as [4]

$$(2.15) \quad T_{\alpha\beta;\gamma} = (\delta_k T_{\alpha\beta}) e_{\gamma}^k + T_{\mu\beta} H_{\mu\alpha\gamma} + T_{\alpha\mu} H_{\mu\beta\gamma},$$

where $\delta_k = \partial_k - G_k^r \partial_r$. Similarly, the v -scalar derivative of the adapted components $T_{\alpha\beta}$ of the tensor T_j^i of (1, 1) type is defined as [4]

$$(2.16) \quad T_{\alpha\beta;\gamma} = L(\partial_k T_{\alpha\beta}) e_{\gamma}^k + T_{\mu\beta} V_{\mu\alpha\gamma} + T_{\alpha\mu} V_{\mu\beta\gamma}.$$

Thus, $T_{\alpha\beta;\gamma}$ and $T_{\alpha\beta;\gamma}$ are the adapted components of $T_{j|k}^i$ and $T_j^i|_k$ respectively i. e.,

$$(2.17) \quad T_{j|k}^i = T_{\alpha\beta;\gamma} e_{(\alpha)}^i e_{(\beta)j} e_{(\gamma)k},$$

$$(2.18) \quad LT_j^i|_k = T_{\alpha\beta;\gamma} e_{(\alpha)}^i e_{(\beta)j} e_{(\gamma)k}.$$

92 A covariant vector field σ_i is called a gradient vector, if there exists a scalar field
93 $\sigma = \sigma(x)$ satisfying $\sigma_i = \partial_i \sigma$. Then, we have

94 **Lemma 2.5.** A covariant vector field $\sigma_i = \sigma_{\alpha} e_{\alpha i}$ is locally a gradient vector, if and
95 only if the scalar components σ_{α} , $\alpha = 1, 2, 3, 4, 5$.

$$(2.19) \quad \sigma_{\alpha;\beta} = \sigma_{\beta;\alpha}, \quad \alpha, \beta = 1, 2, 3, 4, 5.$$

96

$$(2.20) \quad \left\{ \begin{array}{l} \sigma_{1;\alpha} = \sigma_{\alpha;1} = 0, \quad \alpha = 1, 2, 3, 4, 5 \\ \sigma_{2;2} = -\sigma_2 H + \sigma_3 (J + J' + J'') + \sigma_4 (H' + I' + M') \\ \quad + \sigma_5 (H'' + I'' + K''), \\ \sigma_{3;3} = -\sigma_2 I - \sigma_3 J - \sigma_4 I' - \sigma_5 I'', \\ \sigma_{4;4} = -\sigma_2 K - \sigma_3 J' - \sigma_4 H' - \sigma_5 K'', \\ \sigma_{5;5} = -\sigma_2 M - \sigma_3 J'' - \sigma_4 M' - \sigma_5 H'', \\ \sigma_{2;3} = \sigma_{3;2} = \sigma_2 (J + J' + J'') - \sigma_3 I - \sigma_4 K' - \sigma_5 N, \\ \sigma_{2;4} = \sigma_{4;2} = \sigma_2 (H' + I' + M') - \sigma_3 K' - \sigma_4 K - \sigma_5 N', \\ \sigma_{2;5} = \sigma_{5;2} = \sigma_2 (H'' + I'' + K'') - \sigma_3 N - \sigma_4 N' - \sigma_5 M, \\ \sigma_{3;4} = \sigma_{4;3} = -\sigma_2 K' - \sigma_3 I' - \sigma_4 J' - \sigma_5 M'', \\ \sigma_{3;5} = \sigma_{5;3} = -\sigma_2 N - \sigma_3 I'' - \sigma_4 M'' - \sigma_5 J'', \\ \sigma_{4;5} = \sigma_{5;4} = -\sigma_2 N' - \sigma_3 M'' - \sigma_4 K'' - \sigma_5 M'. \end{array} \right.$$

97 *Proof.* It is obvious that σ_i is locally a gradient vector if and only if it satisfies
98 (a) $\partial_j \sigma_i - \partial_i \sigma_j = 0$, (b) $\partial_j \sigma_i = 0$. These are equivalent, respectively, to (2.10)
99 $\sigma_{i;j} (= \partial_j \sigma_i - \sigma_k F_{ij}^k) = \sigma_{j;i}$, (2.11) $\sigma_i|_j = -\sigma_k C_{ij}^k$. We examine the scalar compo-
100 nents σ_{α} of σ_i . Then equations (2.17) and (2.18) give $\sigma_{i;j} = \sigma_{\alpha;\beta} e_{\alpha i} e_{\beta j}$, $\sigma_i|_j =$
101 $\sigma_{\alpha;\beta} e_{\alpha i} e_{\beta j}$ respectively. Then the equations (2.10) and (2.11) are written, respec-
102 tively, in the forms (2.19) and $\sigma_{\alpha;\beta} = -\sigma_{\gamma} C_{\alpha\beta\gamma}$. This equation together with (2.6)
103 gives (2.20). \square

3 Conformal change of Cartan's connection

We consider a conformal change $L(x, y) \longrightarrow \bar{L}(x, y) = e^{\sigma(x)}L(x, y)$ of a five-dimensional Finsler space $F^5 = (M^5, L(x, y))$ with the fundamental function $L(x, y)$, where $\sigma(x)$ is a scalar function of position x^i alone, called the conformal factor. We shall denote the Finsler space with changed fundamental function $\bar{L}(x, y)$ by $\bar{F}^5 = (M^5, \bar{L}(x, y))$ and quantities of \bar{F}^5 by upper line. The following change of important quantities are known [1].

$$(3.1) \quad \bar{l}_i = e^\sigma l_i, \bar{m}_i = e^\sigma m_i, \bar{n}_i = e^\sigma n_i, \bar{p}_i = e^\sigma p_i, \bar{q}_i = e^\sigma q_i, \bar{g}_{ij} = e^{2\sigma} g_{ij},$$

$$(3.2) \quad \bar{l}^i = e^{-\sigma} l^i, \bar{m}^i = e^{-\sigma} m^i, \bar{n}^i = e^{-\sigma} n^i, \bar{p}^i = e^{-\sigma} p^i, \\ \bar{q}^i = e^{-\sigma} q^i, \bar{g}^{ij} = e^{-2\sigma} g^{ij},$$

$$(3.3) \quad \bar{C}_{ijk} = e^{2\sigma} C_{ijk}, \bar{C}_{jk}^i = C_{jk}^i, \bar{H} = H, \bar{I} = I, \bar{J} = J, \bar{K} = K, \\ \bar{M} = M, \bar{N} = N, \bar{H}' = H', \bar{I}' = I', \bar{J}' = J', \bar{K}' = K', \bar{M}' = M', \\ \bar{N}' = N', \bar{H}'' = H'', \bar{I}'' = I'', \bar{J}'' = J'', \bar{K}'' = K'', \bar{M}'' = M''.$$

Lemma (2.5) leads us to the following useful relations:

Proposition 3.1. *If we put $\sigma_i = \partial_i \sigma(x) = \sigma_\alpha e_\alpha|_i$, then we have the relations*

$$(i) \quad \sigma_{\alpha, \beta} = \sigma_{\beta, \alpha}, \quad \alpha, \beta = 1, 2, 3, 4, 5. \\ (ii) \quad \sigma_{1; \alpha} = \sigma_{\alpha; 1} = 0, \quad \alpha = 1, 2, 3, 4, 5. \\ \sigma_{2; 2} = \sigma_1 + \sigma_6, \\ \sigma_{2; 3} = \sigma_{3; 2} = \sigma_7, \\ \sigma_{2; 4} = \sigma_{4; 2} = \sigma_8, \\ \sigma_{2; 5} = \sigma_{5; 2} = \sigma_9, \\ \sigma_{3; 3} = \sigma_1 + \sigma_{10}, \\ \sigma_{3; 4} = \sigma_{4; 3} = \sigma_{11}, \\ \sigma_{3; 5} = \sigma_{5; 3} = \sigma_{12}, \\ \sigma_{4; 4} = \sigma_1 + \sigma_{13}, \\ \sigma_{4; 5} = \sigma_{5; 4} = \sigma_{14}, \\ \sigma_{5; 5} = \sigma_1 + \sigma_{15},$$

where we have substituted

$$\sigma_6 = -\sigma_1 - \sigma_2 H + \sigma_3 (J + J' + J'') + \sigma_4 (H' + I' + M') + \sigma_5 (H'' + I'' + K''), \\ \sigma_7 = \sigma_2 (J + J' + J'') - \sigma_3 I - \sigma_4 K' - \sigma_5 N, \\ \sigma_8 = \sigma_2 (H' + I' + M') - \sigma_3 K' - \sigma_4 K - \sigma_5 N', \\ \sigma_9 = \sigma_2 (H'' + I'' + K'') - \sigma_3 N - \sigma_4 N' - \sigma_5 M, \\ \sigma_{10} = -\sigma_1 - \sigma_2 I - \sigma_3 J - \sigma_4 I' - \sigma_5 I'', \\ \sigma_{11} = -\sigma_2 K' - \sigma_3 I' - \sigma_4 J' - \sigma_5 M'', \\ \sigma_{12} = -\sigma_2 N - \sigma_3 I'' - \sigma_4 M'' - \sigma_5 J'', \\ \sigma_{13} = -\sigma_1 - \sigma_2 K - \sigma_3 J' - \sigma_4 H' - \sigma_5 K'', \\ \sigma_{14} = -\sigma_2 N' - \sigma_3 M'' - \sigma_4 K'' - \sigma_5 M', \\ \sigma_{15} = -\sigma_1 - \sigma_2 M - \sigma_3 J'' - \sigma_4 M' - \sigma_5 H''.$$

Now we consider the change of Christoffel symbols $\gamma_{ijk} = g_{jr}\gamma_{ik}^r = \frac{1}{2}(\partial_k g_{ij} + \partial_i g_{jk} - \partial_j g_{ki})$ constructed from $g_{ij}(x, y)$ with respect to x^i , then we have

$$(3.4) \quad \bar{\gamma}_{jk}^i = \gamma_{jk}^i + \delta_j^i \sigma_k + \delta_k^i \sigma_j - g_{jk} \sigma^i, \quad (\sigma^i = g^{ij} \sigma_j).$$

Thus the change of the well-known quantities $2G^i = \gamma_{jk}^i y^j y^k = \gamma_{00}^i$ is given by

$$(3.5) \quad 2\bar{G}^i = 2G^i + L^2 (\sigma_1 l^i - \sigma_2 m^i - \sigma_3 n^i - \sigma_4 p^i - \sigma_5 q^i).$$

Differentiating (3.5) with respect to y^j , using proposition (3.1) and equation (2.14), we get

$$(3.6) \quad \bar{G}_j^i = G_j^i + Ll^i (\sigma_1 l_j + \sigma_2 m_j + \sigma_3 n_j + \sigma_4 p_j + \sigma_5 q_j)$$

$$\begin{aligned} & -Lm^i (\sigma_2 l_j + \sigma_6 m_j + \sigma_7 n_j + \sigma_8 p_j + \sigma_9 q_j) \\ & -Ln^i (\sigma_3 l_j + \sigma_7 m_j + \sigma_{10} n_j + \sigma_{11} p_j + \sigma_{12} q_j) \\ & -Lp^i (\sigma_4 l_j + \sigma_8 m_j + \sigma_{11} n_j + \sigma_{13} p_j + \sigma_{14} q_j) \\ & -Lq^i (\sigma_5 l_j + \sigma_9 m_j + \sigma_{12} n_j + \sigma_{14} p_j + \sigma_{15} q_j). \end{aligned}$$

On the other hand, the connection coefficients F_{jk}^i of CT are given by [4]

$$F_{ijk} = g_{jr} F_{ik}^r = \gamma_{ijk} - C_{ijr} G_k^r - C_{jkr} G_i^r + C_{ikr} G_j^r.$$

Then the equations (2.6), (3.4) and (3.6) lead to

$$(3.7) \quad \bar{F}_{jk}^i = F_{jk}^i$$

$$\begin{aligned} & +l^i \left\{ \begin{aligned} & \sigma_1 l_j l_k + \sigma_2 (l_j m_k + l_k m_j) + \sigma_3 (l_j n_k + l_k n_j) + \sigma_4 (l_j p_k + l_k p_j) + \sigma_5 (l_j q_k + l_k q_j) \\ & + \sigma_6 m_j m_k + \sigma_7 (m_j n_k + m_k n_j) + \sigma_8 (m_j p_k + m_k p_j) + \sigma_9 (m_j q_k + m_k q_j) + \sigma_{10} n_j n_k \\ & + \sigma_{11} (n_j p_k + n_k p_j) + \sigma_{12} (n_j q_k + n_k q_j) + \sigma_{13} p_j p_k + \sigma_{14} (p_j q_k + p_k q_j) + \sigma_{15} q_j q_k \end{aligned} \right\} \\ & -m^i \left\{ \begin{aligned} & \sigma_2 l_j l_k + \sigma_6 (l_j m_k + l_k m_j) + \sigma_7 (l_j n_k + l_k n_j) + \sigma_8 (l_j p_k + l_k p_j) + \sigma_9 (l_j q_k + l_k q_j) \\ & + \sigma_{16} m_j m_k + \sigma_{17} (m_j n_k + m_k n_j) + \sigma_{18} (m_j p_k + m_k p_j) + \sigma_{19} (m_j q_k + m_k q_j) + \sigma_{20} n_j n_k \\ & + \sigma_{21} (n_j p_k + n_k p_j) + \sigma_{22} (n_j q_k + n_k q_j) + \sigma_{23} p_j p_k + \sigma_{24} (p_j q_k + p_k q_j) + \sigma_{25} q_j q_k \end{aligned} \right\} \\ & -n^i \left\{ \begin{aligned} & \sigma_3 l_j l_k + \sigma_7 (l_j m_k + l_k m_j) + \sigma_{10} (l_j n_k + l_k n_j) + \sigma_{11} (l_j p_k + l_k p_j) + \sigma_{12} (l_j q_k + l_k q_j) \\ & + \sigma_{26} m_j m_k + \sigma_{27} (m_j n_k + m_k n_j) + \sigma_{28} (m_j p_k + m_k p_j) + \sigma_{29} (m_j q_k + m_k q_j) + \sigma_{30} n_j n_k + \\ & \sigma_{31} (n_j p_k + n_k p_j) + \sigma_{32} (n_j q_k + n_k q_j) + \sigma_{33} p_j p_k + \sigma_{34} (p_j q_k + p_k q_j) + \sigma_{35} q_j q_k \end{aligned} \right\} \\ & -p^i \left\{ \begin{aligned} & \sigma_4 l_j l_k + \sigma_8 (l_j m_k + l_k m_j) + \sigma_{11} (l_j n_k + l_k n_j) + \sigma_{13} (l_j p_k + l_k p_j) + \sigma_{14} (l_j q_k + l_k q_j) \\ & + \sigma_{36} m_j m_k + \sigma_{37} (m_j n_k + m_k n_j) + \sigma_{38} (m_j p_k + m_k p_j) + \sigma_{39} (m_j q_k + m_k q_j) + \sigma_{40} n_j n_k \\ & + \sigma_{41} (n_j p_k + n_k p_j) + \sigma_{42} (n_j q_k + n_k q_j) + \sigma_{43} p_j p_k + \sigma_{44} (p_j q_k + p_k q_j) + \sigma_{45} q_j q_k \end{aligned} \right\} \\ & -q^i \left\{ \begin{aligned} & \sigma_5 l_j l_k + \sigma_9 (l_j m_k + l_k m_j) + \sigma_{12} (l_j n_k + l_k n_j) + \sigma_{14} (l_j p_k + l_k p_j) + \sigma_{15} (l_j q_k + l_k q_j) \\ & + \sigma_{46} m_j m_k + \sigma_{47} (m_j n_k + m_k n_j) + \sigma_{48} (m_j p_k + m_k p_j) + \sigma_{49} (m_j q_k + m_k q_j) + \sigma_{50} n_j n_k \\ & + \sigma_{51} (n_j p_k + n_k p_j) + \sigma_{52} (n_j q_k + n_k q_j) + \sigma_{53} p_j p_k + \sigma_{54} (p_j q_k + p_k q_j) + \sigma_{55} q_j q_k \end{aligned} \right\}, \end{aligned}$$

where

$$\begin{aligned} & \sigma_{16} = -\sigma_2 - \sigma_6 H + \sigma_7 (J + J' + J'') + \sigma_8 (H' + I' + M') + \sigma_9 (H'' + I'' + K''), \\ & \sigma_{17} = -\sigma_3 - \sigma_7 H + \sigma_{10} (J + J' + J'') + \sigma_{11} (H' + I' + M') + \sigma_{12} (H'' + I'' + K''), \\ & \sigma_{18} = -\sigma_4 - \sigma_8 H + \sigma_{11} (J + J' + J'') + \sigma_{13} (H' + I' + M') + \sigma_{14} (H'' + I'' + K''), \\ & \sigma_{19} = -\sigma_5 - \sigma_9 H + \sigma_{12} (J + J' + J'') + \sigma_{14} (H' + I' + M') + \sigma_{15} (H'' + I'' + K''), \\ & \sigma_{20} = \sigma_2 + \sigma_6 I + \sigma_7 (3J + 2J' + 2J'') + \sigma_8 I' + \sigma_9 I'' - 2\sigma_{10} I - 2\sigma_{11} K' - 2\sigma_{12} N, \\ & \sigma_{21} = \sigma_6 K' + \sigma_7 (H' + 2I' + M') + \sigma_8 (J + 2J' + J'') + \sigma_9 M'' - \sigma_{10} K' \\ & \quad - \sigma_{11} (K + I) - \sigma_{12} N' - \sigma_{13} K' - \sigma_{14} N, \end{aligned}$$

$$\begin{aligned}
150 \quad \sigma_{22} &= \sigma_6 N + \sigma_7 (H'' + 2I'' + K'') + \sigma_8 M'' + \sigma_9 (J + J' + 2J'') - \sigma_{10} N \\
151 \quad &\quad - \sigma_{11} N' - \sigma_{12} (I + M) - \sigma_{14} K' - \sigma_{15} N, \\
152 \quad \sigma_{23} &= \sigma_2 + \sigma_6 K + \sigma_7 J' + \sigma_8 (3H' + 2I' + 2M') + \sigma_9 K'' \\
153 \quad &\quad - 2(\sigma_{11} K' + \sigma_{13} K + \sigma_{14} N'), \\
154 \quad \sigma_{24} &= \sigma_6 N' + \sigma_7 M'' + \sigma_8 (H'' + I'' + K'') + \sigma_9 (H' + I' + 2M') - \sigma_{11} N \\
155 \quad &\quad - \sigma_{12} K' - \sigma_{13} N' - \sigma_{14} (M + K), \\
156 \quad \sigma_{25} &= \sigma_2 + \sigma_6 M + \sigma_7 J'' + \sigma_8 M' + \sigma_9 (3H'' + 2I'' + 2K'') - 2(\sigma_{12} N + \sigma_{14} N' + \sigma_{15} M), \\
157 \quad \sigma_{26} &= \sigma_3 + 2\sigma_6 (J + J' + J'') - \sigma_7 (H + 2I) - 2(\sigma_8 K' + \sigma_9 N) - \sigma_{10} (J + J' + J'') \\
158 \quad &\quad - \sigma_{11} (H' + I' + M') - \sigma_{12} (H'' + I'' + K''), \\
159 \quad \sigma_{27} &= -\sigma_2 - \sigma_6 I - \sigma_7 J - \sigma_8 I' - \sigma_9 I'', \\
160 \quad \sigma_{28} &= -\sigma_6 K' + \sigma_8 (J + J'') - \sigma_7 (H' + 2I' + M') - \sigma_9 M'' + \sigma_{10} K' \\
161 \quad &\quad + \sigma_{11} (K - I) + \sigma_{12} N' - \sigma_{13} K' - \sigma_{14} N, \\
162 \quad \sigma_{29} &= -\sigma_6 N - \sigma_7 (H'' + 2I'' + M'') - \sigma_8 M'' + \sigma_9 (J + J') + \sigma_{10} N \\
163 \quad &\quad + \sigma_{11} N' + \sigma_{12} (M - I) - \sigma_{14} K' - \sigma_{15} N, \\
164 \quad \sigma_{30} &= -\sigma_3 - \sigma_7 I - \sigma_{10} J - \sigma_{11} I' - \sigma_{12} I'', \\
165 \quad \sigma_{31} &= -\sigma_4 - \sigma_8 I - \sigma_{11} J - \sigma_{13} I' - \sigma_{14} I'', \\
166 \quad \sigma_{32} &= -\sigma_5 - \sigma_9 I - \sigma_{12} J - \sigma_{14} I' - \sigma_{15} I'', \\
167 \quad \sigma_{33} &= \sigma_3 + \sigma_7 K - 2\sigma_8 K' + \sigma_{11} (H'' - 2I'') + \sigma_{12} K'' - 2\sigma_{13} J' - 2\sigma_{14} M'', \\
168 \quad \sigma_{34} &= \sigma_7 N' - \sigma_8 N - \sigma_9 K' + \sigma_{10} M'' - \sigma_{11} (K'' - I'') + \sigma_{12} (M' - I') - \sigma_{13} M'' \\
169 \quad &\quad - \sigma_{14} (J' + J'') - \sigma_{15} M'', \\
170 \quad \sigma_{35} &= \sigma_3 + \sigma_7 M - 2\sigma_9 N + \sigma_{10} J'' + \sigma_{11} M' + \sigma_{12} (H'' - 2I'') - \sigma_{14} M'' - \sigma_{15} J'', \\
171 \quad \sigma_{36} &= \sigma_4 + 2\sigma_6 (H' + I' + M') - 2\sigma_7 K' + \sigma_8 (H - 2K) - 2\sigma_9 N - \sigma_{11} (J + J' + J'') \\
172 \quad &\quad - \sigma_{13} (H' + I' + M') - \sigma_{14} (H'' + I'' + K''), \\
173 \quad \sigma_{37} &= -\sigma_6 K' + \sigma_7 (H' + M') - \sigma_8 (J + 2J' + J'') - \sigma_9 M'' - \sigma_{10} K' \\
174 \quad &\quad + \sigma_{11} I - K - \sigma_{12} N' + \sigma_{13} K' + \sigma_{14} N, \\
175 \quad \sigma_{38} &= -\sigma_2 - \sigma_6 K - \sigma_7 J' - \sigma_8 H' - \sigma_9 K'', \\
176 \quad \sigma_{39} &= -\sigma_6 N' - \sigma_7 M'' - \sigma_8 (H'' + I'' + 2K'') + \sigma_9 (H' + I') + \sigma_{11} N \\
177 \quad &\quad - \sigma_{12} K' + \sigma_{13} N' - \sigma_{14} (K - M) - \sigma_{15} N', \\
178 \quad \sigma_{40} &= \sigma_4 - 2\sigma_7 K' + \sigma_8 I - 2\sigma_{10} I' + \sigma_{11} (J - 2J') - 2\sigma_{12} M'' + \sigma_{13} I' + \sigma_{14} I'', \\
179 \quad \sigma_{41} &= -\sigma_3 - \sigma_7 K - \sigma_{10} J' - \sigma_{11} H' - \sigma_{12} K'', \\
180 \quad \sigma_{42} &= -\sigma_7 N' + \sigma_8 N - \sigma_9 K' - \sigma_{10} M'' - \sigma_{11} (K'' - I'') - \sigma_{12} (M' + I') + \sigma_{13} M'' \\
181 \quad &\quad + \sigma_{14} (J'' - J') - \sigma_{15} M'', \\
182 \quad \sigma_{43} &= -\sigma_4 - \sigma_8 K - \sigma_{11} J' - \sigma_{13} H' - \sigma_{14} K'', \\
183 \quad \sigma_{44} &= -\sigma_5 - \sigma_9 K - \sigma_{12} J' - \sigma_{14} H' - \sigma_{15} K'', \\
184 \quad \sigma_{45} &= \sigma_4 + \sigma_8 M - \sigma_9 N' + \sigma_{11} J'' - \sigma_{12} M'' + \sigma_{13} M' + \sigma_{14} (H'' - K'') - \sigma_{15} M', \\
185 \quad \sigma_{46} &= \sigma_5 + 2\sigma_6 (H'' + I'' + K'') - 2\sigma_7 N - 2\sigma_8 N' + \sigma_9 (H - 2M) - \sigma_{12} (J + J' + J'') \\
186 \quad &\quad - \sigma_{14} (H' + I' + M') - \sigma_{15} (H'' + I'' + K''), \\
187 \quad \sigma_{47} &= -\sigma_6 N + \sigma_7 (H'' + K'') - \sigma_8 M'' - \sigma_9 (J + J' + J'') - \sigma_{10} N - \sigma_{11} N' \\
188 \quad &\quad - \sigma_{12} (M - I) + \sigma_{14} K' + \sigma_{15} N, \\
189 \quad \sigma_{48} &= -\sigma_6 N' - \sigma_7 M'' + \sigma_8 (H'' + I'') - \sigma_9 (H' + I' + 2M') - \sigma_{11} N \\
190 \quad &\quad + \sigma_{12} K' - \sigma_{13} N' + \sigma_{14} (K - M) + \sigma_{15} N', \\
191 \quad \sigma_{49} &= -\sigma_2 - \sigma_6 M - \sigma_7 J'' - \sigma_8 M' - \sigma_9 H'', \\
192 \quad \sigma_{50} &= \sigma_5 - 2\sigma_7 N + \sigma_9 I - 2\sigma_{10} I'' - 2\sigma_{11} M'' + \sigma_{12} (J - 2J'') + \sigma_{14} I' + \sigma_{15} I'', \\
193 \quad \sigma_{51} &= -\sigma_7 N' - \sigma_8 N + \sigma_9 K' - \sigma_{10} M'' - \sigma_{11} (K'' + I'') - \sigma_{12} (M' - I') \\
194 \quad &\quad - \sigma_{13} M'' - \sigma_{14} (J' - J'') + \sigma_{15} M'', \\
195 \quad \sigma_{52} &= -\sigma_3 - \sigma_7 M - \sigma_{10} J'' - \sigma_{11} M' - \sigma_{12} H'', \\
196 \quad \sigma_{53} &= \sigma_5 - 2\sigma_8 N' + \sigma_9 K - 2\sigma_{11} M'' + \sigma_{12} J' - 2\sigma_{13} K''
\end{aligned}$$

$$\begin{aligned}
& +\sigma_{14}(H' - 2M') + \sigma_{15}K'', \\
\sigma_{54} &= -\sigma_4 - \sigma_8M - \sigma_{11}J'' - \sigma_{13}M' - \sigma_{14}H'', \\
\sigma_{55} &= -\sigma_5 - \sigma_9M - \sigma_{12}J'' - \sigma_{14}M' - \sigma_{15}H''.
\end{aligned}$$

Now we shall deal with the conformally invariant scalar field $S(x, y)$. Its h-covariant derivative $\bar{S}_{\perp i}$ with respect to the changed \bar{CT} is defined by

$$\bar{S}_{\perp i} = \partial_i S - (\partial_j S) \bar{G}_i^j.$$

It is enough for the later use to treat a positively homogeneous scalar field S of degree zero in y^i so that $S_{;1} = 0$. Then from (3.6), we have

$$(3.8) \quad \bar{S}_{\perp i} = S_{;i} + S_{;2}(\sigma_2 l_i + \sigma_6 m_i + \sigma_7 n_i + \sigma_8 p_i + \sigma_9 q_i)$$

$$\begin{aligned}
& + S_{;3}(\sigma_3 l_i + \sigma_7 m_i + \sigma_{10} n_i + \sigma_{11} p_i + \sigma_{12} q_i) \\
& + S_{;4}(\sigma_4 l_i + \sigma_8 m_i + \sigma_{11} n_i + \sigma_{13} p_i + \sigma_{14} q_i) \\
& + S_{;5}(\sigma_5 l_i + \sigma_9 m_i + \sigma_{12} n_i + \sigma_{14} p_i + \sigma_{15} q_i).
\end{aligned}$$

Since $\bar{S}_{\perp i} = \bar{S}_{;1} \bar{l}_i + \bar{S}_{;2} \bar{m}_i + \bar{S}_{;3} \bar{n}_i + \bar{S}_{;4} \bar{p}_i + \bar{S}_{;5} \bar{q}_i$, from the equations (3.2) and (3.8) we have the relations:

$$(3.9) \quad \bar{S}_{;1} = \bar{S}_{\perp i} \bar{l}^i = e^{-\sigma} (S_{;1} + S_{;2}\sigma_2 + S_{;3}\sigma_3 + S_{;4}\sigma_4 + S_{;5}\sigma_5),$$

$$\begin{aligned}
\bar{S}_{;2} &= \bar{S}_{\perp i} \bar{m}^i = e^{-\sigma} (S_{;2} + S_{;2}\sigma_6 + S_{;3}\sigma_7 + S_{;4}\sigma_8 + S_{;5}\sigma_9), \\
\bar{S}_{;3} &= \bar{S}_{\perp i} \bar{n}^i = e^{-\sigma} (S_{;3} + S_{;2}\sigma_7 + S_{;3}\sigma_{10} + S_{;4}\sigma_{11} + S_{;5}\sigma_{12}), \\
\bar{S}_{;4} &= \bar{S}_{\perp i} \bar{p}^i = e^{-\sigma} (S_{;4} + S_{;2}\sigma_8 + S_{;3}\sigma_{11} + S_{;4}\sigma_{13} + S_{;5}\sigma_{14}), \\
\bar{S}_{;5} &= \bar{S}_{\perp i} \bar{q}^i = e^{-\sigma} (S_{;5} + S_{;2}\sigma_9 + S_{;3}\sigma_{12} + S_{;4}\sigma_{14} + S_{;5}\sigma_{15}).
\end{aligned}$$

On the other hand, the v-covariant derivative $\bar{S}|_{\underline{i}}$ with respect to the changed \bar{CT} is defined by $\bar{S}|_{\underline{i}} = \partial_i S = S|_i$. Making use of the relation (3.2), this equation gives

$$(3.10) \quad \bar{S}_{;1} = \bar{L}\bar{S}|_{\underline{i}} \bar{l}^i = 0, \bar{S}_{;2} = \bar{L}\bar{S}|_{\underline{i}} \bar{m}^i = S_{;2},$$

$$\begin{aligned}
\bar{S}_{;3} &= \bar{L}\bar{S}|_{\underline{i}} \bar{n}^i = S_{;3}, \bar{S}_{;4} = \bar{L}\bar{S}|_{\underline{i}} \bar{p}^i = S_{;4}, \\
\bar{S}_{;5} &= \bar{L}\bar{S}|_{\underline{i}} \bar{q}^i = S_{;5}.
\end{aligned}$$

Proposition 3.2. *Let S be a conformally invariant scalar field, which is positively homogeneous of degree zero in y^i . Then the conformal changes of scalar derivatives of S are given by (3.9) and (3.10).*

For the conformal change of the adapted components $h_\alpha, J_\alpha, k_\alpha, h'_\alpha, J'_\alpha, k'_\alpha$ of the six h-connection vectors $h_i, J_i, k_i, h'_i, J'_i, k'_i$, from (3.1) and (2.10), we have

$$\begin{aligned}
\bar{m}_{i\perp j} &= e^\sigma (\sigma_j m_i + m_{i\perp j}), \bar{n}_{i\perp j} = e^\sigma (\sigma_j n_i + n_{i\perp j}), \\
\bar{p}_{i\perp j} &= e^\sigma (\sigma_j p_i + p_{i\perp j}), \bar{q}_{i\perp j} = e^\sigma (\sigma_j q_i + q_{i\perp j})
\end{aligned}$$

which in view of (3.6) and (3.7) leads to

$$\begin{aligned}
(3.11) \quad \bar{h}_j &= h_j + \{\sigma_2 u_2 + \sigma_3 u_3 + \sigma_4 u_4 + \sigma_5 u_5\} l_j \\
& + \{\sigma_6 u_2 + \sigma_7 u_3 + \sigma_8 u_4 + \sigma_9 u_5 + \sigma_{17} + \sigma_{56}\} m_j \\
& + \{\sigma_7 u_2 + \sigma_{10} u_3 + \sigma_{11} u_4 + \sigma_{12} u_5 + \sigma_{20} + \sigma_{57}\} n_j \\
& + \{\sigma_8 u_2 + \sigma_{11} u_3 + \sigma_{13} u_4 + \sigma_{14} u_5 + \sigma_{21} + \sigma_{58}\} p_j \\
& + \{\sigma_9 u_2 + \sigma_{12} u_3 + \sigma_{14} u_4 + \sigma_{15} u_5 + \sigma_{22} + \sigma_{59}\} q_j.
\end{aligned}$$

$$\begin{aligned}
 (b) \quad \bar{J}_j &= J_j + \{\sigma_2 v_2 + \sigma_3 v_3 + \sigma_4 v_4 + \sigma_5 v_5\} l_j \\
 &\quad + \{\sigma_6 v_2 + \sigma_7 v_3 + \sigma_8 v_4 + \sigma_9 v_5 + \sigma_{18} + \sigma_{60}\} m_j \\
 &\quad + \{\sigma_7 v_2 + \sigma_{10} v_3 + \sigma_{11} v_4 + \sigma_{12} v_5 + \sigma_{21} + \sigma_{61}\} n_j \\
 &\quad + \{\sigma_8 v_2 + \sigma_{11} v_3 + \sigma_{13} v_4 + \sigma_{14} v_5 + \sigma_{23} + \sigma_{62}\} p_j, \\
 &\quad + \{\sigma_9 v_2 + \sigma_{12} v_3 + \sigma_{14} v_4 + \sigma_{15} v_5 + \sigma_{24} + \sigma_{63}\} q_j. \\
 (c) \quad \bar{k}_j &= k_j + \{\sigma_2 w_2 + \sigma_3 w_3 + \sigma_4 w_4 + \sigma_5 w_5\} l_j \\
 &\quad + \{\sigma_6 w_2 + \sigma_7 w_3 + \sigma_8 w_4 + \sigma_9 w_5 + \sigma_{19} + \sigma_{64}\} m_j \\
 &\quad + \{\sigma_7 w_2 + \sigma_{10} w_3 + \sigma_{11} w_4 + \sigma_{12} w_5 + \sigma_{22} + \sigma_{65}\} n_j \\
 &\quad + \{\sigma_8 w_2 + \sigma_{11} w_3 + \sigma_{13} w_4 + \sigma_{14} w_5 + \sigma_{24} + \sigma_{66}\} p_j \\
 &\quad + \{\sigma_9 w_2 + \sigma_{12} w_3 + \sigma_{14} w_4 + \sigma_{15} w_5 + \sigma_{25} + \sigma_{67}\} q_j. \\
 (d) \quad \bar{h}'_j &= h'_j + \{\sigma_2 u'_2 + \sigma_3 u'_3 + \sigma_4 u'_4 + \sigma_5 u'_5\} l_j \\
 &\quad + \{\sigma_6 u'_2 + \sigma_7 u'_3 + \sigma_8 u'_4 + \sigma_9 u'_5 + \sigma_{28} + \sigma_{68}\} m_j \\
 &\quad + \{\sigma_7 u'_2 + \sigma_{10} u'_3 + \sigma_{11} u'_4 + \sigma_{12} u'_5 + \sigma_{31} + \sigma_{69}\} n_j \\
 &\quad + \{\sigma_8 u'_2 + \sigma_{11} u'_3 + \sigma_{13} u'_4 + \sigma_{14} u'_5 + \sigma_{33} + \sigma_{70}\} p_j \\
 &\quad + \{\sigma_9 u'_2 + \sigma_{12} u'_3 + \sigma_{14} u'_4 + \sigma_{15} u'_5 + \sigma_{34} + \sigma_{71}\} q_j. \\
 (e) \quad \bar{J}'_j &= J'_j + \{\sigma_2 v'_2 + \sigma_3 v'_3 + \sigma_4 v'_4 + \sigma_5 v'_5\} l_j \\
 &\quad + \{\sigma_6 v'_2 + \sigma_7 v'_3 + \sigma_8 v'_4 + \sigma_9 v'_5 + \sigma_{29} + \sigma_{72}\} m_j \\
 &\quad + \{\sigma_7 v'_2 + \sigma_{10} v'_3 + \sigma_{11} v'_4 + \sigma_{12} v'_5 + \sigma_{32} + \sigma_{73}\} n_j \\
 &\quad + \{\sigma_8 v'_2 + \sigma_{11} v'_3 + \sigma_{13} v'_4 + \sigma_{14} v'_5 + \sigma_{34} + \sigma_{74}\} p_j, \\
 &\quad + \{\sigma_9 v'_2 + \sigma_{12} v'_3 + \sigma_{14} v'_4 + \sigma_{15} v'_5 + \sigma_{35} + \sigma_{75}\} q_j. \\
 (f) \quad \bar{k}'_j &= k'_j + \{\sigma_2 w'_2 + \sigma_3 w'_3 + \sigma_4 w'_4 + \sigma_5 w'_5\} l_j \\
 &\quad + \{\sigma_6 w'_2 + \sigma_7 w'_3 + \sigma_8 w'_4 + \sigma_9 w'_5 + \sigma_{39} + \sigma_{76}\} m_j \\
 &\quad + \{\sigma_7 w'_2 + \sigma_{10} w'_3 + \sigma_{11} w'_4 + \sigma_{12} w'_5 + \sigma_{42} + \sigma_{77}\} n_j \\
 &\quad + \{\sigma_8 w'_2 + \sigma_{11} w'_3 + \sigma_{13} w'_4 + \sigma_{14} w'_5 + \sigma_{44} + \sigma_{78}\} p_j \\
 &\quad + \{\sigma_9 w'_2 + \sigma_{12} w'_3 + \sigma_{14} w'_4 + \sigma_{15} w'_5 + \sigma_{45} + \sigma_{79}\} q_j.
 \end{aligned}$$

where we have put

$$\begin{aligned}
 \sigma_{56} &= -\sigma_6 (J + J' + J'') + \sigma_7 I + \sigma_8 K' + \sigma_9 N, \\
 \sigma_{57} &= -\sigma_7 (J + J' + J'') + \sigma_{10} I + \sigma_{11} K' + \sigma_{12} N, \\
 \sigma_{58} &= -\sigma_8 (J + J' + J'') + \sigma_{11} I + \sigma_{13} K' + \sigma_{14} N, \\
 \sigma_{59} &= -\sigma_9 (J + J' + J'') + \sigma_{12} I + \sigma_{14} K' + \sigma_{15} N, \\
 \sigma_{60} &= -\sigma_6 (H' + I' + M') + \sigma_7 K' + \sigma_8 K + \sigma_9 N', \\
 \sigma_{61} &= -\sigma_7 (H' + I' + M') + \sigma_{10} K' + \sigma_{11} K + \sigma_{12} N', \\
 \sigma_{62} &= -\sigma_8 (H' + I' + M') + \sigma_{11} K' + \sigma_{13} K + \sigma_{14} N', \\
 \sigma_{63} &= -\sigma_9 (H' + I' + M') + \sigma_{12} K' + \sigma_{14} K + \sigma_{15} N', \\
 \sigma_{64} &= -\sigma_6 (H'' + I'' + K'') + \sigma_7 N + \sigma_8 N' + \sigma_9 M, \\
 \sigma_{65} &= -\sigma_7 (H'' + I'' + K'') + \sigma_{10} N + \sigma_{11} N' + \sigma_{12} M, \\
 \sigma_{66} &= -\sigma_8 (H'' + I'' + K'') + \sigma_{11} N + \sigma_{13} N' + \sigma_{14} M, \\
 \sigma_{67} &= -\sigma_9 (H'' + I'' + K'') + \sigma_{12} N + \sigma_{14} N' + \sigma_{15} M, \\
 \sigma_{68} &= \sigma_6 K' + \sigma_7 I' + \sigma_8 J' + \sigma_9 M'', \\
 \sigma_{69} &= \sigma_7 K' + \sigma_{10} I' + \sigma_{11} J' + \sigma_{12} M'', \\
 \sigma_{70} &= \sigma_8 K' + \sigma_{11} I' + \sigma_{13} J' + \sigma_{14} M'', \\
 \sigma_{71} &= \sigma_9 K' + \sigma_{12} I' + \sigma_{14} J' + \sigma_{15} M'', \\
 \sigma_{72} &= \sigma_6 N + \sigma_7 I'' + \sigma_8 M'' + \sigma_9 J'', \\
 \sigma_{73} &= \sigma_7 N + \sigma_{10} I'' + \sigma_{11} M'' + \sigma_{12} J'', \\
 \sigma_{74} &= \sigma_8 N + \sigma_{11} I'' + \sigma_{13} M'' + \sigma_{14} J'', \\
 \sigma_{75} &= \sigma_9 N + \sigma_{12} I'' + \sigma_{14} M'' + \sigma_{15} J'', \\
 \sigma_{76} &= \sigma_6 N' + \sigma_7 M'' + \sigma_8 K'' + \sigma_9 M',
 \end{aligned}$$

$$\begin{aligned}
266 \quad & \sigma_{77} = \sigma_7 N' + \sigma_{10} M'' + \sigma_{11} K'' + \sigma_{12} M', \\
267 \quad & \sigma_{78} = \sigma_8 N' + \sigma_{11} M'' + \sigma_{13} K'' + \sigma_{14} M', \\
268 \quad & \sigma_{79} = \sigma_9 N' + \sigma_{12} M'' + \sigma_{14} K'' + \sigma_{15} M'.
\end{aligned}$$

Thus the adapted components $\bar{h}_\alpha, \bar{J}_\alpha, \bar{k}_\alpha, \bar{h}'_\alpha, \bar{J}'_\alpha, \bar{k}'_\alpha$, of $\bar{h}'_i, \bar{J}'_i, \bar{k}'_i$ in $\bar{F}^5 = (M^5, \bar{L}(x, y))$ are given by

$$\begin{aligned}
(3.12) \quad (a) \quad & \bar{h}_1 = e^{-\sigma} \{h_1 + \sigma_2 u_2 + \sigma_3 u_3 + \sigma_4 u_4 + \sigma_5 u_5\}, \\
& \bar{h}_2 = e^{-\sigma} \{h_2 + \sigma_6 u_2 + \sigma_7 u_3 + \sigma_8 u_4 + \sigma_9 u_5 + \sigma_{17} + \sigma_{56}\}, \\
& \bar{h}_3 = e^{-\sigma} \{h_3 + \sigma_7 u_2 + \sigma_{10} u_3 + \sigma_{11} u_4 + \sigma_{12} u_5 + \sigma_{20} + \sigma_{57}\}, \\
& \bar{h}_4 = e^{-\sigma} \{h_4 + \sigma_8 u_2 + \sigma_{11} u_3 + \sigma_{13} u_4 + \sigma_{14} u_5 + \sigma_{21} + \sigma_{58}\}, \\
& \bar{h}_5 = e^{-\sigma} \{h_5 + \sigma_9 u_2 + \sigma_{12} u_3 + \sigma_{14} u_4 + \sigma_{15} u_5 + \sigma_{22} + \sigma_{59}\}. \\
\\
(b) \quad & \bar{J}_1 = e^{-\sigma} \{J_1 + \sigma_2 v_2 + \sigma_3 v_3 + \sigma_4 v_4 + \sigma_5 v_5\}, \\
270 \quad & \bar{J}_2 = e^{-\sigma} \{J_2 + \sigma_6 v_2 + \sigma_7 v_3 + \sigma_8 v_4 + \sigma_9 v_5 + \sigma_{18} + \sigma_{60}\}, \\
271 \quad & \bar{J}_3 = e^{-\sigma} \{J_3 + \sigma_7 v_2 + \sigma_{10} v_3 + \sigma_{11} v_4 + \sigma_{12} v_5 + \sigma_{21} + \sigma_{61}\}, \\
272 \quad & \bar{J}_4 = e^{-\sigma} \{J_4 + \sigma_8 v_2 + \sigma_{11} v_3 + \sigma_{13} v_4 + \sigma_{14} v_5 + \sigma_{23} + \sigma_{62}\}, \\
273 \quad & \bar{J}_5 = e^{-\sigma} \{J_5 + \sigma_9 v_2 + \sigma_{12} v_3 + \sigma_{14} v_4 + \sigma_{15} v_5 + \sigma_{24} + \sigma_{63}\}. \\
(c) \quad & \bar{k}_1 = e^{-\sigma} \{k_1 + \sigma_2 w_2 + \sigma_3 w_3 + \sigma_4 w_4 + \sigma_5 w_5\}, \\
275 \quad & \bar{k}_2 = e^{-\sigma} \{k_2 + \sigma_6 w_2 + \sigma_7 w_3 + \sigma_8 w_4 + \sigma_9 w_5 + \sigma_{19} + \sigma_{64}\}, \\
276 \quad & \bar{k}_3 = e^{-\sigma} \{k_3 + \sigma_7 w_2 + \sigma_{10} w_3 + \sigma_{11} w_4 + \sigma_{12} w_5 + \sigma_{22} + \sigma_{65}\}, \\
277 \quad & \bar{k}_4 = e^{-\sigma} \{k_4 + \sigma_8 w_2 + \sigma_{11} w_3 + \sigma_{13} w_4 + \sigma_{14} w_5 + \sigma_{24} + \sigma_{66}\}, \\
278 \quad & \bar{k}_5 = e^{-\sigma} \{k_5 + \sigma_9 w_2 + \sigma_{12} w_3 + \sigma_{14} w_4 + \sigma_{15} w_5 + \sigma_{25} + \sigma_{67}\}. \\
(d) \quad & \bar{h}'_1 = e^{-\sigma} \{h'_1 + \sigma_2 u'_2 + \sigma_3 u'_3 + \sigma_4 u'_4 + \sigma_5 u'_5\}, \\
280 \quad & \bar{h}'_2 = e^{-\sigma} \{h'_2 + \sigma_6 u'_2 + \sigma_7 u'_3 + \sigma_8 u'_4 + \sigma_9 u'_5 + \sigma_{28} + \sigma_{68}\}, \\
281 \quad & \bar{h}'_3 = e^{-\sigma} \{h'_3 + \sigma_7 u'_2 + \sigma_{10} u'_3 + \sigma_{11} u'_4 + \sigma_{12} u'_5 + \sigma_{31} + \sigma_{69}\}, \\
282 \quad & \bar{h}'_4 = e^{-\sigma} \{h'_4 + \sigma_8 u'_2 + \sigma_{11} u'_3 + \sigma_{13} u'_4 + \sigma_{14} u'_5 + \sigma_{33} + \sigma_{70}\}, \\
283 \quad & \bar{h}'_5 = e^{-\sigma} \{h'_5 + \sigma_9 u'_2 + \sigma_{12} u'_3 + \sigma_{14} u'_4 + \sigma_{15} u'_5 + \sigma_{34} + \sigma_{71}\}. \\
(e) \quad & \bar{J}'_1 = e^{-\sigma} \{J'_1 + \sigma_2 v'_2 + \sigma_3 v'_3 + \sigma_4 v'_4 + \sigma_5 v'_5\}, \\
285 \quad & \bar{J}'_2 = e^{-\sigma} \{J'_2 + \sigma_6 v'_2 + \sigma_7 v'_3 + \sigma_8 v'_4 + \sigma_9 v'_5 + \sigma_{29} + \sigma_{72}\}, \\
286 \quad & \bar{J}'_3 = e^{-\sigma} \{J'_3 + \sigma_7 v'_2 + \sigma_{10} v'_3 + \sigma_{11} v'_4 + \sigma_{12} v'_5 + \sigma_{32} + \sigma_{73}\}, \\
287 \quad & \bar{J}'_4 = e^{-\sigma} \{J'_4 + \sigma_8 v'_2 + \sigma_{11} v'_3 + \sigma_{13} v'_4 + \sigma_{14} v'_5 + \sigma_{34} + \sigma_{74}\}, \\
288 \quad & \bar{J}'_5 = e^{-\sigma} \{J'_5 + \sigma_9 v'_2 + \sigma_{12} v'_3 + \sigma_{14} v'_4 + \sigma_{15} v'_5 + \sigma_{35} + \sigma_{75}\}. \\
(f) \quad & \bar{k}'_1 = e^{-\sigma} \{k'_1 + \sigma_2 w'_2 + \sigma_3 w'_3 + \sigma_4 w'_4 + \sigma_5 w'_5\}, \\
290 \quad & \bar{k}'_2 = e^{-\sigma} \{k'_2 + \sigma_6 w'_2 + \sigma_7 w'_3 + \sigma_8 w'_4 + \sigma_9 w'_5 + \sigma_{39} + \sigma_{76}\}, \\
291 \quad & \bar{k}'_3 = e^{-\sigma} \{k'_3 + \sigma_7 w'_2 + \sigma_{10} w'_3 + \sigma_{11} w'_4 + \sigma_{12} w'_5 + \sigma_{42} + \sigma_{77}\}, \\
292 \quad & \bar{k}'_4 = e^{-\sigma} \{k'_4 + \sigma_8 w'_2 + \sigma_{11} w'_3 + \sigma_{13} w'_4 + \sigma_{14} w'_5 + \sigma_{44} + \sigma_{78}\}, \\
293 \quad & \bar{k}'_5 = e^{-\sigma} \{k'_5 + \sigma_9 w'_2 + \sigma_{12} w'_3 + \sigma_{14} w'_4 + \sigma_{15} w'_5 + \sigma_{45} + \sigma_{79}\}.
\end{aligned}$$

For the conformal change of the adapted components $u_\alpha, v_\alpha, w_\alpha, u'_\alpha, v'_\alpha, w'_\alpha$, of six v-connection vectors $u_i, v_i, w_i, u'_i, v'_i, w'_i$, we again use (3.1) and (2.11). Thus we get $\bar{u}_j = e^\sigma u_j, \bar{v}_j = e^\sigma v_j, \bar{w}_j = e^\sigma w_j, \bar{u}'_j = e^\sigma u'_j, \bar{v}'_j = e^\sigma v'_j, \bar{w}'_j = e^\sigma w'_j$, which lead to

$$\begin{aligned}
(3.13) \quad (a) \quad & \bar{u}_1 = u_1 = 0, \bar{u}_2 = u_2, \bar{u}_3 = u_3, \bar{u}_4 = u_4, \bar{u}_5 = u_5, \\
294 \quad (b) \quad & \bar{v}_1 = v_1 = 0, \bar{v}_2 = v_2, \bar{v}_3 = v_3, \bar{v}_4 = v_4, \bar{v}_5 = v_5, \\
295 \quad (c) \quad & \bar{w}_1 = w_1 = 0, \bar{w}_2 = w_2, \bar{w}_3 = w_3, \bar{w}_4 = w_4, \bar{w}_5 = w_5, \\
296 \quad (d) \quad & \bar{u}'_1 = u'_1 = 0, \bar{u}'_2 = u'_2, \bar{u}'_3 = u'_3, \bar{u}'_4 = u'_4, \bar{u}'_5 = u'_5,
\end{aligned}$$

$$\begin{aligned} (e) \quad & \overline{v}'_1 = v'_1 = 0, \overline{v}'_2 = v'_2, \overline{v}'_3 = v'_3, \overline{v}'_4 = v'_4, \overline{v}'_5 = v'_5, \\ (f) \quad & \overline{w}'_1 = w'_1 = 0, \overline{w}'_2 = w'_2, \overline{w}'_3 = w'_3, \overline{w}'_4 = w'_4, \overline{w}'_5 = w'_5. \end{aligned}$$

From (3.11) and (3.13), we have the following:

Theorem 3.3. *The adapted components of all the six v -connection vectors of five-dimensional Finsler space are invariant under any conformal change.*

Theorem 3.4. *The h -connection vector h_i of F^5 is invariant under σ -conformal change if and only if*

$$\begin{aligned} (i) \quad & \sigma_2 u_2 + \sigma_3 u_3 + \sigma_4 u_4 + \sigma_5 u_5 = 0, \\ (ii) \quad & \sigma_6 u_2 + \sigma_7 u_3 + \sigma_8 u_4 + \sigma_9 u_5 + \sigma_{17} + \sigma_{56} = 0, \\ (iii) \quad & \sigma_7 u_2 + \sigma_{10} u_3 + \sigma_{11} u_4 + \sigma_{12} u_5 + \sigma_{20} + \sigma_{57} = 0, \\ (iv) \quad & \sigma_8 u_2 + \sigma_{11} u_3 + \sigma_{13} u_4 + \sigma_{14} u_5 + \sigma_{21} + \sigma_{58} = 0, \\ (v) \quad & \sigma_9 u_2 + \sigma_{12} u_3 + \sigma_{14} u_4 + \sigma_{15} u_5 + \sigma_{22} + \sigma_{59} = 0. \end{aligned}$$

Theorem 3.5. *The h -connection vector J_i of F^5 is invariant under σ -conformal change if and only if*

$$\begin{aligned} (i) \quad & \sigma_2 v_2 + \sigma_3 v_3 + \sigma_4 v_4 + \sigma_5 v_5 = 0, \\ (ii) \quad & \sigma_6 v_2 + \sigma_7 v_3 + \sigma_8 v_4 + \sigma_9 v_5 + \sigma_{18} + \sigma_{60} = 0, \\ (iii) \quad & \sigma_7 v_2 + \sigma_{10} v_3 + \sigma_{11} v_4 + \sigma_{12} v_5 + \sigma_{21} + \sigma_{61} = 0, \\ (iv) \quad & \sigma_8 v_2 + \sigma_{11} v_3 + \sigma_{13} v_4 + \sigma_{14} v_5 + \sigma_{23} + \sigma_{62} = 0, \\ (v) \quad & \sigma_9 v_2 + \sigma_{12} v_3 + \sigma_{14} v_4 + \sigma_{15} v_5 + \sigma_{24} + \sigma_{63} = 0. \end{aligned}$$

Theorem 3.6. *The h -connection vector k_i of F^5 is invariant under σ -conformal change if and only if*

$$\begin{aligned} (i) \quad & \sigma_2 w_2 + \sigma_3 w_3 + \sigma_4 w_4 + \sigma_5 w_5 = 0, \\ (ii) \quad & \sigma_6 w_2 + \sigma_7 w_3 + \sigma_8 w_4 + \sigma_9 w_5 + \sigma_{19} + \sigma_{64} = 0, \\ (iii) \quad & \sigma_7 w_2 + \sigma_{10} w_3 + \sigma_{11} w_4 + \sigma_{12} w_5 + \sigma_{22} + \sigma_{65} = 0, \\ (iv) \quad & \sigma_8 w_2 + \sigma_{11} w_3 + \sigma_{13} w_4 + \sigma_{14} w_5 + \sigma_{24} + \sigma_{66} = 0, \\ (v) \quad & \sigma_9 w_2 + \sigma_{12} w_3 + \sigma_{14} w_4 + \sigma_{15} w_5 + \sigma_{25} + \sigma_{67} = 0. \end{aligned}$$

Theorem 3.7. *The h -connection vector h'_i of F^5 is invariant under σ -conformal change if and only if*

$$\begin{aligned} (i) \quad & \sigma_2 u'_2 + \sigma_3 u'_3 + \sigma_4 u'_4 + \sigma_5 u'_5 = 0, \\ (ii) \quad & \sigma_6 u'_2 + \sigma_7 u'_3 + \sigma_8 u'_4 + \sigma_9 u'_5 + \sigma_{28} + \sigma_{68} = 0, \\ (iii) \quad & \sigma_7 u'_2 + \sigma_{10} u'_3 + \sigma_{11} u'_4 + \sigma_{12} u'_5 + \sigma_{31} + \sigma_{69} = 0, \\ (iv) \quad & \sigma_8 u'_2 + \sigma_{11} u'_3 + \sigma_{13} u'_4 + \sigma_{14} u'_5 + \sigma_{33} + \sigma_{70} = 0, \\ (v) \quad & \sigma_9 u'_2 + \sigma_{12} u'_3 + \sigma_{14} u'_4 + \sigma_{15} u'_5 + \sigma_{34} + \sigma_{71} = 0. \end{aligned}$$

Theorem 3.8. *The h -connection vector J'_i of F^5 is invariant under σ -conformal change if and only if*

$$\begin{aligned} (i) \quad & \sigma_2 v'_2 + \sigma_3 v'_3 + \sigma_4 v'_4 + \sigma_5 v'_5 = 0, \\ (ii) \quad & \sigma_6 v'_2 + \sigma_7 v'_3 + \sigma_8 v'_4 + \sigma_9 v'_5 + \sigma_{29} + \sigma_{72} = 0, \\ (iii) \quad & \sigma_7 v'_2 + \sigma_{10} v'_3 + \sigma_{11} v'_4 + \sigma_{12} v'_5 + \sigma_{32} + \sigma_{73} = 0, \\ (iv) \quad & \sigma_8 v'_2 + \sigma_{11} v'_3 + \sigma_{13} v'_4 + \sigma_{14} v'_5 + \sigma_{34} + \sigma_{74} = 0, \\ (v) \quad & \sigma_9 v'_2 + \sigma_{12} v'_3 + \sigma_{14} v'_4 + \sigma_{15} v'_5 + \sigma_{35} + \sigma_{75} = 0. \end{aligned}$$

337 **Theorem 3.9.** *The h-connection vector k'_i of F^5 is invariant under σ -conformal*
 338 *change if and only if*

$$\begin{aligned}
 339 & \quad (i) \quad \sigma_2 w'_2 + \sigma_3 w'_3 + \sigma_4 w'_4 + \sigma_5 w'_5 = 0, \\
 340 & \quad (ii) \quad \sigma_6 w'_2 + \sigma_7 w'_3 + \sigma_8 w'_4 + \sigma_9 w'_5 + \sigma_{39} + \sigma_{76} = 0, \\
 341 & \quad (iii) \quad \sigma_7 w'_2 + \sigma_{10} w'_3 + \sigma_{11} w'_4 + \sigma_{12} w'_5 + \sigma_{42} + \sigma_{77} = 0, \\
 342 & \quad (iv) \quad \sigma_8 w'_2 + \sigma_{11} w'_3 + \sigma_{13} w'_4 + \sigma_{14} w'_5 + \sigma_{44} + \sigma_{78} = 0, \\
 343 & \quad (v) \quad \sigma_9 w'_2 + \sigma_{12} w'_3 + \sigma_{14} w'_4 + \sigma_{15} w'_5 + \sigma_{45} + \sigma_{79} = 0.
 \end{aligned}$$

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