

EXISTENCE, UNIQUENESS AND CONTINUOUS DEPENDENCE IN THERMOELASTICITY BY USING A SEMIGROUP OF OPERATORS

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Abstract

In this paper we apply the theory of semigroups of operators in order to obtain the existence and uniqueness of solutions for the mixed initial-boundary value problems in thermoelasticity of micropolar bodies. The continuous dependence of the solutions upon initial data and supply terms is also proved.

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1 Introduction

The origin of the theories of micropolar bodies goes back to E. and F. Cosserat. The authors have introduced an additional degree of kinematic freedom (*microrotation*) to develop a new continuum theory. In the paper [7], Mindlin has proved that the linear theory of a Cosserat continuum is a special case of the theory of media with microstructure. The linear theory of the Cosserat elastic solid is often called "theory of asymmetric elasticity", [8], or "micropolar theory of elasticity", [2]. A Cosserat medium is a continuum, each point of which has the degrees of a rigid body. The deformation of such medium is described by the variables $u_i = u_i(X, t)$, $\varphi_i = \varphi_i(X, t)$, $(X, t) \in B \times [0, t_0]$, where u_i is the displacement and φ_i is the microrotation field. The theories of micropolar bodies do not represent a material length scale, but are quite sufficient for a large number of solid mechanics applications. In this paper we establish an existence and uniqueness for the solutions of the initial-boundary value problem in the context of the thermoelasticity of micropolar bodies. The paper also investigates the continuous dependence upon the initial data and supply terms of the solutions of the above problem. An inhomogeneous and anisotropic elastic material is considered and the initial-boundary value problem is transformed in an

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abstract temporally homogeneous evolutionary equation in a Hilbert space. By using the results of the semigroups theory of linear operators, the existence, uniqueness and continuous dependence results are derived. The proofs are given for the first boundary value problem, but the results are same if the boundary conditions are replaced by those from the second or the third problem.

2 Notations and basic equations

Let B be an open region of three-dimensional Euclidian space occupied by the reference configuration of a micropolar body. We assume that B is regular and we denote the closure of B by \bar{B} . The boundary ∂B of B is closed and bounded. We use a fixed system of rectangular Cartesian axes and adopt Cartesian tensor notation. Points in B are denoted by x_j and $t \in [0, \infty)$ is time. Also, the spatial argument and the time argument of a function will be omitted when there is no likelihood of confusion. In the following we consider the theory of thermoelasticity of micropolar bodies as it established in the paper [6]. The basic equations in that theory are as follows

$$t_{ij,j} + \varrho F_i = \varrho \ddot{u}_i, \quad m_{ij,j} + \varepsilon_{ijk} t_{jk} + \varrho M_i = I_{ij} \ddot{\varphi}_i; \quad (1)$$

$$\varrho T_0 \dot{\eta} = q_{i,i} + \varrho r; \quad (x, t) \in B \times [0, \infty) \quad (2)$$

The equations (1) are the motion equations and (2) is the energy equations.

In the above equations we are used the following notations: u_i - the components of displacement, φ_i -the components of microrotatia, t_{ij} -the components of stress, m_{ij} -the components of couple stress, q_i -the components of the heat conduction vector, η -the specific entropy, ϱ -the constant reference density, T_0 -the constant reference temperature, F_i -the components of body force per unit mass, M_i -the components of body couple force per unit mass, r -the heat supply per unit mass and unit time, I_{ij} -the components of inertia and ε_{ijk} -the alternating symbol.

A superposed dot denotes the differentiation with respect to time, t , and a subscript preceded by a comma denotes the differentiation with respect to the corresponding spatial coordinate.

When the reference solid has a centre of symmetry at each point, but is otherwise non-isotropic, then the constitutive equations are

$$\begin{aligned} t_{ij} &= A_{ijmn} \varepsilon_{mn} + B_{ijmn} \gamma_{mn} - E_{ij} \theta, \\ m_{ij} &= B_{mnij} \varepsilon_{mn} + C_{ijmn} \gamma_{mn} - D_{ij} \theta, \\ q_i &= K_{ij} \theta_{,j}, \\ \varrho \eta &= a \theta + E_{ij} \varepsilon_{ij} + D_{ij} \gamma_{ij}, \quad (x, t) \in B \times [0, \infty) \end{aligned} \quad (3)$$

In the above equations we are used the following geometrical equations

$$\varepsilon_{ij} = u_{j,i} + \varepsilon_{jik} \varphi_k, \quad \gamma_{ij} = \varphi_{j,i} \quad (4)$$

The tensor coefficients in the equations (3) are subject to the symmetry conditions

$$A_{ijmn} = A_{mnij}, \quad C_{ijmn} = C_{mnij}, \quad K_{ij} = K_{ji} \quad (5)$$

The entropy production inequality implies

$$K_{ij}\theta_i\theta_j \geq 0. \quad (6)$$

To the system of field equations (1)-(5) we adjoin the following prescribed boundary conditions

$$u_i(x_k, t) = 0, \varphi_i(x_k, t) = 0, \theta_i(x_k, t) = 0, \quad (x_k, t) \in \partial B \times [0, \infty), \quad (7)$$

and the initial conditions

$$\begin{aligned} u_i(x_k, 0) &= a_i(x_k), \dot{u}_i(x_k, 0) = b_i(x_k), \varphi_i(x_k, 0) = c_i(x_k), \\ \dot{\varphi}_i(x_k, 0) &= d_i(x_k), \theta(x_k, 0) = \theta^0(x_k), \quad (x_k) \in B \end{aligned} \quad (8)$$

where a_i, b_i, c_i, d_i and θ^0 are prescribed functions. Introducing (3) and (4) in (1) and (2), we obtain the following system

$$\begin{aligned} \rho \ddot{u}_i &= [A_{ijmn}(u_{n,m} + \varepsilon_{nmk}\varphi_k) + B_{ijmn}\varphi_{n,m} - E_{ij}\theta]_{,j} + \rho F_i, \\ I_{ij}\ddot{\varphi}_i &= [B_{mnij}(u_{n,m} + \varepsilon_{nmk}\varphi_k) + C_{ijmn}\varphi_{n,m} - D_{ij}\theta]_{,j} + \\ &+ \varepsilon_{ijk}[A_{jkmn}(u_{n,m} + \varepsilon_{nms}\varphi_s) + B_{jkmn}\varphi_{n,m} - E_{jk}\theta] + \rho M_i, \\ aT_0\dot{\theta} &= -T_0[E_{ij}(v_{j,i} + \varepsilon_{jik}\psi_k) + D_{ij}\psi_{j,i}] + (K_{ij}\theta_{,j})_{,i} + \rho r, \end{aligned} \quad (9)$$

where $v_i = \dot{u}_i, \psi_i = \dot{\varphi}_i$. By a solution of the mixed initial boundary value problem of the micropolar thermoelasticity in the cylinder $\Omega_0 = B \times [0, \infty)$ we mean an ordered array (u_i, φ_i, θ) which satisfies the system (9) for all $(x, t) \in \Omega_0$, the boundary conditions (7) and the initial conditions (8).

Throughout this section we shall use the following assumptions on the material properties

$$\begin{aligned} i) \quad &\rho > 0, T_0 > 0, I_{ij} > 0, a > 0; \\ ii) \quad &K_{ij}\xi_i\xi_j \geq k_0\xi_i\xi_i, k_0 > 0, \forall \xi_i; \\ iii) \quad &A_{ijmn}\xi_{ij}\xi_{mn} + 2B_{ijmn}\xi_{ij}\eta_{mn} + C_{ijmn}\eta_{ij}\eta_{mn} \geq \\ &\geq a_0(\xi_{ij}\xi_{ij} + \eta_{ij}\eta_{ij}), a_0 > 0, \forall \xi_{ij}, \eta_{ij}. \end{aligned}$$

The above assumptions are in agreement with the usual restrictions imposed in the Mechanics of continua in order to obtain the existence and uniqueness of solution. For instance, the condition *ii)* represents a considerable strengthening of the consequence (6) of the entropy production inequality. We shall use the vectorial notations $\mathbf{u} = (u_i), \mathbf{v} = (v_i), \varphi = (\varphi_i), \psi = (\psi_i), i = 1, 2, 3$. Let us define

$$\begin{aligned} X = \{ \mathbf{W} = (\mathbf{u}, \mathbf{v}, \varphi, \psi, \theta) : \mathbf{u} \in \mathbf{H}_0^1(B), \mathbf{v} \in \mathbf{H}^0(B), \\ \varphi \in \mathbf{H}_0^1(B), \psi \in \mathbf{H}^0(B), \theta \in H^0(B) \} \end{aligned} \quad (10)$$

where $H_0^m(B)$ and $H^m(B)$ are the familiar Sobolev spaces, [10], and we used the notations $\mathbf{H}^m(B) = [H^m(B)]^3, \mathbf{H}_0^m(B) = [H_0^m(B)]^3$. We wish to transform our

initial-boundary value problem, defined by (9), (7) and (8) into a temporally homogeneous abstract equation in the Hilbert space X . Thus, we define the operators

$$\begin{aligned} A_i \mathbf{W} &= v_i, \quad B_i \mathbf{W} = \frac{1}{\varrho} [A_{ijmn}(u_{n,m} + \varepsilon_{nmk} \varphi_k) + B_{ijmn} \varphi_{n,m} - E_{ij} \theta]_{,j}, \\ C_i \mathbf{W} &= \psi_i, \quad D_i \mathbf{W} = \frac{1}{I_{ij}} [B_{ijmn}(u_{n,m} + \varepsilon_{nmk} \varphi_k) + C_{ijmn} \varphi_{n,m} - \\ &\quad - D_{ij} \theta]_{,j} + \varepsilon_{ijk} (A_{jkmn}(u_{n,m} + \varepsilon_{nms} \varphi_s) + B_{jkmn} \varphi_{n,m} - E_{jk} \theta), \\ E \mathbf{W} &= -\frac{1}{a} [E_{ij}(v_{j,i} + \varepsilon_{jik} \psi_k) + D_{ij} \psi_{j,i}] + \frac{1}{aT_0} (K_{ij} \theta)_{,i}. \end{aligned} \quad (11)$$

Let \mathcal{L} be the operator

$$\mathcal{L} = (\mathbf{A}\mathbf{W}, \mathbf{B}\mathbf{W}, \mathbf{C}\mathbf{W}, \mathbf{D}\mathbf{W}, E\mathbf{W}) \quad (12)$$

where $\mathbf{A} = (A_i)$, $\mathbf{B} = (B_i)$, $\mathbf{C} = (C_i)$, $\mathbf{D} = (D_i)$, $i = 1, 2, 3$, with the domain

$$D = D(\mathcal{L}) = \{\mathbf{W} \in X : \mathcal{L}\mathbf{W} \in X, \mathbf{v} = 0, \psi = 0 \text{ on } \partial B\}. \quad (13)$$

The closure of $D(\mathcal{L})$ is obviously the space X and hence $D(\mathcal{L})$ is dense in X . $D(\mathcal{L})$ is not empty, it contains at least $[C_0^\infty(B)]^{13}$. Thus, we reduce the initial-boundary value problem (9), (7), (8) to the abstract initial value problem on the Hilbert space X

$$\frac{d\mathbf{W}}{dt} = \mathcal{L}\mathbf{W} + \mathcal{F}(t), \quad \mathbf{W}(0) = \mathbf{W}_0, \quad 0 \leq t \leq t_0, \quad (14)$$

where

$$\begin{aligned} \mathcal{F}(t) &= (\mathbf{0}, \mathbf{F}, \mathbf{0}, \mathbf{M}, r), \quad \mathbf{W}_0 = (\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \theta^0), \\ \mathbf{F} &= (F_i), \quad \mathbf{M} = (M_i), \quad \mathbf{a} = (a_i), \quad \mathbf{b} = (b_i), \quad \mathbf{c} = (c_i), \quad \mathbf{d} = (d_i). \end{aligned}$$

3 Basic results

Let X_* be the Hilbert space X equipped with the norm induced by the inner product

$$\begin{aligned} \langle \mathbf{W}, \bar{\mathbf{W}} \rangle_* &= \int_B [\varrho v_i \bar{v}_i + I_{ij} \psi_i \bar{\psi}_i + a \theta \bar{\theta} + A_{ijmn} \varepsilon_{ij} \bar{\varepsilon}_{mn} + \\ &\quad + 2B_{ijmn} (\varepsilon_{ij} \bar{\gamma}_{mn} + \bar{\varepsilon}_{ij} \gamma_{mn}) + C_{ijmn} \gamma_{ij} \bar{\gamma}_{mn}] dv \end{aligned} \quad (15)$$

By taking into account the hypothesis *i*), *ii*), *iii*) we obtain

$$\begin{aligned} |\mathbf{W}|_*^2 &= \langle \mathbf{W}, \mathbf{W} \rangle_* = \int_B [\varrho v_i v_i + I_{ij} \psi_i \psi_i + a \theta^2 + A_{ijmn} \varepsilon_{ij} \varepsilon_{mn} + \\ &\quad + 2B_{ijmn} \varepsilon_{ij} \gamma_{mn} + C_{ijmn} \gamma_{ij} \gamma_{mn}] dv \geq \\ &\geq \int_B [\varrho v_i v_i + I_{ij} \psi_i \psi_i + a \theta^2 + \\ &\quad a_0 (\varepsilon_{ij} \varepsilon_{mn} + \gamma_{ij} \gamma_{mn})] dv \geq c_1 |\mathbf{W}|_X^2 \end{aligned} \quad (16)$$

On the other hand, by using the first Korn inequality, [3], and (15) we can prove that

$$|\mathbf{W}|_*^2 \leq c_2 |\mathbf{W}|_X^2$$

such that, in view of (16) we have

$$c_1 |\mathbf{W}|_X^2 \leq |\mathbf{W}|_*^2 \leq c_2 |\mathbf{W}|_X^2,$$

hence the norm $|\cdot|_*$ is a norm equivalent to the original norm in X .

Lemma 1. *The operator \mathcal{L} is dissipative, that is*

$$\langle \mathcal{L}\mathbf{W}, \mathbf{W} \rangle_* \leq 0, \text{ for all } \mathbf{W} \in D(\mathcal{L}).$$

Proof. According to the relations (11) we have

$$\begin{aligned} \langle \mathcal{L}\mathbf{W}, \mathbf{W} \rangle_* = & \int_B \{v_i [A_{ijmn}(u_{n,m} + \varepsilon_{nmk}\varphi_k) + B_{ijmn}\varphi_{n,m} - E_{ij}\theta]_{,j} + \\ & + \psi_i [B_{ijmn}(u_{n,m} + \varepsilon_{nmk}\varphi_k) + C_{ijmn}\varphi_{n,m} - D_{ij}\theta]_{,j} + \\ & + \psi_i \varepsilon_{ijk} (A_{jkmn}(u_{n,m} + \varepsilon_{nms}\varphi_s) + B_{jkmn}\varphi_{n,m} - E_{jk}\theta)_{,j} + \\ & + \theta \left[\frac{1}{T_0} [(K_{ij}\theta_{,j})_{,i} - (E_{ij}(v_{j,i} + \varepsilon_{jik}\psi_k) + D_{ij}\psi_{j,i})] + \right. \\ & \left. + A_{ijmn}(u_{n,m} + \varepsilon_{nmk}\varphi_k)(v_{j,i} + \varepsilon_{jis}\psi_s) + C_{ijmn}\psi_{n,m}\varphi_{j,i} + \right. \\ & \left. + B_{ijmn}[(u_{j,i} + \varepsilon_{jis}\varphi_s)\psi_{n,m} + (v_{n,m} + \varepsilon_{nms}\psi_s)\varphi_{j,i}] \right\} dv \end{aligned}$$

We make now use of the Green-Gauss formula and the boundary conditions (7) such that it results

$$\langle \mathcal{L}\mathbf{W}, \mathbf{W} \rangle_* = -\frac{1}{T_0} \int_B K_{ij}\theta_{,i}\theta_{,j} dv. \quad (17)$$

On the basis of the inequality *iii*), from (17) we obtain

$$\langle \mathcal{L}\mathbf{W}, \mathbf{W} \rangle_* \leq -\frac{k_0}{T_0} \int_B \theta_{,i}\theta_{,j} dv, \quad (18)$$

such that the proof of Lemma 1 is complete.

Lemma 2. *The operator \mathcal{L} satisfies the range condition, that is*

$$R(\lambda I - \mathcal{L}) = X, \quad \lambda > 0. \quad (19)$$

Proof. Assume that $\hat{\mathbf{W}} = (\hat{\mathbf{u}}, \hat{\mathbf{v}}, \hat{\varphi}, \hat{\psi}, \hat{\theta}) \in X$. Then we must show that for all $\hat{\mathbf{W}} \in X$ the equation

$$\lambda \mathbf{W} - \mathcal{L}\mathbf{W} = \hat{\mathbf{W}} \quad (20)$$

has at least a solution \mathbf{W} in $D(\mathcal{L})$. By eliminating the functions v_i and ψ_i in (20), we obtain the following system of equations in the variables u_i , v_i and θ

$$\begin{aligned}\mathcal{L}_i\omega &= \lambda^2 u_i - \frac{1}{\rho} [A_{ijmn}(u_{n,m} + \varepsilon_{nmk}\varphi_k) + B_{ijmn}\varphi_{n,m} - E_{ij}\theta]_{,j} = g_i, \\ \mathcal{L}_{i+3}\omega &= \lambda^2 \varphi_i - \frac{1}{I_{ij}} [B_{ijmn}(u_{n,m} + \varepsilon_{nmk}\varphi_k) + C_{ijmn}\varphi_{n,m} - D_{ij}\theta]_{,j} + \\ &\quad + \varepsilon_{ijk} [A_{jkmn}(u_{n,m} + \varepsilon_{nms}\varphi_s) + B_{jkmn}\varphi_{n,m} - E_{jk}\theta] = g_{i+3}, \\ \mathcal{L}_7\omega &= \lambda^2 \theta - \frac{1}{\alpha T_0} (K_{ij}\theta_{,j})_{,i} + \frac{1}{\alpha} [E_{ij}(v_{j,i} + \varepsilon_{jik}\psi_k) + D_{ij}\psi_{j,i}] = g_7,\end{aligned}\quad (21)$$

where

$$\begin{aligned}\omega &= (\mathbf{u}, \varphi, \theta), \quad g_i = \lambda \hat{u}_i + \hat{v}_i, \quad g_{i+3} = \lambda \hat{\varphi}_i + \hat{\psi}_i, \\ g_7 &= \hat{\theta} + \frac{1}{\alpha} [E_{ij}(\hat{v}_{j,i} + \varepsilon_{jik}\hat{\psi}_k) + D_{ij}\hat{\psi}_{j,i}].\end{aligned}\quad (22)$$

Let $\langle \cdot, \cdot \rangle$ denote the conveniently weighted $[L_2(B)]^7$ inner product and consider the bilinear form

$$\begin{aligned}Q[\omega, \bar{\omega}] &= \langle \mathbf{L}\omega, \bar{\omega} \rangle = \langle (\mathcal{L}_i\omega, \mathcal{L}_{i+3}\omega, \mathcal{L}_7\omega), (\bar{u}_i, \bar{\varphi}_i, \bar{\theta}) \rangle = \\ &= \int_B [\rho \bar{u}_i \mathcal{L}_i\omega + I_{ij} \bar{\psi}_j \mathcal{L}_{i+3}\omega + \frac{\alpha}{\lambda} \bar{\theta} \mathcal{L}_7\omega] dv\end{aligned}\quad (23)$$

Using the Green-Gauss formula and the boundary conditions (7), it results

$$\begin{aligned}Q[\omega, \omega] &= \int_B [\rho u_i u_i + I_{ij} \varphi_i \varphi_j + \alpha \theta^2 + \\ &\quad A_{ijmn}(u_{n,m} + \varepsilon_{nmk}\varphi_k)(v_{j,i} + \varepsilon_{jis}\psi_s) + \\ &\quad 2B_{ijmn}(u_{j,i} + \varepsilon_{jis}\varphi_s)\varphi_{n,m} + C_{ijmn}\varphi_{n,m}\varphi_{j,i}] dv + \\ &\quad \frac{1}{\lambda T_0} \int_B K_{ij}\theta_{,i}\theta_{,j} dv,\end{aligned}\quad (24)$$

for any $\omega = (\mathbf{u}, \varphi, \theta) \in Y$, $Y \equiv \mathbf{H}_0^1(B) \times \mathbf{H}_0^1(B) \times H_0^1(B)$.

Due to the hypothesis *i*), *ii*), *iii*) and the first Korn inequality it follows that

$$Q[\omega, \omega] \geq C_1 |\omega|_Y^2, \quad \text{for all } \omega = (\mathbf{u}, \varphi, \theta) \in Y, \quad (25)$$

where C_1 is a positive, conveniently chosen, constant and the norm $|\omega|_Y$ is defined by

$$|\omega|_Y = |(\mathbf{u}, \varphi, \theta)|_Y = |\mathbf{u}|_{\mathbf{H}^1(B)} + |\varphi|_{\mathbf{H}^1(B)} + |\theta|_{H^1(B)}.$$

In the usual way, we can prove that

$$Q[\omega, \omega] \leq C_1 |\omega|_Y^2,$$

hence the bilinear form $Q[\omega, \bar{\omega}]$ determines a norm equivalent to the original norm in Y . Since the bilinear form $Q[\omega, \bar{\omega}]$ is continuous in $Y \times Y$ we deduce that there exist a linear bounded transformation T from Y into itself such that we have

$$Q[\omega, \bar{\omega}] = \langle \omega, T\bar{\omega} \rangle_Y, \text{ for any } (\omega, \bar{\omega}) \in Y \times Y. \quad (26)$$

Since

$$\langle \omega, T\omega \rangle_Y = Q[\omega, \omega] \geq C_1 |\omega|_Y^2, \quad (27)$$

we deduce

$$|T\omega|_Y \geq C_1 |\omega|_Y, \quad \omega \in Y. \quad (28)$$

Let $R(T)$ be the range of T . The linear transformation T is one to one. We need to prove that $T\omega = 0$ implies that $\omega = 0$. Indeed, if there is $\omega_0 \in Y$ such that $T\omega_0 = 0$, then (26) implies $Q[\omega_0, \omega_0] = 0$ and then the inequality (25) proves that $\omega_0 = 0$. Therefore, there exists $T^{-1} : R(T) \rightarrow Y$. Now we prove that $R(T)$ is dense in Y . We assume to the contrary that there is $\omega_0 \in Y \setminus R(T)$, $\omega_0 \neq 0$ such that $\langle \omega_0, T\bar{\omega} \rangle_Y = 0$ for any $\bar{\omega} \in Y$. But from (26) we deduce that $Q[\omega_0, \omega_0] = 0$ such that with aid of (25) we deduce that $\omega_0 = 0$. This contradicts the initial assumptions and therefore we obtain that $R(T)$ is dense in Y . So we can continue T^{-1} to Y , such that

$$T^{-1} : Y \rightarrow Y \text{ and } |T^{-1}| \leq C_1^{-1}.$$

Let \mathbf{z} be in $R(T)$ and ω the only function in Y such that $\mathbf{z} = T\omega$. We define the functional \mathcal{K} by $\mathcal{K}(\mathbf{z}) = \langle \mathbf{g}, \omega \rangle$. Obviously, we have

$$|\mathcal{K}(\mathbf{z})| \leq |\mathbf{g}|_{\mathbf{H}_0^{-1}(B)} |\omega|_Y \leq C_1^{-1} |\mathbf{g}|_{\mathbf{H}_0^{-1}(B)} |\mathbf{z}|_Y,$$

and then we deduce that \mathcal{K} is a linear bounded functional defined over $R(T)$ such that

$$|\mathcal{K}| \leq C_1^{-1} |\mathbf{g}|_{\mathbf{H}_0^{-1}(B)}.$$

We can continue \mathcal{K} in the whole space Y , in such a way that the continued functional \mathcal{K} shall have the same norm. On the other hand, since Y is a Hilbert space, the Riesz-Frechet theorem, [10], prove that there exists a unique $\omega \in Y$ such that

$$\mathcal{K}(\tilde{\omega}) = \langle \omega, \tilde{\omega} \rangle_Y, \text{ for any } \tilde{\omega} \in Y, \quad |\omega|_Y = |\mathcal{K}| \leq C_1^{-1} |\mathbf{g}|_{\mathbf{H}_0^{-1}(B)}. \quad (29)$$

If we choose $\tilde{\omega} = T\bar{\omega}$, then from (26) and (29), it follows that the unique $\omega \in Y$ satisfies the equation

$$Q[\omega, \bar{\omega}] = \langle \mathbf{g}, \bar{\omega} \rangle, \text{ for all } \bar{\omega} \in Y. \quad (30)$$

From the relations $\lambda u_i - \hat{u}_i = v_i$, $\lambda \varphi_i - \hat{\varphi}_i = \psi_i$ and $\lambda \theta - \hat{\theta} = \tau$, it follows that $\mathbf{v} \in \mathbf{H}_0^1(B)$, $\psi \in \mathbf{H}_0^1(B)$ and $\tau \in H_0^1(B)$. Therefore we deduce that $\mathbf{W} = (\mathbf{u}, \mathbf{v}, \varphi, \psi, \theta)$ is in $D(\mathcal{L})$ and the proof of the Lemma 2 is complete.

Theorem 1. *The operator \mathcal{L} defined by the relations (12) generates a C_0 -semigroup of contractions on X .*

Proof. This result follows immediately from the Lummer-Phillips theorem, [9].

In order to study the existence and uniqueness of the solution for the inhomogeneous equation (14), we use the following result:

Theorem 2. *Let \mathcal{L} be the infinitesimal generator of a C_0 -contractive semigroup $T(t)$ on X . If $\mathcal{F}(s)$ is continuously differentiable on $[0, t_0]$, then the initial value problem (14) has, for every $\mathbf{W}_0 \in D(\mathcal{L})$, the unique solution*

$$\mathbf{W}(t) = T(t)\mathbf{W}_0 + \int_0^t T(t-s)\mathcal{F}(s)ds, \quad t \in [0, t_0], \quad (31)$$

such that

$$\mathbf{W}(t) \in C^1([0, t_0]; X) \cap C^0([0, t_0]; D(\mathcal{L})).$$

On the basis of the above theorem, we deduce:

Theorem 3. *Let us assume that the thermoelastic coefficients, which are continuously differentiable, satisfy the conditions i), ii), iii). Moreover, we assume that $\mathbf{F} \in C^1([0, t_0]; \mathbf{L}_2(B))$, $\mathbf{M} \in C^1([0, t_0]; \mathbf{L}_2(B))$, $r \in C^1([0, t_0]; L_2(B))$ and $\mathbf{W}_0 = (\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \theta^0) \in D(\mathcal{L})$. Then there exists an unique solution of the initial-boundary value problem (9), (7), (8) such that*

$$(\mathbf{u}, \dot{\mathbf{u}}, \varphi, \dot{\varphi}, \theta) \in [C^1([0, t_0]; X) \cap C^0([0, t_0]; D(\mathcal{L}))]^{13}.$$

The following theorem establish the continuous dependence of the solution of our problem upon the initial data and supply terms. Let (u_i, φ_i, θ) be the difference of two solutions of the problem defined by (9), (7), (8) but corresponding to the differences of the initial data and to the differences of body forces, body couples and heat supplies, $\mathbf{W}_0 = (\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \theta^0)$, $(\mathbf{F}, \mathbf{M}, r)$, respectively.

Theorem 4. *Let us assume that the thermoelastic coefficients, which are continuously differentiable, satisfy the conditions i), ii), iii).*

Moreover, we assume that $\mathbf{F} \in C^1([0, t_0]; \mathbf{L}_2(B))$, $\mathbf{M} \in C^1([0, t_0]; \mathbf{L}_2(B))$, $r \in C^1([0, t_0]; L_2(B))$ and $\mathbf{a} \in \mathbf{H}^1(B)$, $\mathbf{b} \in \mathbf{H}^0(B)$, $\mathbf{c} \in \mathbf{H}^1(B)$, $\mathbf{d} \in \mathbf{H}^0(B)$ and $\theta \in H^1(B)$. If $(\mathbf{u}, \varphi, \theta)$ is the difference of two solutions of the problem (9), (7), (8),

then there exists a positive constant M such that

$$\begin{aligned} & |\mathbf{u}|_{\mathbf{H}^1(B)} + |\dot{\mathbf{u}}|_{\mathbf{H}^0(B)} + |\varphi|_{\mathbf{H}^1(B)} + |\dot{\varphi}|_{\mathbf{H}^0(B)} + |\theta|_{H^0(B)} \leq \\ & \leq M\{|\mathbf{a}|_{\mathbf{H}^1(B)} + |\mathbf{b}|_{\mathbf{H}^0(B)} + |\mathbf{c}|_{\mathbf{H}^1(B)} + |\mathbf{d}|_{\mathbf{H}^0(B)} + |\theta^0|_{H^0(B)} + \\ & \quad + \int_0^t [|\mathbf{F}(s)|_{\mathbf{H}^0(B)} + |\mathbf{M}(s)|_{\mathbf{H}^0(B)} + |r(s)|_{H^0(B)}] ds\}. \end{aligned} \quad (32)$$

Proof. On the basis of the equations (9), (7), (8) we can deduce the following identity

$$\begin{aligned} & \int_B [\rho \dot{u}_i \dot{u}_i + I_{ij} \dot{\varphi}_i \dot{\varphi}_j + a \theta^2 + A_{ijmn}(u_{n,m} + \varepsilon_{nmk} \varphi_k)(u_{j,i} + \varepsilon_{jis} \varphi_s) + \\ & \quad + 2B_{ijmn}(u_{j,i} + \varepsilon_{jis} \varphi_s) \varphi_{n,m} \\ & \quad + C_{ijmn} \varphi_{n,m} \varphi_{j,i}] dv + 2 \int_0^t \int_B K_{ij} \theta_{,i} \theta_{,j} dv ds = \\ & = \int_B [\rho \dot{a}_i \dot{a}_i + I_{ij} \dot{c}_i \dot{c}_j + a(\theta^0)^2 + A_{ijmn}(a_{n,m} + \varepsilon_{nmk} c_k)(a_{j,i} + \varepsilon_{jis} c_s) + \\ & \quad + 2B_{ijmn}(a_{j,i} + \varepsilon_{jis} c_s) c_{n,m} + C_{ijmn} c_{n,m} c_{j,i}] dv + \\ & \quad + 2 \int_0^t \int_B [F_i u_i + M_i \varphi_i + \frac{1}{T_0} r \theta] dv ds, \quad s \in [0, t_0]. \end{aligned} \quad (33)$$

By using the Schwarz's inequality, the hypothesis *i*), *ii*), *iii*) and the first Korn's inequality, from the identity (33) we deduce a Gronwall inequality that demonstrates the estimate (32).

Remark. A similar procedure can be used in the case when the boundary conditions (7) are replaced by the other boundary conditions and the above results are still valid.

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