MAGNETIC DYNAMICS AROUND ELECTRICAL CIRCUITS

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Abstract

§1 specifies some properties of the Biot-Savart-Laplace integral along curves. §2 reports on the results of Sabba Stefănescu regarding the magnetic fields produced by piecewise rectilinear electrical circuits. Most relevant for the present discussion are the examples of magnetic fields with homoclinic behaviour, magnetic fields with open lines and examples of algebraic magnetic surfaces. §3 describes a new magnetic dynamics around a spire of a coil.

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1 Biot-Savart-Laplace integral and the memory of segments and semilines

Let γ be a curve arc. If $M \in R^3\backslash\gamma$, then the *Biot-Savart-Laplace integral* is the vector field

$$\vec{F}_\gamma(M) = \int_\gamma \frac{\vec{v} \times \overrightarrow{PM}}{PM^3} d\tau_P,$$

where $\vec{v}$ is a unit vector field tangent to γ, and $P \in \gamma$ is the variable point.

If $\mathcal{T} : R^3 \to R^3$ is a translation, $\mathcal{R} : R^3 \to R^3$ is an orthogonal transformation and $\mathcal{J} = \mathcal{T} \circ \mathcal{R}$ is an isometry, then

$$\vec{F}_{\mathcal{T}\gamma}(\mathcal{T}M) = \vec{F}_\gamma(M)$$

$$\vec{F}_{\mathcal{R}\gamma}(\mathcal{R}M) = \mathcal{R}\vec{F}_\gamma(M)$$

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$$\vec{F}_{\mathcal{J}\gamma}(\mathcal{J}M) = \mathcal{R}\vec{F}_\gamma(M), \forall M \in R^3 \setminus \gamma.$$

These relations reflects the influence of the isometries of the space R^3 upon the vector field $\vec{F}$.

If γ is a straight line, or a curve unbounded at both ends, or a closed curve,then the field $\vec{F}$ differs by a multiplicative constant from the magnetic field $\vec{H}$ around the circuit γ generated by the current $\vec{v}$.

Suppose γ is a segment, or a straight semiline of direction $\vec{v}$. Fixing the origin $O \in \gamma$, the set γ is described by the vectorial equation

$$\vec{OP} = t\vec{v}, \quad t \in I,$$

where I is either $[0,b]$, or $[0,\infty)$, or $(-\infty,0]$. Denoting $M(x,y,z)$ we have $\vec{OM}= x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{PM}=\vec{OM} - \vec{OP}$. It is obtained $\vec{PM}\ \vec{M} = \vec{v}\,\vec{OM}$, and

$$PM = \|\vec{PM}\| = (OM^2 - 2t(\vec{OM},\vec{v}) + t^2)^{1/2}.$$

It follows

$$\vec{F}(M) = \vec{v}\times \vec{OM} \int_I \frac{dt}{(OM^2 - 2t(\vec{OM},\vec{v}) + t^2)^{3/2}}.$$

For obtaining the primitive necessary for the computation of the integral we use the substitution

$$(OM^2 - 2t(\vec{OM},\vec{v}) + t^2)^{1/2} = t + s$$

and we find

$$\vec{F}(M) = \vec{v}\times \vec{OM}\ \frac{2}{0M^2 + s^2 + 2s(\vec{OM},\vec{v})}\Bigg|_{s=s_1}^{s=s_2},$$

where s_1 corresponds to the lower bound, and s_2 corresponds to the upper bound of the interval I. If $I = (-\infty, 0]$, it is obtained

$$\vec{F}(M) = \frac{\vec{v}\times \vec{OM}}{OM(OM + (\vec{OM},\vec{v}))}.$$

If $I = [0,\infty)$, we find

$$\vec{F}(M) = \frac{\vec{v}\times \vec{OM}}{OM(OM + (\vec{OM},\vec{v}))}.$$

If $I = [0,b]$, then

$$\vec{F}(M) = 2\vec{v}\times \vec{OM}\left(\frac{1}{OM^2 + s_2^2 + 2s_2(\vec{OM},\vec{v})} - \frac{1}{OM(OM + (\vec{OM},\vec{v}))}\right),$$

where

$$s_2 = -b + (OM^2 - 2b(\vec{OM},\vec{v}) + b^2)^{1/2}.$$

Let Γ be a straight line and γ be either a segment or a semiline of Γ. The zeros set of the vector field $\vec{F}_\gamma$ is $\Gamma \setminus \gamma$. The nonconstant field lines of $\vec{F}_\gamma$ are circles situated in planes orthogonal to Γ. While $\vec{F}_\gamma$ is not a properly magnetic field, the vector field $\vec{F}_\Gamma$ is a magnetic field. The field $\vec{F}_\Gamma$ has no zeros, and its fields lines are circles situated in planes orthogonal to Γ. Consequently the phase portraits of $\vec{F}_\gamma$ and $\vec{F}_\Gamma$ consists of the same orbits on $R^3 \setminus \Gamma$. This property can be called *the memory of segments and semilines.*

Open problem: There exist other configurations Γ with the property that $\vec{F}_\gamma$ and $\vec{F}_\Gamma$ have the same phase portrait on $R^3 \setminus \Gamma$?

2 Some results and conjectures of Sabba Ştefănescu

In the opinion of Sabba Ştefănescu [1]–[9], the geometry of the phase portrait of a magnetic field generated by piecewise rectilinear electrical circuits is useful for building nuclear reactors (fusion of atomic nucleus), for guidance of charge particles in the terrestrial magnetic field etc. This is a reason to study the phase portraits of the magnetic fields produced by the elementary configurations.

The paper [1] studies the phase portrait of the magnetic field around a symmetric twice-bended electrical circuit (Fig.1).

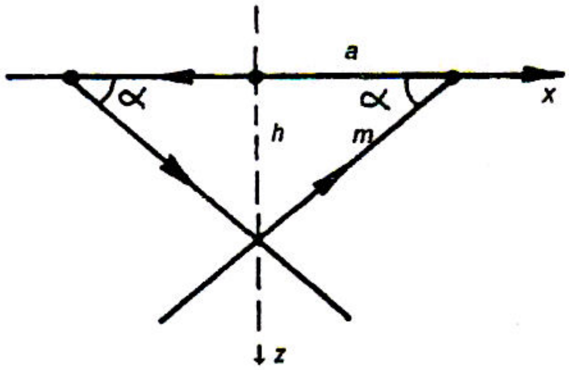

Fig. 1

Results: - the explicit expressions of the components of the magnetic field $\vec{H}$ are

$$H_x = \frac{y \sin\alpha}{r_2[r_2 + (x-a)\cos\alpha - z\sin\alpha]} - \frac{y\sin\alpha}{r_1[r_1 - (x+a)\cos\alpha - z\sin\alpha]}$$

$$H_y = \frac{z}{r_1(r_1 - x - a)} - \frac{z}{r_2(r_2 - x + a)} - \frac{(x-a)\sin\alpha + z\cos\alpha}{r_2[r_2 + (x-a)\cos\alpha - z\sin\alpha]} + \\ + \frac{(x+a)\sin\alpha - z\cos\alpha}{r_1[r_1 - (x+a)\cos\alpha - z\sin\alpha]}$$

$$H_z = \frac{-y}{r_1(r_1 - x - a)} + \frac{y}{r_2(r_2 - x + a)} + \frac{y\cos\alpha}{r_2[r_2 + (x-a)\cos\alpha - z\sin\alpha]} + \\ + \frac{y\cos\alpha}{r_1[r_1 - (x+a)\cos\alpha - z\sin\alpha]},$$

$$r_1^2 = (x+a)^2 + y^2 + z^2,\ r_2^2 = (x-a)^2 + y^2 + z^2;$$

- the set of equilibrium points is the ellipse $r_1 + r_2 = 2m$ without the point $(0, 0, h)$;
- the explicit formulas for the field lines are given and the algebraic first integral is

$$\varphi(x,y,z) = \frac{(r_1 - r_2)^2 (r_1 + r_2 - 2a)(r_1 + r_2 + 2a)}{(r_3 - r_1 + m)(r_3 + r_1 - m)(r_3 - r_2 + m)(r_3 + r_2 - m)},$$

$$r_1^2 = (x+a)^2 + y^2 + z^2, \quad r_2^2 = (x-a)^2 + y^2 + z^2,$$

$$r_3^2 = x^2 + y^2 + (z - a\tan\alpha)^2,\ a = m\cos\alpha;$$

- the homoclinic behaviour of the magnetic field generated by a symmetric twice–bended electrical circuit is proved.

Let us explicitate the set of equilibrium points in the plane $y = 0$. In this condition we have

$$H_x = 0, \quad H_z = 0$$

$$H_y = \frac{z}{r_1(r_1 - x - a)} - \frac{z}{r_2(r_2 - x + a)} + \frac{(x+a)\sin\alpha - z\cos\alpha}{r_1(r_1 - (x+a)\cos\alpha - z\sin\alpha)} -$$

$$- \frac{(x-a)\sin\alpha + z\cos\alpha}{r_2(r_2 + (x-a)\cos\alpha - z\sin\alpha)},$$

$$r_1^2 = (x+a)^2 + z^2, \quad r_2^2 = (x-a)^2 + z^2.$$

On the other hand, we can write

$$H_y(x,0,z) = \frac{1}{z} + \frac{r_1\sin\alpha}{z((x+a)\sin\alpha - z\cos\alpha)} + \frac{1}{(x+a)\sin\alpha - z\cos\alpha} -$$

$$-\frac{1}{z} - \frac{r_2\sin\alpha}{z((x-a)\sin\alpha + z\cos\alpha)} - \frac{1}{(x-a)\sin\alpha + z\cos\alpha} = 0.$$

It follows

$$\frac{r_1\sin\alpha + z}{z}\frac{1}{(x+a)\sin\alpha - z\cos\alpha} = \frac{r_2\sin\alpha + z}{z}\frac{1}{(x-a)\sin\alpha + z\cos\alpha},$$

$$(r_1\sin\alpha + z)((x-a)\sin\alpha + z\cos\alpha) = (r_2\sin\alpha + z)((x+a)\sin\alpha - z\cos\alpha),$$

$$(x-a)r_1\sin^2\alpha + r_1 z\sin\alpha\cos\alpha + z(x-a)\sin\alpha + z^2\cos\alpha =$$

$$= (x+a)r_2\sin^2\alpha - r_2 z\sin\alpha\cos\alpha + z(x+a)\sin\alpha - z^2\cos\alpha,$$

$$(x-a)r_1\sin^2\alpha + r_1 z\sin\alpha\cos\alpha - az\sin\alpha + z^2\cos\alpha =$$

$$= (x+a)r_2\sin^2\alpha - r_2 z\sin\alpha\cos\alpha + az\sin\alpha - z^2\cos\alpha,$$

$$x(r_1 - r_2)\sin^2\alpha - a(r_1 + r_2)\sin^2\alpha + (r_1 + r_2)z\sin\alpha\cos\alpha - 2az\sin\alpha + 2z^2\cos\alpha = 0$$

$$r_1 - r_2 = \frac{2ax}{m},\ r_1 + r_2 = 2m,\ \sin\alpha = \frac{h}{m},\ \cos\alpha = \frac{a}{m}$$

$$\frac{2ax^2}{m}\frac{h^2}{m^2} - a\frac{h^2}{m^2}2m + 2mz\frac{ha}{m^2} - 2az\frac{h}{m} + 2z^2\frac{a}{m} = 0$$

$$\frac{x^2h^2}{m^2} - h^2 + z^2 = 0,\ \frac{x^2}{m^2} + \frac{z^2}{h^2} = 1.$$

Equilibrium set: Ellipse, $r_1 + r_2 = 2m$ without the point $(0, 0, h)$.

The preceding results are sufficient to describe the magnetic traps around a symmetric twice–bended electrical circuit.

The paper [2] analyses the phase portrait of the magnetic field generated by a general twice-bended electrical circuit (Fig.2).

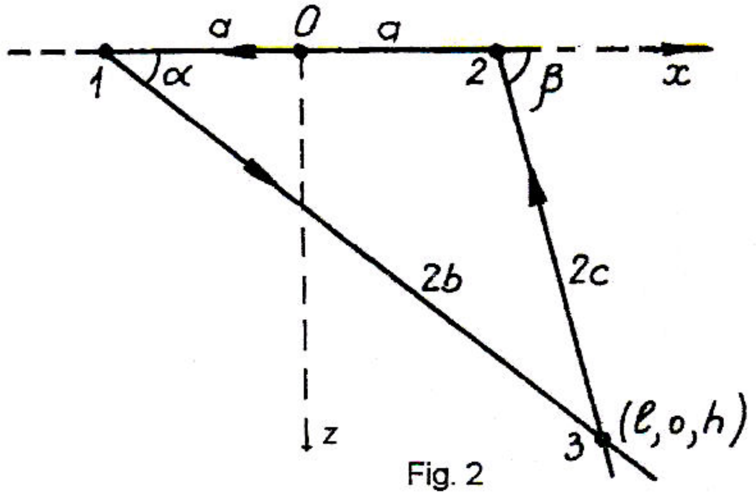

Fig. 2

Imposing

$$\vec{v} = \begin{cases} -\vec{\imath}\cos\beta + \vec{k}\sin\beta & \text{on} \quad 23 \\ -\vec{\imath} & \text{on} \quad 12 \\ -\vec{\imath}\cos\alpha + \vec{k}\sin\alpha & \text{on} \quad 13, \end{cases}$$

Sabba Ştefănescu has obtained the following results:

- the components of the magnetic field $\vec{H}$ are

$$H_x = \frac{-y\sin\alpha}{r_1[r_1 - (x+a)\cos\alpha - z\sin\alpha]} + \frac{y\sin\beta}{r_2[r_2 - (x-a)\cos\beta - z\sin\beta]}$$

$$H_y = \frac{z}{r_1(r_1 - x - a)} - \frac{z}{r_2(r_2 - x + a)} + \frac{(x+a)\sin\alpha - z\cos\alpha}{r_1[r_1 - (x+a)\cos\alpha - z\sin\alpha]} -$$
$$-\frac{(x-a)\sin\beta - z\cos\beta}{r_2[r_2 - (x-a)\cos\beta - z\sin\beta]}$$

$$H_z = \frac{-y}{r_1(r_1 - x - a)} + \frac{y}{r_2(r_2 - x + a)} + \frac{y\cos\alpha}{r_1[r_1 - (x+a)\cos\alpha - z\sin\alpha]} +$$
$$+\frac{-y\cos\beta}{r_2[r_2 - (x-a)\cos\beta - z\sin\beta]},$$

$r_1^2 = (x+a)^2 + y^2 + z^2$, $r_2^2 = (x-a)^2 + y^2 + z^2$, $r_3^2 = (x-l)^2 + y^2 + (z-h)^2$;

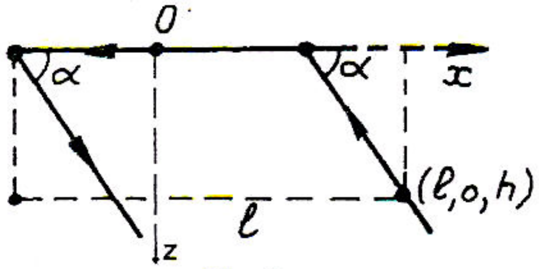

Fig. 3

- the magnetic lines are situated on the algebraic magnetic surfaces

$$\frac{(r_1 - r_2 - 2b + 2c)^2 (r_1 + r_2 - 2a)(r_1 + r_2 + 2a)}{(r_3 - r_2 + 2c)(r_3 + r_2 - 2c)(r_3 - r_1 + 2b)(r_3 + r_1 - 2b)} = \text{const}$$

of degree 4.

Remarks. 1) For $\beta = 180^o - \alpha$ we obtain the symmetrical case solved in [1].

2) For $\beta = \alpha$ we obtain a special configuration (Fig.3) and all the results of S.Ştefănescu are true.

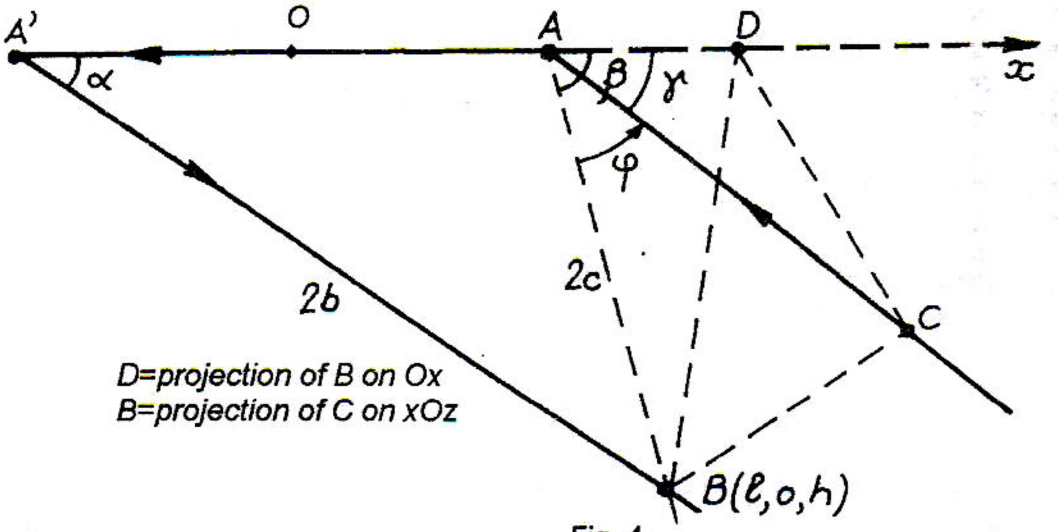

Fig. 4

3) Particular case, $\alpha = \beta = \frac{\pi}{2}$,

$$\begin{aligned} H_x &= \frac{y}{r_2(r_2 - z)} - \frac{y}{r_1(r_1 - z)} \\ H_y &= \frac{z}{r_1(r_1 - x - a)} - \frac{z}{r_2(r_2 - x + a)} - \frac{x - a}{r_2(r_2 - z)} + \frac{x + a}{r_1(r_1 - z)} \\ H_z &= \frac{-y}{r_1(r_1 - x - a)} + \frac{y}{r_2(r_2 - x + a)}. \end{aligned}$$

4) Automatically we have a generalization for the configuration of Fig.4. In this case

$$\frac{h}{2c} = \sin\beta; \ \frac{2c}{AC} = \cos\varphi\, ; AC = \frac{2c}{\cos\varphi}, \ AD = 2c\cos\beta, \ \cos\gamma = \frac{AD}{AC}.$$

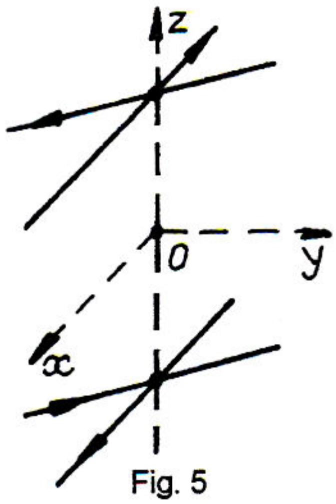

Fig. 5

The paper [3] analyses the magnetic lines around two pairs of rectilinear electrical circuits situated in parallel planes, the currents having the sense in Fig.5.

Results: - explicit computation of a family of algebraic magnetic surfaces of degree 8,

$$4y^2z^2\frac{[\nu^2(1+\mu^2)x^2+\mu^2\nu^2y^2+\nu^2z^2+h^2(1+\mu^2)]^2}{D_1^2D_2^2D_3^2D_4^2} = \text{const.},$$

$$D_1^2=(z+h)^2+(-x\nu+y\mu)^2, \quad D_2^2=(z-h)^2-(-x\nu+y\mu)^2$$

$$D_3^2=(z+h)^2+(x\nu+y\mu)^2, \quad D_4^2=(z-h)^2+(x\nu+y\mu)^2;$$

- the magnetic lines in the plane $z=0$ are open curves;
- generally, a magnetic line has an open helicoidal shape and can be defined on the magnetic algebraic surfaces by Abelian integrals;
- a magnetic line can be everywhere dense in a magnetic surface.

The paper [4] studies the magnetic lines around a directilinear dicurrent (Fig.6).

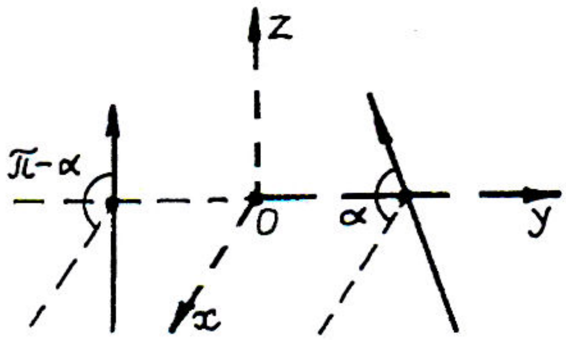

Fig. 6

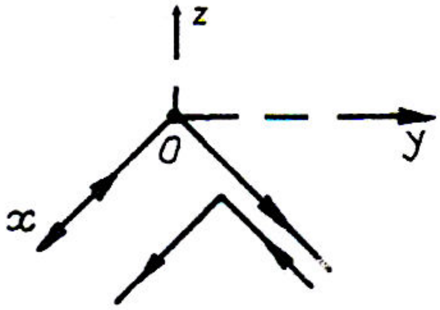

Fig. 7

Results: - explicit computation of an algebraic first integral,

$$\varphi(x,y,z)=\frac{(zx+ay)^2+z^2(y^2+z^2)}{(y^2+z^2)^2}$$

- the magnetic lines are helicoidal open curves situated on ruled algebraic surfaces of degree 4; they are determined by elliptical integrals.

The paper [5] develops further the ideas of the paper [4];

Results: - the differential system of the magnetic lines admits a family of algebraic ruled magnetic surfaces of degree 4,

- the magnetic lines are open helicoidal curves that can be defined on the above magnetic surfaces by elliptical integrals of the first and second kind. The step of these helices, measured on an arbitrary generatrix of an algebraic magnetic surface, is constant and characteristic for the considered surface.

In the papers [3]-[5] the magnetic fields are generated by infinite lines of electrical currents.

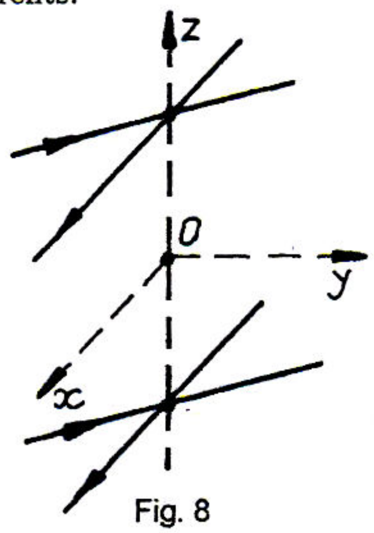

Fig. 8

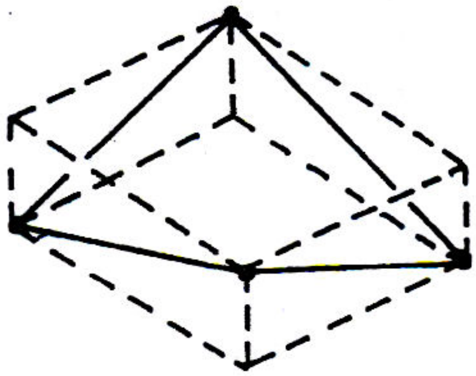

Fig. 9

The paper [6] discusses some configurations which are strongly connected with the circuits feasible in the laboratories, but with open magnetic lines (Fig.7). Two new concepts are introduced: 1) the needle- current as the limit of an angular current whose angle tends to zero, while the current intensity tends towards infinite; 2) the U-dicurrent, as the limit of a piecewise rectilinear U-shaped current, with two vertices, in which the distance between the parallel sides tends towards zero, while the current intensity tends to infinite. In this case the phase portrait contains open helicoidal magnetic lines.

Now we select some ideas in the commentary of the paper [7]. The basic configuration analysed in this paper consists of four infinite rectilinear currents of the same intensity forming two complete equipollent angles situated in two parallel planes and having their centers on the same vertical Oz (Fig.8). In case of nonplanar electrical circuits, the shape of magnetic lines becomes particularly intricate, sometimes apparently chaotic as far as the electrical circuit generating the magnetic field deviates from planarity (e.g. a skewed square circuits, Fig.9).

From the theoretical point of view it is very interesting to know whether in the complicated structure of magnetic phase portraits, especially of those due to nonplanar electrical circuits, one can detect simpler structural elements. The investigations undertaken in this direction lead to the identification, in a sequence of particular cases, of the existence of certain algebraic first integrals for the differential equations of magnetic lines. These first integrals determine pencils of algebraic surfaces on which there are wound magnetic lines, generally transcedental ones. The complete determination of magnetic lines may be reduced in these cases to Abelian quadratures and in their turn, these quadratures, under entirely special conditions, may be reduced to elliptical integrals and even to elementary transcedental ones.

- **Sabba Ştefănescu Conjecture.** *The differential system describing the lines of a magnetic field generated by piecewise rectilinear electrical circuits admits an algebraic first integral.*

Open problem. *Which are the Cartesian implicit equations and the shape of the magnetic lines around n equal rectilinear currents placed arbitrarily in space? A known solution:* $n = 2$.

The commentary of Sabba Ştefănescu made in paper [8] contains also a lot of interesting ideas. First he mention that some controversies in Romanian technical world are due to the wrong idea that magnetic lines around filiform electrical circuits are always closed. Also, the existence of open magnetic lines was almost unanimously ascribed to the fact the system of two equal infinite rectilinear currents cannot constitute by any means a closed circuits. This objection is fully grounded. But, if it is admitted that the two currents are closed at infinity through filiform rectilinear junctions, one may prove that these junctions determine, within large finite distance, a negligible magnetic field, and consequently, this part of the field cannot modify the quasi-helicoidal form of the magnetic lines [5]. Other examples of open magnetic lines [3]-[7] raised similar objections. The careful investigation of these problems led Sabba Stefanescu to the conclusion that the closing/nonclosing of the magnetic lines is due not to the continuity/discontinuity of the generating electrical field $\vec{v}$ or circuits but to the fact that the circuits are/are not planar. Given the complexity of the differential system governing the magnetic lines the mathematical proof of this assertion appears as difficult and till now has not been approached so far.

The basic configuration exploited in the paper [8] is an electrical circuit of skew square type (obtained by connecting in series four diagonals of the faces of an upright prism whose basis is a square) and it has sure magnetic lines of the open helicoidal type (Fig.9). During the decrease of each angle towards zero, the magnetic lines remain open, of infinite length, but their turns become progressively closer.

The paper [10] of S.Bobbio shows that confinement of thermonuclear plasma requires stationary and quasi-stationary magnetic fields, and consequently the ideas of Sabba Ştefănescu are very modern and topical. Also, S.Bobbio mentions the **Grad conjecture** (confirmed by all numerical experiments performed recently): ***without symmetry is not possible the existence of a rational magnetic surface, immersed in a set of irrational surfaces*** (see [10] p.138). This conjecture is strongly related to **Sabba Stefanescu conjecture** (see also, [9], [11]- [17]).

3 Magnetic dynamics around a spire of a coil

A spire of a coil can be modelled by a four times bended electrical circuit Γ as in Fig.10, where $a, b \in (0, \infty)$. In this case we shall take

$$\vec{v} = \begin{cases} -\vec{k} & \text{on} \quad 65 \\ \vec{j} & \text{on} \quad 54 \\ -\vec{i} & \text{on} \quad 43 \\ -\vec{j} & \text{on} \quad 32 \\ \vec{k} & \text{on} \quad 21. \end{cases}$$

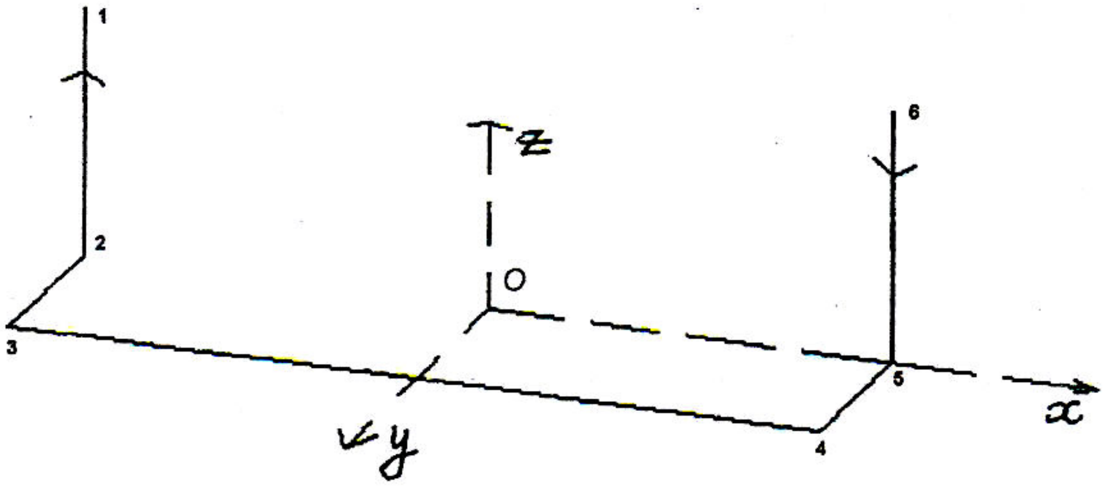

Fig.10

The magnetic field produced by the configuration Γ and the piecewise constant versor $\vec{v}$ is the sum of the following vector fields

$$\begin{aligned}
\vec{F}_{65}(M) &= \frac{-(x-a)\vec{j} + y\vec{i}}{r_2(r_2 - z)}, \quad r_2^2 = (x-a)^2 + y^2 + z^2 \\
\vec{F}_{54}(M) &= \frac{-z\vec{i} + (x-a)\vec{k}}{r_3(r_3 - y + b)} + \frac{z\vec{i} - (x-a)\vec{k}}{r_3(r_3 - y)}, \quad r_3^2 = (x-a)^2 + (y-b)^2 + z^2 \\
\vec{F}_{43}(M) &= \frac{-z\vec{j} + (y-b)\vec{k}}{r_3(r_3 - x + a)} + \frac{z\vec{j} - (y-b)\vec{k}}{r_4(r_4 - x - a)}, \quad r_4^2 = (x+a)^2 + (y-b)^2 + z^2 \\
\vec{F}_{32}(M) &= \frac{z\vec{i} - (x+a)\vec{k}}{r_4(r_4 - y + b)} + \frac{-z\vec{i} + (x+a)\vec{k}}{r_4(r_4 - y)} \\
\vec{F}_{21}(M) &= \frac{(x+a)\vec{j} - y\vec{i}}{r_1(r_1 - z)}, \; r_1^2 = (x+a)^2 + y^2 + z^2.
\end{aligned}$$

It follows the magnetic field

$$\vec{H} = \vec{F}_{65} + \vec{F}_{54} + \vec{F}_{43} + \vec{F}_{32} + \vec{F}_{21}$$

of components

$$H_x = -\frac{y}{r_1(r_1 - z)} + \frac{y}{r_2(r_2 - z)} + \frac{z}{r_3(r_3 - y)} - \frac{z}{r_3(r_3 - y + b)} - \frac{z}{r_4(r_4 - y)} + \frac{z}{r_4(r_4 - y + b)}$$

$$H_y = \frac{x + a}{r_1(r_1 - z)} - \frac{x - a}{r_2(r_2 - z)} - \frac{z}{r_3(r_3 - x + a)} + \frac{z}{r_4(r_4 - x - a)}$$

$$H_z = -\frac{x - a}{r_3(r_3 - y)} + \frac{x - a}{r_3(r_3 - y + b)} + \frac{y - b}{r_3(r_3 - x + a)} + \frac{x + a}{r_4(r_4 - y)} - \frac{x + a}{r_4(r_4 - y + b)} - \frac{y - b}{r_4(r_4 - x - a)}$$

defined on $R^3 \setminus \Gamma$.

From practical point of view, only a suitable selection of the values of the parameters a and b describes a magnetic field associated to a spire of a coil. Indeed, for a proper spire, the parameters a, b cannot be neither small nor large, since then we would have the following degenerate cases:

1) $$\lim_{a \to 0} H_x = 0,\ \lim_{a \to 0} H_y = 0,\ \lim_{a \to 0} H_z = 0;$$

2) $$\lim_{b \to 0} H_x = \frac{y}{r_2(r_2 - z)} - \frac{y}{r_1(r_1 - z)}$$

$$\lim_{b \to 0} H_y = \frac{z}{r_1(r_1 - x - a)} - \frac{z}{r_2(r_2 - x + a)} - \frac{x - a}{r_2(r_2 - z)} + + \frac{x + a}{r_1(r_1 - z)}$$

$$\lim_{b \to 0} H_z = \frac{y}{r_2(r_2 - x + a)} - \frac{y}{r_1(r_1 - x - a)};$$

3) $$\lim_{a \to \infty} H_x = 0,\ \lim_{a \to \infty} H_y = \frac{2z}{(y - b)^2 + z^2},\ \lim_{a \to \infty} H_z = -\frac{2(y - b)}{(y - b)^2 + z^2};$$

4) $$\lim_{b \to \infty} H_x = \frac{y}{r_2(r_2 - z)} - \frac{y}{r_1(r_1 - z)}$$

$$\lim_{b \to \infty} H_y = \frac{x + a}{r_1(r_1 - z)} - \frac{x - a}{r_2(r_2 - z)}$$

$$\lim_{b \to \infty} H_z = 0.$$

Theorem. *Any orbit of $\vec{H}$ determined by a point of the form $(0, y_0, z_0)$ is included in the plane yOz (Equivalently, the plane $yOz : x = 0$ is a magnetic surface).*

Proof. We find $H_x(0, y, z) = 0$.

The phase portrait of this magnetic field is suggested by the Figs. 11,12 drawn using a PC program for the case $a = 2$, $b = 1$. In the Fig. 11 we have the magnetic line determined by the initial point $(1,0,0)$, while in the Fig. 12 there appear the field lines corresponding to $(0,0,0)$ respectively to $(0.2,0,0)$.

Since rot $\vec{H} = 0$, the following theorem is true.

Theorem. *The nonclassical magnetic dynamics around* Γ *is described by the potential dynamical system with 3 degrees of freedom*

$$\frac{d^2x}{dt^2} = \frac{\partial f}{\partial x}, \frac{d^2y}{dt^2} = \frac{\partial f}{\partial y}, \frac{d^2z}{dt^2} = \frac{\partial f}{\partial z}$$

where $f = \frac{1}{2}(H_x^2 + H_y^2 + H_z^2)$ *is the energy of the magnetic field* $\vec{H}$.

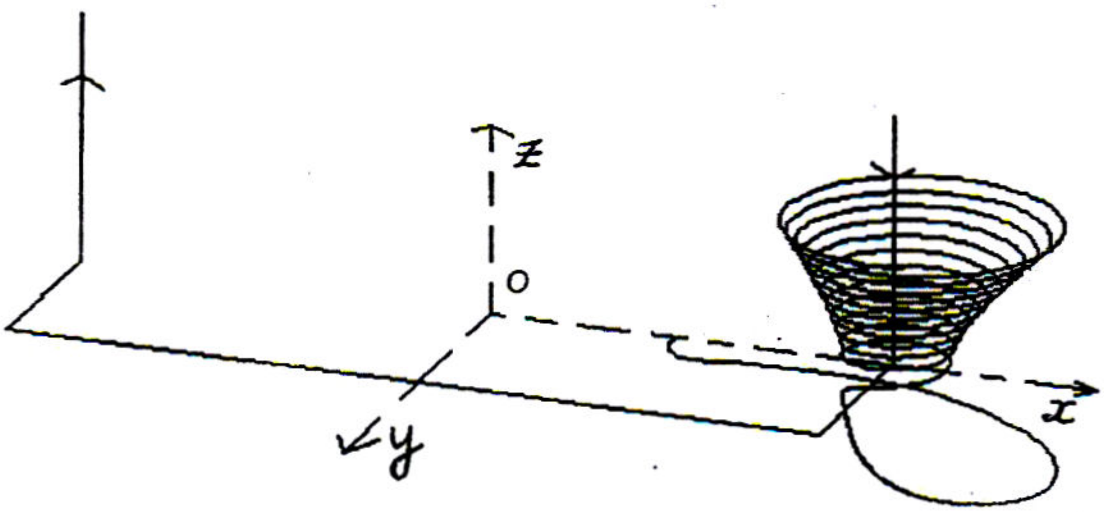

Fig.11

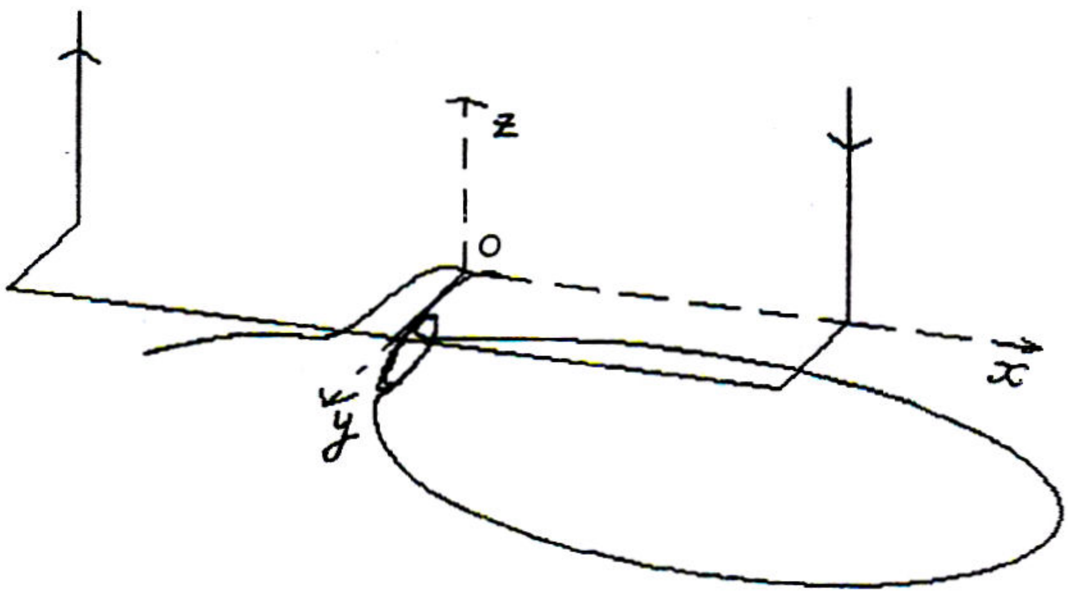

Fig.12

The preceding second order differential system is *conservative* since the restriction of the *Hamiltonian*

$$\mathcal{H} = \frac{1}{2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right] - f(x,y,z).$$

to each solution is a constant.

In fact the trajectories (solutions) of this dynamical system group in three classes:

- the set of original magnetic lines with the energy $\mathcal{H} = 0$,
- a set of trajectories with the energy $\mathcal{H} = \text{const} < 0$,
- a set of trajectories with the energy $\mathcal{H} = \text{const} > 0$,

The preceding dynamical system represents a new Lorentz world-force law since the next theorem is true.

Theorem. *Every trajectory of the dynamical system*

$$\frac{d^2x}{dt^2} = \frac{\partial f}{\partial x}, \ \frac{d^2y}{dt^2} = \frac{\partial f}{\partial y}, \ \frac{d^2z}{dt^2} = \frac{\partial f}{\partial z},$$

which has the total constant energy $\mathcal{H} > -f$, is a reparametrized geodesic of the Riemann-Jacobi metric

$$g_{ij} = (\mathcal{H} + f)\delta_{ij}, \ i,j = 1,2,3.$$

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