

FINSLER CONNECTION IN THE HIGHER ORDER GEOMETRY

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Abstract

The first part of this work is a natural extension of a recent paper by M.Anastasiu ([1]). In §1 the Finsler connections are defined by local components. In §2 a Finsler connection appears as a pair (N, ∇) , where N is a nonlinear connection on the jet bundle of order k , $Osc^k M$ and ∇ is a linear connection in the pull-back bundle of the tangent bundle by the projection map $\pi^k : Osc^k M \rightarrow M$. Next, the notion of linear connection of Cartan type is generalized and two examples are given.

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1 Introduction

Let M be a real, smooth manifold of dimension n and $(Osc^k M, \pi^k, M)$ its k -osculator bundle, $k \in \mathbb{N}^*$ (the jets bundle of order k of the manifold M). Then $Osc^k M$ is a real, smooth manifold of dimension $n(k+1)$. We set $E = Osc^k M$. For $k = 1$, $Osc^k M$ can be identified, in a canonical way, with the tangent bundle TM .

Let (x^i) be the local coordinates in a local chart $U \subset M$. The local coordinates on $(\pi^k)^{-1}(U) \subset Osc^k M$ will be denoted by $(x^i, y^{(1)i}, \dots, y^{(k)i})$.

For each $u \in E$, let $\{\frac{\partial}{\partial x^i} |_u, \frac{\partial}{\partial y^{(1)i}} |_u, \dots, \frac{\partial}{\partial y^{(k)i}} |_u\}$ be the natural basis of the tangent spaces $T_u E$.

As $(\pi^k)_* : (TE, \tau_E, E) \rightarrow (TM, \tau, M)$ is a π^k morphism of vector bundles, it results that its kernel is a vector subbundle of the bundle (TE, τ_E, E) . This will be denoted by $V_1 E$ and will be called the vertical subbundle of the TE . The fibers of $V_1 E$ determine an integrable distribution $V_1 : u \in E \mapsto V_1(u) \subset T_u E$ that has the dimension kn , called vertical distribution.

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For each $u \in E$ we consider the linear mapping $J_u : T_u E \rightarrow T_u E$ defined in the natural basis as follows:

$$(1.1) \quad \begin{aligned} J_u\left(\frac{\partial}{\partial x^i} \mid u\right) &= \frac{\partial}{\partial y^{(1)i}} \mid u, \dots, J_u\left(\frac{\partial}{\partial y^{(k-1)i}} \mid u\right) = \frac{\partial}{\partial y^{(k)i}} \mid u, \\ J_u\left(\frac{\partial}{\partial y^{(k)i}} \mid u\right) &= 0 \end{aligned}$$

and extended by linearity. The map $J : TE \rightarrow TE$ is called k -tangent structure. Consider $V_k = J^k(V_1)$ and the map $V_k : u \in E \rightarrow V_k(u)$ is an integrable, n -dimensional distribution.

Let be $(\pi^k)^*(TM) = \{(u, X) \in Osc^k M \times TM, \pi^k(u) = \tau(X)\}$ and the map $(\pi^k)^*(\tau) : (\pi^k)^*(TM) \rightarrow Osc^k M$ defined by $(\pi^k)^*(\tau)(u, X) = u$. That is, $((\pi^k)^*(TM), (\pi^k)^*(\tau), Osc^k M)$ is a vector bundle over E , called the pull-back bundle of the tangent bundle (TM, τ, M) by the map π^k . This vector bundle is isomorphic with the vertical bundle $(V_k E, \tau_E|_{V_k E}, E)$.

A section in this bundle is locally of the form

$$S : u = (x, y^{(1)}, \dots, y^{(k)}) \mapsto S^i(x, y^{(1)}, \dots, y^{(k)}) \overline{\frac{\partial}{\partial x^j}}$$

with $\overline{\frac{\partial}{\partial x^j}}$ the natural basis in $T_{\pi^k(u)} M$. This will be called π^k -vector field of TM and will be identified to a d-vector field on E , which has the components (S^i) . The $\mathcal{F}(E)$ -module of the π^k -vector fields will be denoted by $\Gamma((\pi^k)^*(TM))$. There exists a remarkable π^k -vector field on TM : $C : u \mapsto (u, \pi_1^k(u))$ and it can be identified to the Liouville vector field $\overset{1}{\Gamma} = y^{(1)i} \frac{\partial}{\partial y^{(k)i}}$

Let be $\overset{2}{\Gamma} = y^{(1)i} \frac{\partial}{\partial y^{(k-1)i}} + 2y^{(2)i} \frac{\partial}{\partial y^{(k)i}}, \dots, \overset{k}{\Gamma} = y^{(1)i} \frac{\partial}{\partial y^{(1)i}} + \dots + 2y^{(2)i} \frac{\partial}{\partial y^{(2)i}} + ky^{(k)i} \frac{\partial}{\partial y^{(k)i}}$. The vector fields $\overset{1}{\Gamma}, \overset{2}{\Gamma}, \dots, \overset{k}{\Gamma}$ are called Liouville vector fields of the k -osculator bundle.

A nonlinear connection in the k -osculator bundle is a subbundle NE , of the tangent bundle of E such that the Whitney sum

$$(1.2) \quad TE = NE \oplus V_1 E,$$

holds. A nonlinear connection induces a regular distribution \bar{N} on E of dimension n , which is supplementary to the vertical distribution V_1 . For each $u \in E$ the map $\pi_{*,u}^k|_{N(u)} : N(u) \rightarrow T_{\pi^k(u)} M = ((\pi^k)^*(TM))_u$ is an isomorphism of linear spaces. Its inverse, which is denoted by $(l_h)_u$, will be called horizontal lift. Using the notation $\frac{\delta}{\delta x^i} \mid u = (l_h)_u(\overline{\frac{\partial}{\partial x^i}})$ we obtain a basis for $N(u)$. This is expressed in the natural basis of the tangent space $T_u E$ as follows:

$$(1.3) \quad \frac{\delta}{\delta x^i} \mid u = \frac{\partial}{\partial x^i} \mid u - N_{(1)}^j \binom{j}{i}(u) \frac{\partial}{\partial y^{(1)j}} \mid u - \dots - N_{(k)}^j \binom{j}{i}(u) \frac{\partial}{\partial y^{(k)j}} \mid u.$$

The functions $(N_{(1)}^j, \dots, N_{(k)}^j)$ are called the coefficients of the nonlinear connection N .

We use the following notations: $\frac{\delta}{\delta y^{(\alpha)i}} = J^\alpha\left(\frac{\delta}{\delta x^i}\right)$. If $N_0 = N, N_1 = J(N_0), \dots, N_{k-1} =$

$J(N_{k-2})$ we obtain the following decomposition in the Whitney sum:

$$(1.4) \quad TE = N_0E \oplus N_1E \oplus \cdots \oplus N_{k-1}E \oplus V_kE.$$

For each $u \in E$,

$$(1.5) \quad \left\{ \frac{\delta}{\delta x^i} \Big|_u, \frac{\delta}{\delta y^{(1)i}} \Big|_u, \dots, \frac{\delta}{\delta y^{(k-1)i}} \Big|_u, \frac{\partial}{\partial y^{(k)i}} \Big|_u \right\}$$

is a non-holonomic basis for T_uE that is adapted to the decomposition (1.4). The dual basis of the basis (1.5) is expressed in the dual natural basis using a system of functions (M_i^j, \dots, M_i^j) which are called the dual coefficients of the nonlinear connection N .

The Liouville vector fields are expressed in the adapted basis as follows:

$$(1.6) \quad \begin{aligned} \overset{1}{\Gamma} &= z^{(1)i} \frac{\partial}{\partial y^{(k)i}} \\ \overset{2}{\Gamma} &= z^{(1)i} \frac{\delta}{\delta y^{(k-1)i}} + 2z^{(2)i} \frac{\partial}{\partial y^{(k)i}}, \dots, \\ \overset{k}{\Gamma} &= z^{(1)i} \frac{\delta}{\delta y^{(1)i}} + 2z^{(2)i} \frac{\delta}{\delta y^{(2)i}} + \dots + kz^{(k)i} \frac{\partial}{\partial y^{(k)i}}, \end{aligned}$$

where

$$(1.7) \quad \begin{aligned} z^{(1)i} &= y^{(1)i}, 2z^{(2)i} = 2y^{(2)i} + M_j^i y^{(1)j}, \dots, \\ kz^{(k)i} &= ky^{(k)i} + (k-1) \underset{(1)}{M_j^i} y^{(k-1)j} + \dots + \underset{(k-1)}{M_j^i} y^{(1)j}. \end{aligned}$$

Let $((\pi^k)^*(TM))^{(k)}$ the Whitney sum of the pull-back bundle on itself of k -times. We call connection map on the k -osculator bundle $([2, 3])$ a π^k epimorphism of vector bundles $K = (\overset{(1)}{K}, \overset{(2)}{K}, \dots, \overset{(k)}{K}) : TE \rightarrow ((\pi^k)^*(TM))^{(k)}$ that satisfies:

$$\begin{aligned} \overset{(k)}{K} \circ J^\alpha &= \overset{(k-\alpha)}{K}, \quad \forall \alpha \in \{1, 2, \dots, k-1\}, \\ \overset{(k)}{K} \circ J^k &= \pi_*^k. \end{aligned}$$

In the paper [3] we have proved that every nonlinear connection N is the kernel of a connection map.

Definition 1.1 A Finsler connection on the k -osculator bundle is a system $F\Gamma = (N, F_{ij}^m, C_{ij}^m, \alpha = \overline{1, k})$, where N is a nonlinear connection on the k -osculator bundle,

the functions F_{ij}^m are defined on every domain of local chart and which at a change of coordinates are modified like as the coefficients of a linear connection on M and C_{ij}^m are the local components of a d-tensor field of type (1,2).

To a Finsler connection $F\Gamma$ we associate a linear connection D on E , which is expressed in the adapted basis as follows:

$$(1.8) \quad \begin{aligned} D_{\frac{\delta}{\delta x^j}} \frac{\delta}{\delta x^i} &= F_{ij}^m \frac{\delta}{\delta x^m}, D_{\frac{\delta}{\delta x^j}} \frac{\delta}{\delta y^{(\alpha)i}} = F_{ij}^m \frac{\delta}{\delta y^{(\alpha)m}}, \\ D_{\frac{\delta}{\delta y^{(\beta)j}}} \frac{\delta}{\delta x^i} &= C_{ij}^m \frac{\delta}{\delta x^m}, D_{\frac{\delta}{\delta y^{(\beta)j}}} \frac{\delta}{\delta y^{(\alpha)i}} = C_{ij}^m \frac{\delta}{\delta y^{(\alpha)m}}. \end{aligned}$$

The linear connection D is called N -linear connection. It preserves by parallelism the distributions $N_0, N_1, \dots, N_{k-1}, V_k$ and the k -tangent structure J is parallel with respect to it.

For a Finsler connection $F\Gamma$ we denote by D_j^i the h -tensor fields of deflection and by d_j^i the v_α -tensor fields of deflection. These are given by:

$$(1.9) \quad \begin{aligned} D_{\frac{\delta}{\delta x^j}} \Gamma &= D_j^i \frac{\delta}{\delta y^{(1)i}} + 2 D_j^i \frac{\delta}{\delta y^{(k-1)i}} + \dots + k D_j^i \frac{\delta}{\delta y^{(k)i}}, \\ D_{\frac{\delta}{\delta y^{(\alpha)j}}} \Gamma &= d_j^i \frac{\delta}{\delta y^{(1)i}} + 2 d_j^i \frac{\delta}{\delta y^{(k-1)i}} + \dots + k d_j^i \frac{\delta}{\delta y^{(k)i}}. \end{aligned}$$

2 Finsler Connection on the k -osculator bundle. Characterizations.

Theorem 2.1 *There exists a one to one correspondence between the set of Finsler connections $F\Gamma$ and the set of pairs (N, ∇) , with N a nonlinear connection on E and ∇ a linear connection in the pull-back bundle $(\pi^k)^*(TM)$.*

Proof. If $F\Gamma$ is specified by $(N, F_{ij}^m, C_{ij}^m, \alpha = \overline{1, k})$, we define $\nabla : \chi(E) \times \Gamma((\pi^k)^*(TM)) \rightarrow \Gamma((\pi^k)^*(TM))$, by

$$(2.1) \quad \nabla_{\frac{\delta}{\delta x^k}} \frac{\partial}{\partial x^j} = F_{jk}^i \frac{\partial}{\partial x^i}, \quad \nabla_{\frac{\delta}{\delta y^{(\alpha)k}}} \frac{\partial}{\partial x^j} = C_{jk}^i \frac{\partial}{\partial x^i}.$$

and we extend it by linearity. So, for a Finsler connection $F\Gamma$ we have a pair (N, ∇) .

Conversely, let (N, ∇) be a pair like in hypothesis. In the natural basis ∇ takes the form:

$$(2.2) \quad \nabla_{\frac{\partial}{\partial x^k}} \frac{\partial}{\partial x^j} = \Gamma_{jk}^i \frac{\partial}{\partial x^i}, \quad \nabla_{\frac{\partial}{\partial y^{(\alpha)k}}} \frac{\partial}{\partial x^j} = \Gamma_{jk}^i \frac{\partial}{\partial x^i}.$$

If we consider the system of functions (defined on every neighborhood coordinates):

$$\begin{aligned} F_{jl}^i &= \Gamma_{jl}^i - \Gamma_{js}^i N_l^s - \dots - \Gamma_{js}^i N_l^s, \\ C_{jl}^i &= \Gamma_{jl}^i - \Gamma_{js}^i N_l^s - \dots - \Gamma_{js}^i N_l^s, \end{aligned}$$

by a direct calculation we obtain that $F\Gamma = (N, F_{ij}^m, C_{ij}^m, \alpha = \overline{1, k})$ is a Finsler connection on E . In the adapted basis the linear connection ∇ is expressed as (2.1),

where F_{jl}^i and C_{jl}^i are given by the previous formula. Concluding, the correspondence $F\Gamma \leftrightarrow (N, \nabla)$ is one to one. \square .

Definition 2.1 A linear connection ∇ in the pull-back bundle $(\pi^k)^*(TM)$ is said to be *regular* if the subspace $H(u) = \{X_u \in T_u E, \nabla_{X_u} C = 0\}$ of $T_u E$ is supplementary to $V_k(u)$ for every $u \in E$.

Proposition 2.1 *Every regular connection ∇ in the pull-back bundle $(\pi^k)^*(TM)$ induces a nonlinear connection N on E .*

Proof. As ∇ is a regular connection we obtain a distribution $H : u \in E \rightarrow H(u) = \{X_u \in T_u E, \nabla_{X_u} C = 0\}$ of dimension kn supplementary to the distribution V_k and so $T_u E = H(u) \oplus V_k(u), \forall u \in E$. We denote by v_k the vertical projector induced by the previous decomposition. For every $u \in E$ the map $(l_{v_k})_u : ((\pi^k)^*(TM))_u \rightarrow V_k(u)$ defined by $(l_{v_k})_u(\frac{\partial}{\partial x^i}) = \frac{\partial}{\partial y^{(k)i}}|_u$ and extended by linearity is an isomorphism of vector spaces. We denote by $K_u^{(k)} : V_k(u) \rightarrow ((\pi^k)^*(TM))_u$ the inverse map of $(l_{v_k})_u$ and we extend it to $T_u E$ by $K_u := K_u^{(k)} \circ v_k$. In this way we obtain a morphism of vector bundles with the base E , $K : TE \rightarrow (\pi^k)^*(TM)$ for which $H = Ker K$. Let $K^{(k-1)} = K^{(k)} \circ J, \dots, K^{(1)} = K^{(k)} \circ J^{k-1}$. The map $K = (K^{(1)}, \dots, K^{(k)}) : TE \rightarrow ((\pi^k)^*(TM))^{(k)}$ is a connection map. Its kernel is a nonlinear connection N .

Next, we give a characterization for the regular connections.

Theorem 2.2 *There exists a one to one correspondence between the set of regular connections ∇ in the pull-back bundle and the set of Finsler connections $F\Gamma$ satisfying:*

$$(*) \quad D_i^j = d_i^j = \dots = d_i^{(1k-1)} = 0$$

and

$$(**) \quad \det(d_i^j) \neq 0.$$

Proof. Let ∇ be a regular connection. According to the definition and the previous proposition we obtain a kn -dimensional distribution H and a connection map $K = (K^{(1)}, \dots, K^{(k)})$. Let $N = Ker K$. For the pair (N, ∇) we have according to the Theorem 2.1. a Finsler connection $F\Gamma$. We prove that for this the conditions (*) and (**) hold.

For $X_u \in T_u E$ we have:

$$(2.3) \quad \nabla_{X_u} C = (X^i D_i^j + (K^i X)^i d_i^j + \dots + (K^{(k-1)} X)^i d_i^{(1k-1)} + (K^i X)^i d_i^{(k)}) \frac{\partial}{\partial x^j} |_u.$$

The condition $H(u) \subset Ker K_u^{(k)}$ assures that (*) is true and $H(u) \cap V_k(u) = \{0\}$ assures that (**) hold.

Conversely, let $F\Gamma = (N, F_{ij}^m, C_{ij}^m, \alpha = \overline{1, k})$, be a Finsler connection satisfying (*) and (**). According to (2.3), the condition (*) implies that if $X_u \in H_u$ then $X_u \in Ker K_u^{(k)}$ and so $H_u \subset Ker K_u^{(k)}$. The condition (**) assure that $H(u) \cap V_k(u) = \{0\}$ and so $H_u = K_u^{(k)}$.

Definition 2.2 Let N be a linear connection on E . An N -linear connection on E is said to be of *Cartan type* if:

$$(2.4) \quad N(u) = \{X_u \in T_u E, D_{X_u}^{(\alpha)} \Gamma = 0 \ \forall \alpha = \overline{1, k}\} \quad \forall u \in E.$$

Theorem 2.3 Let N be a nonlinear connection on E and D an N -linear connection. Then D is a linear connection of Cartan type, if and only if

$$(2.5) \quad D_j^i = 0 \text{ and } \det(d_i^j) \neq 0.$$

Proof. Let $X_u \in T_u E$. According to (1.9) we obtain the following formulae:

$$\begin{aligned} D_{X_u} \overset{1}{\Gamma} &= (X^i D_i^j + (K^1 X)^i d_i^j + \dots + \dots + (K^k X)^i d_i^j) \frac{\partial}{\partial y^{(k)^j}} \Big|_u = \\ &= A^j \frac{\partial}{\partial y^{(k)^j}} \Big|_u, \\ D_{X_u} \overset{2}{\Gamma} &= A^j \frac{\delta}{\delta y^{(k-1)^j}} \Big|_u + 2(X^i D_i^j + (K^1 X)^i d_i^j + \dots \\ &+ (K^k X)^i d_i^j) \frac{\partial}{\partial y^{(k)^j}} \Big|_u = A^j \frac{\delta}{\delta y^{(k-1)^j}} \Big|_u + 2 A^j \frac{\partial}{\partial y^{(k)^j}} \Big|_u, \\ &\dots \\ D_{X_u} \overset{k}{\Gamma} &= A^j \frac{\delta}{\delta y^{(1)^j}} \Big|_u + 2 A^j \frac{\delta}{\delta y^{(2)^j}} \Big|_u + \dots + (k-1) A^j \frac{\delta}{\delta y^{(k-1)^j}} \Big|_u + \\ &+ k(X^i D_i^j + (K^1 X)^i d_i^j + \dots + (K^k X)^i d_i^j) \frac{\partial}{\partial y^{(k)^j}} \Big|_u = \\ &= A^j \frac{\delta}{\delta y^{(1)^j}} \Big|_u + 2 A^j \frac{\delta}{\delta y^{(2)^j}} \Big|_u + \dots + k A^j \frac{\partial}{\partial y^{(k)^j}} \Big|_u. \end{aligned}$$

The conditions $D_{X_u} \overset{\alpha}{\Gamma} = 0, \ \forall \alpha = \overline{1, k}$ are equivalent with the equations $A^j = 0 \ \forall \alpha = \overline{1, k} \ \forall j = \overline{1, n}$ and these are equivalent with:

$$(2.6) \quad X^i (D_i^j + (K^1 X)^i d_i^j + \dots + (K^k X)^i d_i^j) \begin{pmatrix} (11) & (21) & \dots & (k1) \\ d_i^j & d_i^j & \dots & d_i^j \\ \cdot & \cdot & \cdot & \cdot \\ (1k) & (2k) & \dots & (kk) \\ d_i^j & d_i^j & \dots & d_i^j \end{pmatrix} = 0.$$

Let N be a nonlinear connection and D be an N -linear connection which satisfies (2.3). We have that $X \in N$ if and only if $K^1 X = K^2 X = \dots = K^k X = 0$. Then (2.5) becomes $X^i (D_i^j + (K^1 X)^i d_i^j + \dots + (K^k X)^i d_i^j) = 0$ from where we obtain that the h -tensor fields of deflections are vanishing. If we assume that $\det(d_i^j) \neq 0$ then exists $X_u \in T_u E$ with $K_u X_u \neq 0$ such that (2.5) holds. So, we have $X_u \notin N(u)$ with $D_{X_u} \overset{\alpha}{\Gamma} = 0, \ \forall \alpha = \overline{1, k}$. This means that $N(u) \subset \{X_u \in T_u E, D_{X_u} \overset{\alpha}{\Gamma} = 0 \ \forall \alpha = \overline{1, k}\}$. \square

3 Examples of linear connections of Cartan type.

Let (M, g) be a Riemann manifold, γ_{jk}^i the local coefficients of the Levi-Civita connection.

Theorem 3.1 *There exists a unique nonlinear connection N on E such that $F\Gamma = (N, F_{ij}^m = \gamma_{ij}^m \circ \pi^k, C_{ij}^m = 0)$ is a linear connection of Cartan type on E .*

Proof. We prove that on every domain of local chart we can define a system of functions (N_j^i, \dots, N_j^i) which is uniquely determined by the conditions (2.5) of the Theorem 2.3. By a change of coordinates these functions will change according to the rule which allows us to say that and these functions are the coefficients of a nonlinear connection N on E . (In the $k = 2$ case these are determined by M. de Leon and M.E. Vasquez-Abal in [4]). In the same time, we determine a system of functions (M_j^i, \dots, M_j^i) which will be called the dual coefficients of the nonlinear connection N .

These coefficients are as follows:

$$N_j^i(x, y^{(1)}, \dots, y^{(k)}) = \gamma_{mj}^i(x) y^{(1)m} = M_j^i(x, y^{(1)}, \dots, y^{(k)}).$$

Let $z^{(2)i}$ defined by (1.7).

$$N_j^i(x, y^{(1)}, \dots, y^{(k)}) = \left(\frac{\partial}{\partial x^j} - N_j^p \frac{\partial}{\partial y^{(1)p}} \right) z^{(2)p} + \gamma_{mj}^i(x) z^{(2)m}.$$

Also, we consider:

$$M_j^i = N_j^i + N_m^i M_j^m \text{ and } z^{(3)i} \text{ like as in (1.7).}$$

Using the previous formulae we can define:

$$N_j^i(x, y^{(1)}, \dots, y^{(k)}) = \left(\frac{\partial}{\partial x^j} - N_j^p \frac{\partial}{\partial y^{(1)p}} - N_j^p \frac{\partial}{\partial y^{(2)p}} \right) z^{(2)p} + \gamma_{mj}^i(x) z^{(3)m}.$$

$$\text{Let } M_j^i = N_j^i + N_m^i M_j^m + N_m^i M_j^m \text{ and } z^{(4)i} \text{ defined by (1.7).}$$

Next, in the same manner we obtain (N_j^i, \dots, N_j^i) , (M_j^i, \dots, M_j^i) and $z^{(5)i}, \dots, z^{(k)i}$.

Finally, with the previous functions we define

$$N_j^i(x, y^{(1)}, \dots, y^{(k)}) = \left(\frac{\partial}{\partial x^j} - N_j^p \frac{\partial}{\partial y^{(1)p}} - N_j^p \frac{\partial}{\partial y^{(2)p}} - \dots - N_j^p \frac{\partial}{\partial y^{(k-1)p}} \right) z^{(k)p} + \gamma_{mj}^i(x) z^{(k)m}.$$

By a direct calculation we can verify that at a change of coordinates on E the system of functions (N_j^i, \dots, N_j^i) satisfies (4.9) from [6] and so these are the local coefficients

of a nonlinear connection on E . The Finsler connection $F\Gamma$ given by this theorem is of Cartan type because:

a) the h -tensors of deflections are vanishing (by the construction of the functions

N_j^i we have that D_j^i are vanishing);

b) $\det(d_j^{(\alpha\beta)}) \neq 0$ (for $\alpha > \beta$ we have $d_j^{(\alpha\beta)} = 0$, $d_j^{(\alpha\alpha)} = \delta_j^\alpha$ and so $\det(d_j^{(\alpha\beta)}) = 1$). \square

Let $F^n = (M, F)$ be a Finsler space and $(N_j^i, F_{ij}^m, C_{ij}^m)$ the Cartan connection of the Finsler space F^n ([5, p.113]).

Theorem 3.2 *There exists a unique nonlinear connection N on E such that $F\Gamma = (N, F_{ij}^m := F_{ij}^m \circ \pi_1^k, C_{ij}^m = C_{ij}^m \circ \pi_1^k, C_{ij}^m = 0 \quad \alpha \geq 2)$ is a linear connection of Cartan type on E .*

Proof. The proof follows the previous theorem line. Thus, on every domain of local chart, we define the systems of functions $(N_j^{(1)}, \dots, N_j^{(k)})$ and $(M_j^{(1)}, \dots, M_j^{(k)})$ which will be the coefficients and the dual coefficients, respectively, of a nonlinear connection N . From the vanishing of the first h -tensor of deflection we obtain $N_j^{(1)}(x, y^{(1)}, \dots, y^{(k)}) = N_j^{(1)}(x, y^{(1)}) \circ \pi_1^k$. To determine the next coefficients a method like in the previous theorem is used. \square

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