NEW MODIFIED LIE ADMISSIBLE STATISTICS

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Abstract

In the present paper we construct the new modified Lie admissible statistics for open quantum systems, by using the Lie-admissible von Neumann equation and non-canonical commutation relation. For the special case of the Lie-admissible complex time model we obtain a new non- canonical quantum statistics. In the same context we consider also the problem of quantum measurement and the "loss of coherence". Finally it has been studied the variation of entropy (i.e its increase or decrease) within such a nonunitary time evolution of the density operator.

1 Introduction

Before three years ago Jannussis and Skaltsas ([1]) have introduced the Lie-admissible Liouville-von Neumann equation for the density operator of open quantum systems, according to Santilli's Lie-admissible theory ([2]). This equation has a general form.

All the equations of Liouville type which are known to the authors, linear or non linear, are particular cases of it. It has been proved also that the models which exist and dealing with the problem of the quantum measurement are partial cases of the above mentioned Lie- admissible model and have Lie-admissible character.

The time evolution of the density operator is in general nonunitary. In fact, this is easily implied from section 5 of ref. [1], where the simple case of the lifting operator $T = 1 - i\lambda$, λ real parameter leads to the so-called Lie-admissible complex time model ([3]).

From the results of ref. [1], it is evident that the quantum measurement problem is ultimately an irreversible process, due to the interaction between quantum system and apparatus ([4]), and therefore it possesses a Lie-admissible structure in its most

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general possible formulation. According to ref. [1, 2] we can define the Lie-admissible von Neumann equation for the density operator ρ, i.e.:

$$i\hbar\frac{d\rho}{dt} = [H,\rho] = HT\rho - \rho T^{+}H, \tag{1}$$

with the commutation relation

$$qTp - pT^{+}q = i\hbar\hat{I}, \tag{2}$$

where $H(q,p)$ is the usual Hamilton operator of the system and $T \neq T^{+}$ describes in general nonconservative interactions and I is the new unity.

At this point we can exploit the theory of Santilli's Hadronic Mechanics ([5]) and define two units, i.e.:

$$\hat{I} = I^{>}\;,\; \hat{I} = {}^{<}I. \tag{3}$$

In the framework of the general Lie-admissible theory the commutation relation (2) is broken in two relations of the form:

$$qTp - pT^{+}q = i\hbar\left\{\begin{array}{c} I^{>} \\ {}^{<}I \end{array}\right. , \tag{4}$$

where

$$I^{>} = T^{-1}\;,\; {}^{<}I = \left(T^{+}\right)^{-1}. \tag{5}$$

Due to the Hermitian character of the density operator $\rho(q,p,t)$ and the non unitary character of the time evolution, it becomes evident a kind of contradiction between the eq. (1) and (2), in as much as the operators q, p are not anymore Hermitian, their time evolutions, according to the genotopic Heisenberg equations of motion ([5]), i.e.:

$$i\hbar\frac{dq}{dt} = qTH - HT^{+}q, \tag{6}$$

$$i\hbar\frac{dp}{dt} = pTH - HT^{+}p \tag{7}$$

are still Hermitian.

We can avoid this contradiction, only when the commutation relations (4) take the form:

$$qTp - pT^{+}q = i\hbar\frac{1}{\sqrt{TT^{+}}}, \tag{8}$$

fact which is implied from the physical reality which characterizes the science of Physics. For the special case $T = T^{+}$ and $HT \neq TH$ which corresponds to the Lie-isotopic formulation, we have:

$$qTp - pTq = \frac{i\hbar}{T}, \tag{9}$$

which is correct, as it becomes clear from ref. [5]. Therefore the Lie-admissible statistics leads to a new modified Lie-admissible statistics, which is described from the equations:

$$i\hbar\frac{d\rho}{dt} = HT\rho - \rho T^{+}H, \tag{10}$$

$$qTp - pT^{+}q = i\frac{\hbar}{\sqrt{TT^{+}}}. \tag{11}$$

Based on the above equations the organization of the paper is arranged as follows. In sect. 2 we consider the new deformed Heisenberg quantum mechanics for $T = 1+i\lambda$. In sect. 3 we construct the deformed Hamilton operator. In sect. 4 we study the quantum measurement problem as an irreversible phenomenon and also the variation of entropy. Sect. 5 is devoted to concluding remarks.

2 New Lie-deformed Heisenberg Quantum Mechanics

According to the Lie-admissible commutation relation (11), we initiate the operator T_{jk}:

$$q_j T_{jk} p_k - p_k T^{+}_{jk} q_j = i\hbar\frac{\delta_{jk}}{\sqrt{T_{jk}T^{+}_{jk}}}, \tag{12}$$

$$[q_j, q_k] = q_j q_k - q_k q_j = 0 \quad , \quad [p_j, p_k] = p_j p_k - p_k p_j = 0, \tag{13}$$

where T_{jk} are fixed elements. For the special case

$$T_{jk} = 1 + i\lambda_{jk}\ , \ T^{+}_{jk} = 1 - i\lambda_{jk}\ , \ \lambda_{jk} = \lambda_k\delta_{jk}\ ,$$

λ_{jk} real, we obtain the new Heisenberg ring ([6]):

$$q_j(1 + i\lambda_{jk}\delta_{jk})p_k - p_k(1 - i\lambda_{jk}\delta_{jk})q_j = \frac{i\hbar\delta_{jk}}{\left(1 + \lambda^2_{jk}\delta_{jk}\right)^{\frac{1}{2}}}, \tag{14}$$

$$[q_j, p_k] = 0, \quad j \neq k \quad , [q_j, q_k] = [p_j, p_k] = 0. \tag{15}$$

For $\lambda_{jk} = 0$ it is reduced to the usual Heisenberg ring of the canonical quantum mechanics. The new operators q_j and p_k of the above ring take the following form in q and p- representation, respectively:

$$q_j \to q_j \quad , \quad p_k = \frac{\hbar}{2\lambda_k\sqrt{1+\lambda_k^2}}\frac{1}{q_k}\left(1 - e^{2i\theta_k q_k\frac{\partial}{\partial q_k}}\right), \tag{16}$$

$$p_k \to p_k \quad , \quad q_j = \frac{\hbar}{2\lambda_j\sqrt{1+\lambda_j^2}}\frac{1}{p_j}\left(1 - e^{-2i\theta_j p_j\frac{\partial}{\partial_j p_j}}\right), \tag{17}$$

where

$$\theta_k = \arctan \lambda_k \tag{18}$$

and the new operators q_j, p_k are Hermitian.

For simplicity we limit ourselves in one-dimension, e.g. to the new operators $q_1 = q$, $p_1 = p$ with the following commutation relation:

$$q(1 + i\lambda)p - (1 - i\lambda)q = \frac{i\hbar}{\sqrt{1+\lambda^2}} \tag{19}$$

and the corresponding representations are:

$$q \to q,\ p = \frac{\hbar}{2\lambda\sqrt{1+\lambda^2}}\frac{1}{q}\left(1 - e^{2i\theta q\frac{\partial}{\partial q}}\right), \tag{20}$$

$$p \to p,\ q = \frac{\hbar}{2\lambda\sqrt{1+\lambda^2}}\frac{1}{p}\left(1 - e^{-2i\theta p\frac{\partial}{\partial p}}\right), \tag{21}$$

where

$$\theta = \arctan \lambda. \tag{22}$$

The commutation relation (19) can be written:

$$[q,p] + i\lambda\{q,p\} = \frac{i\hbar}{\sqrt{1+\lambda^2}}, \tag{23}$$

where $\{q,p\} = qp + pq$ is the anticommutator. As it is well known ([7]), the commutator $[q,p]$ expresses an internal symmetry and the anticommutator $\{qp + pq\}$ an external one. In the way the appearance of the parameter can be explained as it describes external interactions, e.g. interactions between particles or interactions of the particle with the rest world.

Also the parameter λ is connected with the "chronon" of Caldirola ([8]) which generally describes the time interactions between two physical systems.

For

$$H = \frac{\omega}{2}(qp + pq) = \frac{\hbar\omega}{2\hbar}(qp + pq), \tag{24}$$

the commutation relation (23) takes the form:

$$[q,p] = \frac{i\hbar}{\sqrt{1+\lambda^2}} - \frac{2i\hbar}{\hbar\omega}\lambda H = i\hbar\left(\frac{1}{\sqrt{1+\lambda^2}} - \frac{2\lambda}{\hbar\omega}H\right) \tag{25}$$

and satisfies the non-canonical commutation relation of Heisenberg type ([9]). According to ref. [6], the eigenfunctions of the operators q and p take the form:

$$\varphi_p(q) = Ae_Q\left[\frac{ipq}{\hbar}(1 - i\lambda)\sqrt{1+\lambda^2}\right] \text{ with eigenvalues } p \tag{26}$$

and

$$\varphi_q(p) = Be_{Q^*}\left[-\frac{ipq}{\hbar}(1+i\lambda)\sqrt{1+\lambda^2}\right] \text{ with eigenvalues } q, \tag{27}$$

where A, B are arbitrary constants,

$$Q = \frac{1+i\lambda}{1-i\lambda} \tag{28}$$

and $e_Q(z)$ is the Q-exponential function, i. e.:

$$e_Q(z) = \sum_{n=0}^{\infty} \frac{z^n}{[n]!}\,, \text{ with } [n] = \frac{Q^n-1}{Q-1}. \tag{29}$$

The formulas (26) and (27) for $p = \hbar k$ take the form:

$$f_k(q) = C_0 e_{\frac{1+i\lambda}{1-i\lambda}}\left[ikq(1-i\lambda)\sqrt{1+\lambda^2}\right], \tag{30}$$

$$f_q(k) = C_0 e_{\frac{1-i\lambda}{1+i\lambda}}\left[-ikq(1+i\lambda)\sqrt{1+\lambda^2}\right]. \tag{31}$$

The above expressions for $\lambda = 0$ are reduced exactly to the ordinary exponential functions, i.e., e^{ikq} and e^{-ikq} which are the eigenfunctions of the operators p and q in canonical quantum mechanics. Also the eigenvalue equation for the free particle Hamilton operator is:

$$\frac{p^2}{2m} f_E(q) = E\, f_E(q), \tag{32}$$

where the operator p in the q-representation has the form (30). From the solution (29) we obtain:

$$f_E(q) = const.\, e_{\frac{1+i\lambda}{1-i\lambda}}\left[i\frac{\sqrt{2mE}}{\hbar}q(1-i\lambda)\sqrt{1+\lambda^2}\right] \tag{33}$$

and for

$$E = \frac{\hbar^2 k^2}{2m} \tag{34}$$

we have:

$$f_k(q) = const.\, e_{\frac{1+i\lambda}{1-i\lambda}}\left[ikq(1-i\lambda)\sqrt{1+\lambda^2}\right]. \tag{35}$$

In the following we will study the equation (10) for the case of free particle with $T = 1 - i\lambda$.

3 Deformed Hamiltonian

Let us begin from the case of free particle.
The equation (10) for the case of free particle takes the form:

$$i\hbar\frac{dp}{dt} = \left(\frac{1-i\lambda}{2m}\right)\left[\frac{\hbar}{2\lambda\sqrt{1+\lambda^2}}\frac{1}{q}\left(1 - e^{2i\theta q\frac{\partial}{\partial q}}\right)\right]^2 \rho - \tag{36}$$

$$-\rho\left(\frac{1+i\lambda}{2m}\right)\left[\frac{\hbar}{2\lambda\sqrt{1+\lambda^2}}\frac{1}{q}\left(1-e^{2i\theta q\frac{\partial}{\partial q}}\right)\right]^2,$$

because the deformed Hamilton operator

$$H=\frac{1}{2m}\left[\frac{\hbar}{2\lambda\sqrt{1+\lambda^2}}\frac{1}{q}\left(1-e^{2i\theta q\frac{\partial}{\partial q}}\right)\right]^2 \tag{37}$$

is time-independent, the solution of the above equation takes the form:

$$\begin{aligned}\rho(t)= \quad & \exp\left\{\frac{-it(1-i\lambda)}{\hbar}\left[\frac{\hbar}{2\lambda\sqrt{1+\lambda^2}}\frac{1}{q}\left(1-e^{2i\theta q\frac{\partial}{\partial q}}\right)\right]^2\right\}\rho(0)\cdot \\ & \exp\left\{\frac{it(1+i\lambda)}{\hbar}\left[\frac{\hbar}{2\lambda\sqrt{1+\lambda^2}}\frac{1}{q}\left(1-e^{2i\theta q\frac{\partial}{\partial q}}\right)\right]^2\right\}.\end{aligned} \tag{38}$$

From the above solution we conclude that the special case of free particle with $T=1-i\lambda$ leads to a non unitary Lie-admissible time evolution, with the complex time $t(1-i\lambda)=t-i\lambda t$, where t is the usual time.

The solution (38) in the energy representation

$$\begin{aligned}Hf_k(q) &= \frac{p^2}{2m}f(q)=\frac{1}{2m}\left[\frac{\hbar}{2\lambda\sqrt{1+\lambda^2}}\frac{1}{q}\left(1-e^{2i\theta q\frac{\partial}{\partial q}}\right)\right]^2 f_k(q) \\ &= \frac{\hbar^2k^2}{2m}f_k(q)\end{aligned} \tag{39}$$

reads:

$$\rho_{k'k}(t)=e^{-\frac{\lambda t}{\hbar}\left(\frac{\hbar^2k'^2}{2m}+\frac{\hbar^2k^2}{2m}\right)-\frac{it}{\hbar}\left(\frac{\hbar^2k'^2}{2m}-\frac{\hbar^2k^2}{2m}\right)}\rho_{k'k}(0), \tag{40}$$

with

$$\rho_{k'k}(t)=<k|\rho(t)|k'> \tag{41}$$

and for $k=k'$ yields

$$\rho_{kk}(t)=e^{-\frac{\lambda t}{m}\hbar k^2}\rho_{kk}(0) \tag{42}$$

$$= \quad e^{-\frac{2\lambda t}{\hbar}E(k)}\rho_{kk}(0)\,,\ \text{with } E(k)=\frac{\hbar^2k^2}{2m}. \tag{43}$$

Also, as it has proved in ref. [1], for any Hamilton operator $H(q,p)$ in the q or p- representation according to (20) and (21), when the deformed Hamilton operator which yield is Hermitian, then the Lie-admissible modified equation (10) in the q-representation is written:

$$i\hbar\frac{d\rho(t)}{dt} = H(\lambda)(1-i\lambda)\rho(t) - \rho(t)(1+i\lambda)H(\lambda). \tag{44}$$

If $H(\lambda)$ is time-independent the solution of the above equation has the form:

$$\rho(t) = e^{-\frac{it}{\hbar}(1-i\lambda)H(\lambda)}\rho(0)e^{\frac{it}{\hbar}(1+i\lambda)H(\lambda)}. \tag{45}$$

Furthermore we assume that we can determined the eigenvalues of the $H(\lambda)$ operator, i. e.:

$$H(\lambda)|n>= E_n(\lambda)|>, \tag{46}$$

then, the solution (45) reads:

$$\rho_{nl}(t) = e^{-\lambda\frac{t}{\hbar}\left[E_n(\lambda)+E_l(\lambda)\right] - \frac{it}{\hbar}\left[E_l(\lambda)-E_n(\lambda)\right]}\rho_{nl}(0), \tag{47}$$

with

$$\rho_{nl}(t) =< n|\rho(t)|l > \tag{48}$$

and for $n = l$ yields:

$$\rho_{nn}(t) = e^{-\frac{2\lambda t E_n}{\hbar}}\rho_{nn}(0). \tag{49}$$

From formulas (47) and (48) we see that for $t > 0$ we have dissipative behavior of the function $\rho_{nl}(t)$ for $\lambda > 0$ and antidissipative exponential behavior of ρ_{nl} for $\lambda < 0$.

A remarkable interpretation of the parameter λ is given in ref. [10], in which the equivalence between the Caldirola-Montaldi model ([8]) and the Lie-admissible complex time model is proved for the nonrelativistic and relativistic case. The connection between the parameter and Caldirola "chronon", which describes the interaction time between physical systems, leads to the result that an external interaction exists between the particles and environment.

4 Irreversibility and the Problem of Quantum Measurement

As it is well known in quantum mechanics, we have a fundamental distinction between pure states and mixtures. The former are represented by unit vectors in a Hilbert space and the latter by density matrices. In the standard formulation, the pure states occupy a privileged position, that is, the super position principle holds for them, they obey the Schrödinger time evolution, according to which pure states transform into pure states, and observables correspond to self-adjoint operators mapping again pure states into pure states.

The fundamental distinction between pure states and mixtures is lost during the measurement process, as is also well known, according to the von Newmann classical analysis.

These latter evolution converts pure states into mixtures and is irreversible. It is this dual state evolution and especially the transformation of the pure states into mixtures during measurements, which creates many important problems ([11]). We could avoid this dualism of state evolution if we formulate a physical principle that implies the loss of distinguishability between the pure and mixtures. Various attempts in this direction have been made and an extensive bibliography exists in the field.

One interesting attempt comes from the so called Brussels school theory (BST) proposed by Prigogine and his co-workers ([12]). In a recent paper by Jannussis et.all ([13]) we have proved that the (BST) theory is a partial case of Lie-admissible treatment. In the context of the Lie-admissible formulation, the time evolution itself implies the above mentioned loss of distinguishability.

More concretely, irreversibility arises naturally in the Heisenberg - Santilli nonunitary time evolution, which transforms pure states to mixtures, implying in this way the loss of the distinguishability between them. Also in the ref. [1] we examined various models related to the quantum measurement problem and we proved that accept a common algebraic basis, that of Lie-admissible algebras.

In the following we give illustrations of the above results.

According to ref. [10] and from the solutions (45) and (47), for $\rho^2(0) = \rho(0)$ (that is, if we have a pure states for $t = 0$), then $\rho^2(t) \neq \rho(t)$ (that is, this is converted by the proposed evolution to mixture). From the solution (47) it seems that for $t > 0$ we have *loss of coherence.*

It is now interesting to study the time evolution of the entropy $S(t)$ with the density operator $\rho(t, \lambda)$, i.e.:

$$\rho(t,\lambda) = e^{-\frac{it}{\hbar}(1-i\lambda)H(\lambda)}\rho(0)e^{\frac{it}{\hbar}(1+i\lambda)H(\lambda)}, \tag{50}$$

where

$$H(\lambda) = \frac{\hbar^2}{8m\lambda^2(1+\lambda^2)}\left[\frac{1}{q}\left(1 - e^{2i\theta q\frac{\partial}{\partial q}}\right)\right]^2 + V(q) \tag{51}$$

is the new Hermitian deformed Hamilton operator in the q-representation eq. (20). The operator in ansantz of (18) takes the form:

$$H(\lambda) = \frac{\hbar^2\theta^2}{2m\lambda^2(1+\lambda^2)}\left[\frac{\partial^2}{\partial q^2} + \frac{2i\theta}{2}\left(\frac{\partial}{\partial q}\left(\frac{\partial}{\partial q}p + p\frac{\partial}{\partial q}\right)\frac{\partial}{\partial q}\right) + O(2i\theta)^2\right] + V(q) \tag{52}$$

and also the operator $\rho(t, \lambda)$ in ansantz of λ is written:

$$\rho(t,\lambda) = \rho_c(t) + \frac{\lambda}{1!}\left(\frac{\partial\rho(t,\lambda)}{\partial\lambda}\right)_{\lambda=0} + O(\lambda^2), \tag{53}$$

where

$$\rho_c(t) = \exp\left\{-\frac{itH(0)}{\hbar}\rho\right\}\rho(0)\exp\left\{\frac{itH(0)}{\hbar}\rho\right\} \tag{54}$$

is the usual density operator,

$$\left(\frac{\partial H(\lambda)}{\partial\lambda}\right)_{\lambda=0} = -\frac{i\hbar^2}{2m}\left(\frac{\partial}{\partial q}\left(\frac{\partial}{\partial q}q + q\frac{\partial}{\partial q}\right)\frac{\partial}{\partial q}\right) \tag{55}$$

and

$$\begin{aligned}\left(\frac{\partial\rho}{\partial\lambda}(t,\lambda)\right)_{\lambda=0} &= -\frac{t}{\hbar}\left[\{H(0),\rho_c(t)\} + \right.\\ &\left. +i\left[\int_0^1 e^{-\frac{it}{\hbar}\xi H(0)}\left(\frac{\partial H(\lambda)}{\partial\lambda}\right)_{\lambda=0} e^{\frac{it}{\hbar}\xi H(0)}\,d\xi, \rho_c(t)\right]\right].\end{aligned} \tag{56}$$

In the following we use the standard formula for the entropy:

$$S(t) = -ktr[\rho(t,\lambda)\ln\rho(t,\lambda)] = -k\int\rho(t,\lambda)\ln\rho(t,\lambda)\,d\Omega, \tag{57}$$

where $d\Omega$ is a volume element in phase space and k is the Boltzmann constant. The above entropy for small values of λ takes the form:

$$S(t) = -k\int\left(\rho_c(t) - \lambda R(t)\right)\left(\ln\rho_c(t) - \lambda R(t)\right)\,d\Omega, \tag{58}$$

where

$$R(t) = -\left(\frac{\partial\rho(t,\lambda)}{\partial\lambda}\right)_{\lambda=0}. \tag{59}$$

By using the Taylor expansion for the derivative of the operator

$$\frac{\partial}{\partial\lambda}\ln\left[\rho_c(t) - \lambda R(t)\right],$$

we obtain:

$$\begin{aligned}\ln\left[\rho_c(t) - \lambda R(t)\right] &= \ln\rho_c(t) - \lambda\left[\frac{1}{\rho_c(t)}R(t) + \frac{1}{2}\frac{1}{\rho_c^2(t)}\left[\rho_c(t),R(t)\right] + \right.\\ + &\left.\frac{1}{3}\frac{1}{\rho_c^3(t)}\left[\rho_c(t),\left[\rho_c(t),R(t)\right]\right] + \cdots\right] + O(\lambda^2)\end{aligned} \tag{60}$$

and the formula (58) yields:

$$S(t) = -k\int\left(\rho_c(t) - \lambda R(t)\right)\ln\left(\rho_c(t) - \lambda M(t)\right)\,d\Omega, \tag{61}$$

where

$$M(t) = \frac{1}{\rho_c(t)}R(t) + \frac{1}{2}\frac{1}{\rho_c^2(t)}\left[\rho_c(t),R(t)\right] + \frac{1}{3}\frac{1}{\rho_c^3(t)}\left[\rho_c(t),\left[\rho_c(t),R(t)\right]\right] + \cdots. \tag{62}$$

From (61) we obtain:

$$S(t) = S_0 + k\lambda tr\,[R(t)\ln\rho_c(t) + \rho_c(t)M(t)] + O(\lambda^2), \tag{63}$$

with

$$S_0 = -ktr\rho_c(t)\ln\rho_c(t). \tag{64}$$

From the above result we see that the entropy is dependent on time and the parameter λ. So, for appropriate values of the above variable, the entropy can either increase or decrease. In particular, we must to emphasize the model proposed by Zeh, ([14]) according to which the entropy can increase or decrease in the general description of the nonunitary formulation.

5 Conclusion

In the present paper we have constructed a new modified Lie-admissible statistics, which is described from the eq. (10) and the corresponding commutation relation (11). The Lie-admissible complex time model permit us for first time, with the help of the representations (20) or (21), to construct the deformed operator $H(\lambda)$ (51). This operator remains Hermitian as occurs exactly in the transformation of the classical mechanics to quantum mechanics. The importance also of the results that arise from the study of the above models, contribute to a better description of the quantum measurement problem, and from eq. (21) for $t > 0$, $\lambda > 0$, we have *loss of coherence*.

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