CLASSIFICATION OF NILPOTENT LIE ALGEBRAS OF DIMENSION EIGHT

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Abstract

The problem of the classification of Nilpotent Lie Algebras has been partially solved. The aim of the present paper is to provide the results of the classification of Nilpotent Lie Algebras of dimension eight, whose maximal abelian ideal is of dimension four.

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Key words: Classification, Nilpotent Lie Algebras, Abelian ideal and the classification problem.

1 Introduction

Let $\mathbf{g}$ be a Nilpotent Lie Algebra over a field K of characteristic zero. If $\mathbf{g}_0$ is a maximal abelian ideal of $\mathbf{g}$ then $\dim \mathbf{g}_0 = m$ satisfies the inequalities:

$$\frac{1}{2}\left(\sqrt{8n+1}-1\right) \leq m \leq n$$

Therefore, if $\dim \mathbf{g} = n = 8$, then m can take the values 8, 7, 6, 5, 4.

The classification of Nilpotent Lie Algebras $\mathbf{g}$ of dimension eight, whose maximal abelian ideal has dimensions 7 or 6 has been achieved by [2] , [3] respectively.

The case where $\dim \mathbf{g} = 8$ and $\dim \mathbf{g_0} = 5$ has been given in [1].

This paper contains the classification of Nilpotent Lie Algebras of dimension eight, whose maximal abelian ideal has *dimension four*.

PROBLEM

The problem of classifying Nilpotent Lie Algebras of dimension eight, whose maximal abelian ideal has *dimension four* is analyzed in the following three categories:

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1. **Determine the abelian Nilpotent Lie Algebras.**

2. **Determine the decomposable Nilpotent Lie Algebras.**

3. **Determine the non-decomposable Nilpotent Lie Algebras of this case.**

In order to deal with these questions we have determined a complicated set of computer programs, run in powerful computers of the latest technology. These programs deal with the problem from the very beginning, that is they determine the matrices corresponding to the linear mappings.

Then, from this set of matrices, we select, by using algorithms, those quadruples which satisfy the following relations, corresponding to the above mentioned 3 categories:

1. $[E_1, E_2] = 0$, $[E_1, E_3] = 0$, $[E_1, E_4] = 0$, $[E_2, E_3] = 0$, $[E_2, E_4] = 0$, $[E_3, E_4] = 0$

2. $[E_1, E_2] = E_3$, $[E_1, E_3] = 0$, $[E_1, E_4] = 0$, $[E_2, E_3] = 0$, $[E_2, E_4] = 0$, $[E_3, E_4] = 0$

3. $[E_1, E_2] = E_3$, $[E_1, E_3] = E_4$, $[E_1, E_4] = 0$, $[E_2, E_3] = 0$, $[E_2, E_4] = 0$, $[E_3, E_4] = 0$

Then, another algorithm searches throughout the quadruples and eliminates the isomorphic cases.

At the next stage, one more algorithm examines whether it is necessary to keep parameters with Lie Algebras. If it is so, another program tries and eliminates these parameters, considering the above given relations.

Subsequently, we created a large program which, by using the Jacobi's relation, that is

$$[[e_1, e_2], e_3] + [[e_2, e_3], e_1] + [[e_3, e_1], e_2] = 0$$

gives all the constructive constants $[e_i, e_j]$, $(\forall)\ i, j = 1, 2, ...n - m$, together with their remaining parameters, and it separates the Nilpotent Lie Algebras in consideration according to the number of their non-zero constructive constants.

It also checks whether all these Lie Algebras are non-isomorphic.

In order to solve this problem we consider some cases.

In the presentation of the results we use some special notations. A notations such as "$\mathbf{H = 1 > 2 * a_{22}}$" should be interpreted as follows:

"$\mathbf{H = 1}$": Denotes the step and consequently the amount of remaining nonzero coefficients a_{ij}. They appear in descending order, according to their number of occurences.

" $\blacktriangleright 2\times$ ": Expresses how many times a coefficient appears in an algebra.

"$\mathbf{a_{22}}$": Denotes the parameter in consideration.

",": It is used as a separator.

2 Abelian Lie algebras of dimension 8 whose maximal abelian ideal has dimension 4

Consider the following relations:
$[E_1, E_2] = 0,\ [E_1, E_3] = 0,\ [E_1, E_4] = 0,$
$[E_2, E_3] = 0,\ [E_2, E_4] = 0,\ [E_3, E_4] = 0.$

There exists 1 Lie algebra that has 9 remaining parameters:

$[e_1, e_2] = +a_{13}e_7$
$[e_1, e_3] = +a_{42}e_5 + a_{22}e_6$
$[e_1, e_4] = +a_{52}e_5 + a_{42}e_6 + a_{33}e_7$
$[e_1, e_6] = +e_8$
$[e_2, e_3] = +a_{52}e_5 + a_{42}e_6 + a_{43}e_7$
$[e_2, e_4] = +a_{51}e_5 + a_{52}e_6$
$[e_2, e_5] = +e_8$
$[e_3, e_4] = +a_{63}e_7 + a_{64}e_8$
$[e_3, e_6] = +e_7$
$[e_4, e_5] = +e_7$
$H = 1 \blacktriangleright 3 \times a_{42},\quad H = 2 \blacktriangleright 3 \times a_{52},\quad H = 3 \blacktriangleright 1 \times a_{13},\quad H = 4 \blacktriangleright 1 \times a_{22},$
$H = 5 \blacktriangleright 1 \times a_{33},\quad H = 6 \blacktriangleright 1 \times a_{43},\quad H = 7 \blacktriangleright 1 \times a_{51},\quad H = 8 \blacktriangleright 1 \times a_{63},$
$H = 9 \blacktriangleright 1 \times a_{64},$

3 Decomposable Lie algebras of dimension 8 whose maximal abelian ideal has dimension 4.

Consider the following relations:

$$[E_1, E_2] = E_3,\ [E_1, E_3] = 0,\ [E_1, E_4] = 0,$$
$$[E_2, E_3] = 0,\ [E_2, E_4] = 0,\ [E_3, E_4] = 0,$$

There exists 10 Lie Algebras in this category:

LIE ALGEBRA number 1:
$[e_1, e_2] = +e_3$
$[e_1, e_3] = +a_{12}e_6 - 2.00a_{52}e_7$
$[e_1, e_4] = -a_{52}e_5 + a_{22}e_6 + a_{23}e_7$
$[e_1, e_6] = +e_8$
$[e_2, e_3] = +a_{31}e_5 + a_{41}e_7$
$[e_2, e_4] = +a_{41}e_5 + a_{43}e_7$
$[e_2, e_5] = +e_6$
$[e_3, e_4] = +a_{52}e_6$
$[e_3, e_5] = +e_8$
$[e_4, e_7] = +e_8$
$H = 1 \blacktriangleright 3 \times a_{52},\quad H = 2 \blacktriangleright 2 \times a_{41},\quad H = 3 \blacktriangleright 1 \times a_{12},\quad H = 4 \blacktriangleright 1 \times a_{22},$
$H = 5 \blacktriangleright 1 \times a_{23},\quad H = 6 \blacktriangleright 1 \times a_{31},\quad H = 7 \blacktriangleright 1 \times a_{43},$

LIE ALGEBRA number 2:
$[e_1, e_2] = +e_3$
$[e_1, e_3] = +a_{12}e_6 - 2.00a_{52}e_7$
$[e_1, e_4] = -a_{52}e_5 + a_{22}e_6 + a_{23}e_7$
$[e_1, e_6] = +e_8$
$[e_2, e_3] = +a_{41}e_7$
$[e_2, e_4] = +a_{41}e_5 + a_{43}e_7$
$[e_2, e_5] = +e_6$
$[e_3, e_4] = +a_{52}e_6$
$[e_3, e_5] = +e_8$
$[e_4, e_5] = +e_7$
$[e_4, e_7] = +e_8$
$H = 1 \blacktriangleright 3 \times a_{52},\quad H = 2 \blacktriangleright 2 \times a_{41},\quad H = 3 \blacktriangleright 1 \times a_{12},\quad H = 4 \blacktriangleright 1 \times a_{22},$
$H = 5 \blacktriangleright 1 \times a_{23},\quad H = 6 \blacktriangleright 1 \times a_{43},$

LIE ALGEBRA number 3:

$[e_1, e_2] = +e_3$ $\quad [e_1, e_3] = +a_{14}e_8$
$[e_1, e_4] = -a_{54}e_6 + a_{24}e_8$ $\quad [e_1, e_6] = +e_7$
$[e_2, e_3] = +a_{34}e_8$ $\quad [e_2, e_4] = +a_{43}e_7 + a_{44}e_8$
$[e_2, e_5] = +e_6$ $\quad [e_2, e_6] = +e_8$
$[e_3, e_4] = +a_{54}e_8$ $\quad [e_3, e_5] = +e_7$
$[e_4, e_5] = +e_8$
$H = 1 \blacktriangleright 2 \times a_{54}, \quad H = 2 \blacktriangleright 1 \times a_{14}, \quad H = 3 \blacktriangleright 1 \times a_{24}, \quad H = 4 \blacktriangleright 1 \times a_{34},$
$H = 5 \blacktriangleright 1 \times a_{43}, \quad H = 6 \blacktriangleright 1 \times a_{44},$

LIE ALGEBRA number 4:

$[e_1, e_2] = +e_3$ $\quad [e_1, e_3] = -2.00a_{52}e_6 + a_{41}e_7 + a_{14}e_8$
$[e_1, e_4] = -a_{52}e_5 + a_{22}e_6$ $\quad [e_1, e_6] = +e_8$
$[e_2, e_3] = +a_{41}e_6 + a_{33}e_7$ $\quad [e_2, e_4] = +a_{41}e_5 + a_{43}e_7$
$[e_2, e_5] = +e_6$ $\quad [e_2, e_7] = +e_8$
$[e_3, e_4] = +a_{52}e_6$ $\quad [e_3, e_5] = +e_8$
$[e_4, e_5] = +e_7$ $\quad [e_4, e_6] = +e_8$
$H = 1 \blacktriangleright 3 \times a_{41}, \quad H = 2 \blacktriangleright 3 \times a_{52}, \quad H = 3 \blacktriangleright 1 \times a_{14}, \quad H = 4 \blacktriangleright 1 \times a_{22},$
$H = 5 \blacktriangleright 1 \times a_{33}, \quad H = 6 \blacktriangleright 1 \times a_{43},$

LIE ALGEBRA number 5:

$[e_1, e_2] = +e_3$
$[e_1, e_3] = +(+a_{33} - a_{41} - 2.00a_{52})e_6 + (-a_{33} + a_{41})e_7 + a_{14}e_8$
$[e_1, e_4] = -a_{52}e_5 + a_{22}e_6$ $\quad [e_1, e_6] = +e_8$
$[e_2, e_3] = +(-a_{33} + a_{41})e_6 + a_{33}e_7$ $\quad [e_2, e_4] = +a_{41}e_5 + a_{43}e_7$
$[e_2, e_5] = +e_6$ $\quad [e_2, e_7] = +e_8$
$[e_3, e_4] = +a_{52}e_6$ $\quad [e_3, e_5] = +e_8$
$[e_4, e_5] = +e_7$ $\quad [e_4, e_6] = +e_8$
$[e_4, e_7] = +e_8$
$H = 1 \blacktriangleright 4 \times a_{33}, \quad H = 2 \blacktriangleright 4 \times a_{41}, \quad H = 3 \blacktriangleright 3 \times a_{52}, \quad H = 4 \blacktriangleright 1 \times a_{14},$
$H = 5 \blacktriangleright 1 \times a_{22}, \quad H = 6 \blacktriangleright 1 \times a_{43},$

LIE ALGEBRA number 6:

$[e_1, e_2] = +e_3$ $\quad [e_1, e_3] = +(+a_{41} - a_{53})e_7 + a_{14}e_8$
$[e_1, e_4] = -a_{53}e_6$ $\quad [e_1, e_6] = +e_8$
$[e_2, e_3] = +(+a_{41} - a_{53})e_6 + a_{33}e_7$ $\quad [e_2, e_4] = +a_{41}e_5 + a_{43}e_7$
$[e_2, e_5] = +e_6$ $\quad [e_2, e_6] = +e_7$
$[e_2, e_7] = +e_8$ $\quad [e_3, e_4] = +a_{53}e_7$
$[e_3, e_5] = +e_8$ $\quad [e_4, e_5] = +e_7$
$[e_4, e_6] = +e_8$
$H = 1 \blacktriangleright 4 \times a_{53},, \quad H = 2 \blacktriangleright 3 \times a_{41}, \quad H = 3 \blacktriangleright 1 \times a_{14}, \quad H = 4 \blacktriangleright 1 \times a_{33},$
$H = 5 \blacktriangleright 1 \times a_{43},$

LIE ALGEBRA number 7:

$[e_1,e_2] = +e_3$ $[e_1,e_3] = +(+a_{41} - 2.00a_{53})e_5 + a_{42}e_6 + a_{13}e_7$
$[e_1,e_4] = +a_{21}e_5 + (+a_{41} - a_{53})e_6$ $[e_1,e_5] = +e_7$
$[e_1,e_7] = +e_8$ $[e_2,e_3] = +a_{42}e_5 + a_{32}e_6$
$[e_2,e_4] = +a_{41}e_5 + a_{42}e_6 + a_{44}e_8$ $[e_2,e_6] = +e_7$
$[e_3,e_4] = +a_{53}e_7$ $[e_3,e_6] = +e_8$
$[e_4,e_5] = +e_8$
$H = 1 \blacktriangleright 3 \times a_{41},\quad H = 2 \blacktriangleright 3 \times a_{42},\quad H = 3 \blacktriangleright 3 \times a_{53},\quad H = 4 \blacktriangleright 1 \times a_{13},$
$H = 5 \blacktriangleright 1 \times a_{21},\quad H = 6 \blacktriangleright 1 \times a_{32},\quad H = 7 \blacktriangleright 1 \times a_{44},$

LIE ALGEBRA number 8:

$[e_1,e_2] = +e_3$ $[e_1,e_3] = -2.00a_{53}e_6 + a_{13}e_7$
$[e_1,e_4] = -a_{53}e_5 + a_{22}e_6 + a_{23}e_7$ $[e_1,e_5] = +e_6$
$[e_1,e_7] = +e_8$ $[e_2,e_3] = +a_{52}e_6$
$[e_2,e_4] = +a_{52}e_5 + a_{42}e_6$ $[e_2,e_5] = +e_7$
$[e_3,e_4] = +a_{52}e_6 + a_{53}e_7$ $[e_3,e_5] = +e_8$
$[e_4,e_5] = +e_7$ $[e_4,e_6] = +e_8$
$H = 1 \blacktriangleright 3 \times a_{52},\quad H = 2 \blacktriangleright 3 \times a_{53},\quad H = 3 \blacktriangleright 1 \times a_{13},\quad H = 4 \blacktriangleright 1 \times a_{22},$
$H = 5 \blacktriangleright 1 \times a_{23},\quad H = 6 \blacktriangleright 1 \times a_{42},$

LIE ALGEBRA number 9:

$[e_1,e_2] = +e_3$
$[e_1,e_3] = +(-a_{33} - 2.00a_{53})e_6 + a_{33}e_7 + a_{14}e_8$ $[e_1,e_4] = -a_{53}e_5 + a_{43}e_7$
$[e_1,e_5] = +e_6$ $[e_1,e_7] = +e_8$
$[e_2,e_3] = +(-a_{33} + a_{52} - a_{53})e_6 + a_{33}e_7$
$[e_2,e_4] = +a_{52}e_5 + a_{42}e_6 + a_{43}e_7$ $[e_2,e_5] = +e_7$
$[e_2,e_7] = +e_8$ $[e_3,e_4] = +a_{52}e_6 + a_{53}e_7$
$[e_3,e_5] = +e_8$ $[e_4,e_5] = +e_7$
$[e_4,e_6] = +e_8$ $[e_4,e_7] = +e_8$
$H = 1 \blacktriangleright 4 \times a_{33},\quad H = 2 \blacktriangleright 4 \times a_{53},\quad H = 3 \blacktriangleright 3 \times a_{52},\quad H = 4 \blacktriangleright 2 \times a_{43},$
$H = 5 \blacktriangleright 1 \times a_{14},\quad H = 6 \blacktriangleright 1 \times a_{42},$

LIE ALGEBRA number 10:

$[e_1,e_2] = +e_3$
$$[e_1,e_3] = +\{+(B^3-B^2)a_{33} - (B^2+1)a_{52} + (B^3-2.00)a_{53}\}e_6 + +\{+(B - B^2)a_{33} + Ba_{52} - B^2a_{53}\}e_7$$
$[e_1,e_4] = -a_{53}e_5 + (+Ba_{42} + Ba_{43} - Ba_{54})e_7$ $[e_1,e_5] = +e_6$ $[e_1,e_6] = +e_8$
$[e_1,e_7] = +e_8$ $[e_2,e_3] = +(-Ba_{33} + a_{52} - Ba_{53})e_6 + a_{33}e_7 + a_{34}e_8$
$[e_2,e_4] = +a_{52}e_5 + a_{42}e_6 + a_{43}e_7$ $[e_2,e_5] = +e_7$ $[e_2,e_7] = +Be_8$
$[e_3,e_4] = +a_{52}e_6 + a_{53}e_7 + a_{54}e_8$ $[e_3,e_5] = +e_8$ $[e_4,e_5] = +e_7$
$[e_4,e_6] = +e_8$ $[e_4,e_7] = +Be_8$
$H = 1 \blacktriangleright 5 \times a_{52},\quad H = 2 \blacktriangleright 5 \times a_{53},\quad H = 3 \blacktriangleright 4 \times a_{33},\quad H = 4 \blacktriangleright 2 \times a_{42},$
$H = 5 \blacktriangleright 2 \times a_{43},\quad H = 6 \blacktriangleright 2 \times a_{54},\quad H = 7 \blacktriangleright 1 \times a_{34},$

Nilpotent Lie algebras 1, 7 *and* 10 *have* 7 *parameters.*

Nilpotent Lie algebras $2-5$, 8 *and* 9 *have* 6 *parameters.*

Nilpotent Lie algebra 6 *has* 5 *parameters.*

4 Non decomposable Lie algebras of dimension 8 whose maximal abelian ideal has dimension 4

Consider the following relations:

$[E_1, E_2] = E_3,\ [E_1, E_3] = E_4,\ [E_1, E_4] = 0,$

$[E_2, E_3] = 0,\ [E_2, E_4] = 0,\ [E_3, E_4] = 0.$

There exists 2 Lie Algebras in this category:

LIE ALGEBRA number 1:

$[e_1, e_2] = +e_3 \quad [e_1, e_3] = +e_4 \quad [e_1, e_4] = +a_{12}e_6 + a_{13}e_7$

$[e_1, e_6] = +e_7 \quad [e_1, e_7] = +e_8 \quad [e_2, e_3] = +a_{44}e_6 + a_{24}e_8$

$[e_2, e_4] = +a_{44}e_7 \quad [e_2, e_5] = +e_6 \quad [e_3, e_4] = +a_{44}e_8$

$[e_3, e_5] = +e_7 \quad [e_4, e_5] = +e_8$

$H = 1 \blacktriangleright 3 \times a_{44}, \quad H = 2 \blacktriangleright 1 \times a_{12}, \quad H = 3 \blacktriangleright 1 \times a_{13}, \quad H = 4 \blacktriangleright 1 \times a_{24},$

LIE ALGEBRA number 2:

$[e_1, e_2] = +e_3 \quad [e_1, e_3] = +e_4 \quad [e_1, e_4] = +(+a_{33} - a_{44})e_6 + a_{13}e_7$

$[e_1, e_6] = +e_7 \quad [e_1, e_7] = +e_8 \quad [e_2, e_3] = +a_{33}e_6 + a_{24}e_8$

$[e_2, e_4] = +a_{33}e_7 \quad [e_2, e_5] = +e_6 \quad [e_2, e_6] = +e_8$

$[e_3, e_4] = +a_{44}e_8 \quad [e_3, e_5] = +e_7 \quad [e_4, e_5] = +e_8$

$H = 1 \blacktriangleright 3 \times a_{33}, \quad H = 2 \blacktriangleright 2 \times a_{44}, \quad H = 3 \blacktriangleright 1 \times a_{13}, \quad H = 4 \blacktriangleright 1 \times a_{24},$

Nilpotent Lie algebras 1 and 2 have 4 parameters.

5 Non decomposable Lie algebras of dimension 8 whose maximal abelian ideal has dimension 4.

Consider the following relations:

$[E_1, E_2] = -E_3,\ [E_1, E_3] = -E_4,\ [E_1, E_4] = 0,$

$[E_2, E_3] = 0,\ [E_2, E_4] = 0,\ [E_3, E_4] = 0$

There exists 6 Lie Algebras in this category:

LIE ALGEBRA number 1:

$[e_1, e_2] = -e_3 \quad [e_1, e_3] = -e_4 \quad [e_1, e_4] = +a_{11}e_5 + a_{44}e_7$

$[e_1, e_5] = +e_6 \quad [e_1, e_6] = +e_7 \quad [e_2, e_3] = -a_{33}e_6 + a_{23}e_7$

$[e_2, e_4] = +a_{33}e_7 \quad [e_2, e_7] = +e_8 \quad [e_3, e_4] = +a_{44}e_8$

$[e_3, e_6] = +e_8 \quad [e_4, e_5] = +e_8$

$H = 1 \blacktriangleright 2 \times a_{33}, \quad H = 2 \blacktriangleright 2 \times a_{44}, \quad H = 3 \blacktriangleright 1 \times a_{11}, \quad H = 4 \blacktriangleright 1 \times a_{23},$

LIE ALGEBRA number 2:

$[e_1, e_2] = -e_3 \quad [e_1, e_3] = -e_4$

$[e_1, e_4] = +1.50a_{43}e_5 + a_{44}e_7$

$[e_1, e_5] = +e_6 \quad [e_1, e_6] = +e_7$

$[e_2, e_3] = -0.50a_{43}e_5 - a_{33}e_6 + a_{23}e_7$

$[e_2, e_4] = +0.50a_{43}e_6 + a_{33}e_7 \quad [e_2, e_5] = +e_7 \quad [e_2, e_7] = +e_8$

$[e_3, e_4] = +a_{43}e_7 + a_{44}e_8 \quad [e_3, e_6] = +e_8 \quad [e_4, e_5] = +e_8$

$H = 1 \blacktriangleright 4 \times a_{43}, \quad H = 2 \blacktriangleright 2 \times a_{33}, \quad H = 3 \blacktriangleright 2 \times a_{44}, \quad H = 4 \blacktriangleright 1 \times a_{23},$

LIE ALGEBRA number 3:

$[e_1, e_2] = -e_3 \qquad [e_1, e_3] = -e_4$

$[e_1, e_4] = +(+a_{13} - a_{44})e_5 + (-a_{13} + a_{44})e_6 + a_{13}e_7$

$[e_1, e_5] = +e_6 \qquad [e_1, e_6] = +e_7 \qquad [e_2, e_3] = -a_{33}e_6 + a_{23}e_7$

$[e_2, e_4] = +a_{33}e_7 \qquad [e_2, e_6] = +e_8 \qquad [e_2, e_7] = +e_8$

$[e_3, e_4] = +a_{44}e_8 \qquad [e_3, e_5] = +e_8 \qquad [e_3, e_6] = +e_8$

$[e_4, e_5] = +e_8$

$H = 1 \blacktriangleright 3 \times a_{13}, \quad H = 2 \blacktriangleright 3 \times a_{44}, \quad H = 3 \blacktriangleright 2 \times a_{33}, \quad H = 4 \blacktriangleright 1 \times a_{23},$

LIE ALGEBRA number 4:

$[e_1, e_2] = -e_3 \qquad [e_1, e_3] = -e_4$

$[e_1, e_4] = +1.50a_{43}e_5 - 1.50a_{43}e_6 + (+1.50a_{43} + a_{44})e_7 \qquad [e_1, e_5] = +e_6$

$[e_1, e_6] = +e_7 \qquad [e_2, e_3] = -0.50a_{43}e_5 - a_{33}e_6 + a_{23}e_7$

$[e_2, e_4] = +0.50a_{43}e_6 + a_{33}e_7 \qquad [e_2, e_5] = +e_7 \qquad [e_2, e_6] = +e_8$

$[e_2, e_7] = +e_8 \qquad [e_3, e_4] = +a_{43}e_7 + a_{44}e_8 \qquad [e_3, e_5] = +e_8$

$[e_3, e_6] = +e_8 \qquad [e_4, e_5] = +e_8$

$H = 1 \blacktriangleright 6 \times a_{43}, \quad H = 2 \blacktriangleright 2 \times a_{33} \quad H = 3 \blacktriangleright 2 \times a_{44}, \quad H = 4 \blacktriangleright 1 \times a_{23},$

LIE ALGEBRA number 5:

$[e_1, e_2] = -e_3 \qquad [e_1, e_3] = -e_4$

$[e_1, e_4] = +(+a_{13} - a_{33} - a_{44})e_5 + (-a_{13} + a_{33} + a_{44})e_6 + a_{13}e_7$

$[e_1, e_5] = +e_6 \qquad [e_1, e_6] = +e_7 \qquad [e_1, e_7] = +e_8$

$[e_2, e_3] = -a_{33}e_6 - a_{34}e_7 \qquad [e_2, e_4] = +a_{33}e_7 + a_{34}e_8 \qquad [e_2, e_6] = +e_8$

$[e_2, e_7] = +e_8 \qquad [e_3, e_4] = +a_{44}e_8 \qquad [e_3, e_5] = +e_8$

$[e_3, e_6] = +e_8 \qquad [e_4, e_5] = +e_8$

$H = 1 \blacktriangleright 4 \times a_{33}, \quad H = 2 \blacktriangleright 3 \times a_{13}, \quad H = 3 \blacktriangleright 3 \times a_{44}, \quad H = 4 \blacktriangleright 2 \times a_{34},$

LIE ALGEBRA number 6:

$[e_1, e_2] = -e_3 \qquad [e_1, e_3] = -e_4$

$[e_1, e_4] = +1.50a_{43}e_5 + a_{43}e_6 + (+a_{33} - a_{43} + a_{44})e_7$

$[e_1, e_5] = +e_6 \qquad [e_1, e_6] = +e_7 \qquad [e_1, e_7] = +e_8$

$[e_2, e_3] = -0.50a_{43}e_5 - a_{33}e_6 - a_{34}e_7 \quad [e_2, e_4] = +0.50a_{43}e_6 + a_{33}e_7 + a_{34}e_8$

$[e_2, e_5] = +e_7 \qquad [e_2, e_6] = +e_8 \qquad [e_2, e_7] = +e_8$

$[e_3, e_4] = +a_{43}e_7 + a_{44}e_8 \qquad [e_3, e_6] = +e_8 \qquad [e_4, e_5] = +e_8$

$H = 1 \blacktriangleright 6 \times a_{43} \quad H = 2 \blacktriangleright 3 \times a_{33}, \quad H = 3 \blacktriangleright 2 \times a_{34} \quad H = 4 \blacktriangleright 2 \times a_{44}$

Nilpotent Lie algebras 1 - 6 have 4 parameters.

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