KNIZHNIK–ZAMOLODCHIKOV CONNECTIONS, SPIN CALOGERO–SUTHERLAND OPERATORS, AND GENERALIZED MATSUO–CHEREDNIK MAPS

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Abstract

We consider the generalized spin maps of Matsuo and Cherednik which are the linear isomorphisms from the space of solutions of the Knizhnik-Zamolodchikov equations to the space of eigenfunctions of scalar Calogero-Sutherland operators. It is shown that in the spin case the Matsuo-Cherednik maps are monomorphisms.

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Let $f_1, \dots, f_n$ be orthonormal basis in an Euclidean space $\mathbb{R}^n$ equipped with the standard scalar product $(x, y) = \sum_{i=1}^{n} x_i y_i$, $x, y \in \mathbb{R}^n$. Denote the derivative in the direction f_i, by ∂_i, $i = 1, \dots, n$ and the derivative in the direction $\xi = \sum_{i=1}^{n} a_i f_i$ by ∂_ξ. Let $\Delta = \sum_{i=1}^{n} \partial_i^2$ be the Laplace operator. Consider the irreducible and reduced root system $R \subset \mathbb{R}^n$ and the positive root system $R_+ \subset R$; the Weyl group of the root system will be denoted by $W(R)$ and its generators will be denoted by s_α, $\alpha \in R_+$. Let $H_\alpha = \{z | (\alpha, z) = 0\}$, $\alpha \in R$ and $H = \bigcup_{\alpha \in R} H_\alpha$ be the system of reflection hyperplanes of $W(R)$. Their complexifications will be denoted by the same letters. Denote

$$H_{\alpha,m} = \left\{ z \in \mathbf{C}^n \mid (\alpha, z) = \frac{2\pi i m}{\mu} \right\}, \qquad m \in \mathbf{Z}, \quad \mu \in \mathbf{C},$$

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$$\tilde{H} = \bigcup_{\alpha \in R_+, m \in \mathbf{Z}} H_{\alpha,m}.$$

Let $K[W(R)]$ be a group algebra of Weyl group $W(R)$ for the field $K = I\!R, \mathbf{C}$. Consider the space of infinitely derivable functions $\mathcal{F}_K$ on $K^n \setminus H$ or on $\mathbf{C}^n \setminus \tilde{H}$ with values in $K[W(R)]$.

We will consider the action on $\mathcal{F}_K$ of Knizhnik–Zamolodchikov (KZ) operators in Cherednik–Matsuo form (see [1], [2]):

$$\nabla_\xi(k,\mu) = \partial_\xi - \frac{1}{2}\left(\sum_{\alpha \in R} \mu k_\alpha (\alpha,\xi)\Big(f(\mu(\alpha,z))\sigma_\alpha + \sigma_\alpha \varepsilon_\alpha\Big)\right) - e_\xi(\lambda) \tag{1}$$

for $\xi \in K$ and spin Calogero–Sutherland (CS) operators

$$L_{CS}(k,\mu) = -\Delta + \sum_{\alpha \in R_+} \mu^2(\alpha,\alpha)k_\alpha^2\big(f^2(\mu(\alpha,z)) - 1\big) + k_\alpha f'\big(\mu(\alpha,z)\sigma_\alpha\big), \tag{2}$$

where μ is an arbitrary parameter, $\mu \in K$, k_α is a $W(R)$-invariant function on the root system R, $f(x) = \coth z$, f' is its derivative, σ_α is the operator of multiplication by reflection $s_\alpha \in K[W(R)]$ and the operators ε_α and $e_\xi(\lambda)$ on $K[W(R)]$ are defined by the equations

$$\varepsilon_\alpha(w) = \begin{cases} w, & w^{-1}\alpha \in R_+, \\ -w, & w^{-1}\alpha \in R_-, \end{cases}$$

$$e_\eta(\lambda)(w) = (w\lambda, \eta) \cdot w, \quad w \in W.$$

Let ${}^*\nabla_\xi(k,\mu)$ be formal adjoint operator for the operator $\nabla_\xi(k,\mu)$ defined by

$${}^*\nabla_\xi(k,\mu) = -\partial_\xi - \frac{1}{2}\left(\sum_{\alpha \in R} \mu k_\alpha (\alpha,\xi)\Big(f(\mu(\alpha,z))\sigma_\alpha + \sigma_\alpha \varepsilon_\alpha\Big)\right) - e_\xi(\lambda).$$

Lemma 1 *The following equation holds:*

$$\sum_{i=1}^n {}^*\nabla_i(k,\mu)\nabla_i(k,\mu) = L_{CS}(k,\mu) + (\lambda,\lambda). \tag{3}$$

Proof is done by direct calculation using the trigonometric analog of the Dunkl identities (see for example [3, Theorem 1]).

Theorem 1 *Let $\Phi(z) \in \mathcal{F}_K$ be the solution of the KZ equation*

$$\nabla_\xi(k,\mu)\Phi(z) = 0, \xi \in K^n, \tag{4}$$

where $\nabla_\xi(k,\mu)$ is defined by (1). Then $\Phi(z)$ is the eigenfunction of the spin CS operator $L_{CS}(k,\mu)$ with the eigenvalue $-(\lambda,\lambda)$

$$L_{CS}(k,\mu)\Phi(z) = -(\lambda,\lambda)\Phi(z). \tag{4'}$$

The proof is an obvious corollary from the Lemma 1.

In [2, 4] the linear maps from the space of solutions of the KZ equation (4) to the space of eigenfunctions of scalar CS operators (more exactly, to the space of eigenstates of quantum problem with Hamiltonian conjugate to the operator $L_{CS}(k,\mu)$ for $\sigma_\alpha = 1$) were defined. In the case of spin operator $L_{CS}(k,\mu)$ the analog of these linear maps are the identity maps. Further these maps will be called the Matsuo-Cherednik maps also. In [2, 4] it was shown also that the Matsuo–Cherednik maps are monomorphisms. More exactly, they are isomorphisms on the space of eigenstates of quantum problem with the corresponding scalar Hamiltonian L_{CS}. In spin case this map evidently is a monomorphism also. In [4, 5] it was noted that the Matsuo–Cherednik maps are connected with one-dimensional complex irreducible representations of Weyl group: trivial and determinantal ones. The spin case considered above corresponds to the left regular representation $W(R)$. It will be shown that the assertion on monomorphism is true for the spin CR operators corresponding to the arbitrary irreducible representation of Weyl group $W(R)$.

Consider the group algebra $\mathbf{C}[W(R)]$ and the decomposition of the identity of this algebra into the sum of orthogonal indecomposable idempotents

$$1 = \delta_1 + \delta_2 + \cdots + \delta_N, \tag{5}$$

where

$$\delta_i\delta_j = 0 \quad \text{for } i \neq j \text{ and } \delta_i^2 = \delta_i.$$

Every left ideal $E_{\delta_i} = \mathbf{C}[W(R)] \cdot \delta_i$ is the space of the complex irreducible representation of the group $W(R)$ and every irreducible representation of the group $W(R)$ is contained among E_{δ_i}, $i = 1, 2, \ldots, N$, with multiplicity equal to the dimension of this representation. The decomposition (5) implies the decomposition of the group algebra into the direct sum

$$C[W(R)] = E_{\delta_1} \oplus \cdots \oplus E_{\delta_N}, \tag{6}$$

that is the decomposition of the left regular representation of $W(R)$ into the sum of irreducible representations of this group. The representation will be called further simply δ_i.

According with the decomposition of (6) the space of functions $\mathcal{F}_\mathbf{C}$ splits into the direct sum

$$\mathcal{F}_\mathbf{C} = \mathcal{F}_{\delta_1} \oplus \cdots \oplus \mathcal{F}_{\delta_N},$$

where $\mathcal{F}_{\delta_i}$ is the space of differentiable functions on $\mathbf{C}^n \setminus H$ or $\mathbf{C}^n \setminus \tilde{H}$ with values in E_{δ_i} and for $F \in \mathcal{F}_\mathbf{C}$ the component $\mathcal{F}_{\delta_i}$ is obtained from F by multiplication to δ_i.

Definition 1 *The linear map*

$$m_{\delta_i} : \mathcal{F}_\mathbf{C} \to \mathcal{F}_{\delta_i}$$

which associates $F_{\delta_i} = F \cdot \delta_i$, with $F \in \mathcal{F}_\mathbf{C}$ is called the Matsuo-Cherednik map corresponding to the irreducible representation δ_i.

Lemma 2 *Every space $\mathcal{F}_{\delta_i}$ is invariant with respect to the action of the spin operator $L_{CS}(k,\mu)$.*

Proof. It is evident, because the action of the Laplace operator, the multiplication by the scalar function and the left multiplication by the s_α, $\alpha \in R$, applied to the function $F \in \mathcal{F}_{\delta_i}$ give an element of the ideal E_{δ_i}. Hence, $L_{CS}(k,\mu)$ conserves $\mathcal{F}_{\delta_i}$. □

Definition 2 *The restriction of the operator $L_{CS}(k,\mu)$ to the space $\mathcal{F}_{\delta_i}$ is called the* spin CS operator *corresponding to the representation δ_i and will be denoted by $L^{\delta_i}_{CS}(k,\mu)$.*

The following theorem follows immediately from Lemmas 1 and 2.

Theorem 2 *Let $\Phi \in \mathcal{F}_{\mathbf{C}}$ be the solution of the KZ equation (4). Then $m_{\delta_i}(\Phi)$, $i = 1,\dots,N$, is the eigenfunction of the spin operator $L^{\delta_i}_{CS}(k,\mu)$ with the eigenvalue $-(\lambda,\lambda)$.*

Now we state the main theorem.

Theorem 3 *Let $\lambda \in \mathbf{C}^n$ be a vector such that for all $w \in W(R)$ the vector $w\lambda - \lambda$ does not belong to the root lattice and consider*

$$k_\alpha \neq \pm\frac{2(\lambda.\alpha)}{(\alpha,\alpha)}, \alpha \in R_+,$$

then the map m_{δ_i} is a monomorphism for any irreducible representation δ_i.

In order to prove the Theorem, some necessary notions, references and lemmas are given below.

The proof contains the following steps (as in the case of one-dimensional representation in [2]):

(1) the construction of the base $\Phi^w(z)$, $w \in W(R)$, of the solutions of the KZ equation (4);

(2) the proof for the assertion: every basic element goes to the non zero eigenfunction $m_{\delta_i}(\Phi^w(z))=\Phi^w_{\delta_i}(z)$ of the spin CS operator corresponding to the representation δ_i;

(3) the proof of the linear independence of the functions $\Phi^w_{\delta_i}(z)$.

The solutions that we will construct differ by their asymptotic behavior for $(\alpha_i, z) \to \infty$, $i = 1,\dots,n$, where α_i is a simple root. Introduce the variables $y_i = e^{(\alpha_i,z)}$ where α_i are simple roots of R, $i = 1,\dots,n$. Let α_i^* be a dual basis in $\mathbf{C}^n$, that is $(\alpha_i^*,\alpha_i) = \delta_{ij}$. Then we have

$$\frac{\partial}{\partial y_i} = \frac{1}{y_i}\partial_{\alpha_i^*}.$$

If we write the equation (4) in the neighborhood of $y_i = 0$, then we obtain

$$\frac{\partial \Phi}{\partial y_i} = \left(\frac{A^0_{\alpha_i^*}}{y_i} + \quad \text{regular part}\right)\Phi,$$

$i = 1, \ldots, n$, where

$$A^0_\xi = \sum_{\alpha \in R_+} k_\alpha(\alpha, \xi)\sigma_\alpha(\varepsilon_\alpha - 1) + e_\xi(\lambda).$$

Using the integrability of KZ equations it follows that A^0_ξ commutes for different $\xi \in \mathbf{C}^n$.

Consider the basis of the group algebra $\mathbf{C}[W(R)]$ consisting of the elements $[w], w \in W(R)$. Order its elements by the length (that is by the minimal number of generators of the Weyl group in their representations).

We need the following technical Lemma.

Lemma 3 *For any positive root α the operator $s_\alpha(\varepsilon_\alpha - 1)$ on $\mathbf{C}[W(R)]$ applied to the basic element $[w]$ gives an element which is proportional to a basic element with lower length. In this basis $s_\alpha(\varepsilon_\alpha - 1)$ has a lower triangular form.*

Proof. Since for positive root α, $w^{-1}\alpha \in R_-$ if and only if the lengths of elements satisfy the inequality $l(s_\alpha w) < l(w)$ (see [6]) we obtain:

$$\sigma_\alpha(\varepsilon_\alpha - 1)[w] = \begin{cases} 0, & \text{if} \quad l(s_\alpha w) > l(w), \\ -2s_\alpha w, & \text{if} \quad l(s_\alpha w) < l(w). \end{cases}$$

□

In the sequel we suppose that $\lambda \in \mathbf{C}^n$ is such that $w\lambda - \lambda$ does not belong to the root lattice for any $w \in W(R)$. Such values of λ are called *regular*.

The following assertion regarding the existence of solutions of KZ equations holds.

Lemma 4 *Let $\lambda \in \mathbf{C}^n$ be regular. Then for every $w \in W(R)$ there exists the solution of the KZ equation (4) of the form*

$$\Psi^w(z) = e^{(w\lambda, z)} \left(\Psi^w_0 + \sum_{\mu > 0} \Psi^w_\mu e^{(\mu, z)} \right), \tag{7}$$

where the summation goes through the positive elements of the root lattice, and Ψ^w_0 for $w \in W(R)$ are eigenfunctions of the operators

$$A^0_\xi \Psi^w_0 = (w\lambda, \xi)\Psi^w_0.$$

Proof. The diagonal elements of the matrix operator A^0_ξ are the numbers $(w\lambda, \xi)$. By the assumption any pair of eigenvalues $(w\lambda, \xi)$ and $(w'\lambda, \xi)$ for $w \neq w'$ does not differ by an integer. The existence of the desired solution follows from the local theory of linear Fuchsian systems (see [7, 8]). □

Lemma 5 *For $\Psi^w_0 = \sum_{w' \in W} c_{w,w'}[w']$ considered in Lemma 4, if $l(w') \geq l(w)$ for $w' \neq w$, then we have $c_{w,w'} = 0$ and*

$$\Psi^w_0 = \sum_{w' \in W, l(w') \leq i(w)} c_{w,w'}[w'].$$

The proof is evident.

Now we will show that for indecomposable idempotent δ_i every solution $\Phi^w(z)$ transforms to the function $\Phi^w_{\delta_i}(z) \neq 0$. For this purpose we analyze the term Ψ^w_0 in (6). The coefficients $c_{w,w'}$ depend from the parameters k_α and λ of the equation KZ (4).

The following Lemma, which is completely analogous to Lemma 4.3.1 of [2], gives the values of the first coefficient and recurrent formula for the others.

Lemma 6 1. *If α is a simple root of the reduced irreducible root system we have*

$$c_{s_\alpha,1}(\lambda, k_\alpha) = \frac{k_\alpha}{(\lambda, \alpha^\vee)},$$

where $\alpha^\vee = \frac{2\alpha}{(\alpha,\alpha)}$.

2. *If α is a simple root and $l(s_\alpha w) > l(w)$ then we have*

$$c_{s_\alpha w,w'}(\lambda, k_\alpha) = c_{w,s_\alpha w'}(\lambda, k_\alpha) + c_{w,w'}(\lambda, k_\alpha) c_{s_\alpha,1}(\lambda, k_\alpha).$$

Now we prove the principal assertion on behavior of principal term of asymptotics of Ψ^w_0 for $y = (y_1, \ldots, y_n) = (0, \ldots, 0)$ in (7).

Lemma 7 *Let w be an element of W and $w = s_{i_1}, \ldots, s_{i_l}$ its representation of minimal length where $s_{i_k} = s_{\alpha_{i_k}}$ is a basis of simple roots of R. Denote as $w_k = s_{i_{k+1}} \cdots s_{i_l}$. Then Ψ^w_0 used in (7) becomes:*

$$\Psi^w_0 = \prod_{k=1}^{l} \left(\frac{k_{\alpha_{i_k}}}{(\lambda, w_k^{-1} \alpha_{i_k})} + s_{i_k} \right) = \prod_{\beta \in R_+ \cap w^{-1} R_-} \left(\frac{k_\beta}{(\lambda, \beta)} + s_\beta \right). \tag{8}$$

Proof. We will prove (8) by induction on length of $w \in W$. For $w = 1$ (7) is fulfilled (the multiplication is done through empty set of indices) and $\Psi^1_0 = 1$. Let further α is a simple root and suppose that $l(\sigma_\alpha w) > l(w) = l$ where $w = s_{i_1} \cdots\ s_{i_l}$ is a minimal length representation. Using the properties of the coefficients $c_{w,w'}$ we obtain:

$$\begin{aligned} \Psi^{s_\alpha w}_0 &= \textstyle\sum_{w' \in W} c_{s_\alpha w,w'}(\lambda, k)[w'] \\ &= \textstyle\sum_{w' \in W} c_{w,s_\alpha w'}(\lambda, k)[w'] \\ &+ \textstyle\sum_{w' \in W} c_{w,w'}(\lambda, k) c_{s_\alpha,1}(w\lambda, k)[w']. \end{aligned}$$

If $s_\alpha w$ changes into w' and using the equation $s_\alpha^2 = 1$ we obtain:

$$\begin{aligned} \Psi^{s_\alpha w}_0 &= \textstyle\sum_{w' \in W} c_{w,w'}(\lambda, k) s_\alpha [w'] \\ &+ \textstyle\sum_{w' \in W} c_{w,w'}(\lambda, k)[w'] \frac{k_\alpha}{(w\lambda, \alpha^\vee)} \\ &= (s_\alpha + \frac{k_\alpha}{(\lambda, w^{-1}\alpha^\vee)}) \Psi^w_0. \end{aligned}$$

Using the induction assumptions we obtain:

$$\Psi^{s_\alpha w}_0 = \left(s_\alpha + \frac{k_\alpha}{(\lambda, w^{-1}\alpha)} \right) \prod_{m=1}^{l} \left(\frac{k_{\alpha_{i_m}}}{(\lambda, w^{-1}\alpha_{i_m})} + s_{i_m} \right).$$

Denote s_α by $s_{i_{l+1}}$. We have

$$\Psi_0^{s_\alpha w} = \prod_{m=1}^{l+1} \left(\frac{k_{\alpha_{i_m}}}{(\lambda, w^{-1}\alpha_{i_m})} + s_{i_m} \right).$$

The proof of the second equality is based on the following assertion: if α is positive (in particular, simple) root then from $l(s_\alpha w) > l(w)$ it follows that $w^{-1}\alpha \in R_+$. □

Corollary 1 *For any indecomposable idempotent δ and for any $\alpha \in R_+$, such that*

$$\frac{k_\alpha}{(\lambda, \alpha)} \neq \pm 1, \tag{9}$$

we have

$$\Psi_0^w \neq 0 \quad and \quad \Psi_{0,\delta}^w = \Psi_0^w \cdot \delta \neq 0.$$

Proof. Let us assume the contrary and suppose that $\Psi_0^w = 0$. Then there exists such minimal length index β in the decomposition of Ψ_0^w that we have

$$\left(\frac{k_\beta}{(\lambda, \beta)} + s_\beta \right) \prod_{\alpha \in R_+ \cap w^{-1} R_- \alpha top \alpha \prec \beta} \left(\frac{k_\alpha}{(\lambda, \alpha)} + s_\alpha \right) = 0. \tag{10}$$

Here $\alpha \prec \beta$ is natural ordering of the elements $\alpha \in R_+ \cap w^{-1}R_-$ having origin in the representation of the elements w in the form $w = s_{i_1} \cdots s_{i_l}$. It signifies that $\frac{k_\alpha}{(\lambda,\alpha)}$ is the eigenvalue of $\sigma_\alpha = s_\alpha$ (in regular representation). But since $s_\beta =^2 1$ its eigenvalues can be ± 1 only. However, in virtue of (9) it is impossible. Consequently, the assumption that $\Psi_0^w = 0$ is not true and we have $\Psi_0^w \neq 0$.

Similarly it is proved that $\Psi_{0,\delta}^w = \Psi_0^w \cdot \delta \neq 0$ (δ is equal to one of δ_i, $i = 1, \ldots, N$). □

Proof of Theorem 3. From the assertion that $\Psi_{0,\delta}^w \neq 0$ for all $w \in W$ and for sufficiently small y_i, that is for $(\alpha_i, z) \to -\infty$ we have for nonzero principal term of asymptotics

$$\Psi_\delta^w(z) = e^{(w\lambda, z)} (\Psi_{0,\delta}^w + \sum_{\mu > 0} \Psi_{\mu,\delta}^w e^{(\mu, z)}).$$

Represent the element $w\lambda \in C^n$ in the form

$$w\lambda = \sum_{i=1}^n p_i^w(\lambda) \cdot \alpha_i.$$

Consider the vector of coefficients

$$p_i^w(\lambda) = (p_1^w(\lambda), \ldots p_n^w(\lambda)).$$

By the hypothesis of the Theorem 3 we have

$$p^w(\lambda) - p^{w'}(\lambda) \neq \mathbf{Z}^n,$$

for all $w \neq w'$, $w, w' \in W$. It follows that there exists a number i such that $p_i^w(\lambda) - p_i^{w'}(\lambda) \notin \mathbf{Z}$ (in particular $p_i^w(\lambda) \neq p_i^{w'}(\lambda)$).

Then in the local coordinates $y_1, \ldots, y_n$ the solution is presented in the form

$$\tilde{\Phi}_\delta^w(y) = \prod_{i=1}^n y_i^{p_i^w(\lambda)} \left(\Psi_{0,\delta}^w + \sum_{\mu>0} \Psi_{\mu,\delta}^w \prod_{i=1}^n y_i^{p_i(\mu)} \right),$$

where $p_i(\mu) > 0$.

The linear independence of the set of monomials $\prod_{i=1}^n y_i^{p_i^w(\mu)}$, $w \in W$, follows from the conditions of the theorem. This fact implies the linear independence of the principal terms of the asymptotics of $\tilde{\Phi}_\delta^w(y)$ that is of $\prod_{i=1}^n y_i^{p_i^w(\lambda)} \cdot \Psi_{0,\delta}^w$ for $w \in W$. The last implies the linear independence of the functions of $\tilde{\Phi}_\delta^w(y)$ and consequently of the functions $\Phi_\delta^w(y)$, $w \in W$. The elements of the basis of the solution of the KZ equation tend to the non zero and linear independent eigenfunctions of the spin CS operator. This proves the monomorphism of generalized Matsuo-Cherednik map for any representation δ and in particular for irreducible representation.

In conclusion we state the hypothesis concerning the generalizations of the shift Opdam operators.

Hypothesis. There exists some map D which closes the diagram for any irreducible representations δ_i and δ_j

$$KZ$$

$$\delta elta_i \swarrow \qquad\qquad \searrow \delta elta_j \quad E_{\delta elta_i}$$

$$E_{\delta elta_i} \qquad \overset{D}{\delta ashrightarrow} \qquad E_{\delta elta_j}$$

Such a D is called the generalized shift operator.

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