DIRAC OPERATORS AND THE WEITZENBÖCK FORMULA FOR $Spin^G(3)$ -STRUCTURES

Ana-Maria Teleman

Abstract

The aims of this article are to present a few properties of the Dirac operators associated to $Spin^G(3)$ -structures and to give a generalised version of the Weitzenböck Formula for $Spin^G(3)$ -structures. This formula is in fact a Bochner-Lichnerowicz type identity, as it can be found in [LM], Theorem II. 8.8., Theorem II. 8.17. and Theorem D.12. and gives identities in which one of the terms is a Dirac laplaceian (in this case the square of a Dirac operator). It can be found also in papers related to the Seiberg-Witten theory (see for instance [OT1], [OT2], [OT3] and [OT4]), with the name of Weitzenböck formula. The definitions related to vector bundles and connections can be found in [FU], [K], [KN] and [We]. The general definitions of the Lie groups $Spin^G(n)$ ($n \in \mathbb{N}^*$), of $Spin^G(n)$-structures in $SO(n)$-principal bundles and of associated Seiberg-Witten monopole equations (for $SO(4)$-coframes bundles) have been introduced and studied in [T1] and [T3]. We shall consider only the case $n = 3$.

Key words: spinor, connection, curvature, Dirac operator.
1991 Mathematics Subject Classification: 53C07, 55R10, 58G30, 22E60.

1 Introduction

Let (X, g) be a Riemannian compact manifold of dimension 3 and E a hermitian vector bundle over X of rank r. We shall assume that the associated vector bundles like $\Lambda^q T^* X$ or $End(E)$ are endowed with the canonically induced respective metrics. We shall use the following

Notations:
$ad(E) = P \times_{ad_{U(r)}} u(r)$, where P is the principal bundle of the unitary frames in E and $u(r)$ the Lie algebra of the unitary group $U(r)$.
$A^0(E) = \{s : X \longrightarrow E;\ s \text{ is a smooth section in } E\}$.
$A^q(X) = A^0(\Lambda^q T^*(X))$.

Editor Gr.Tsagas *Proceedings of the Workshop on Global Analysis, Differential Geometry and Lie Algebras*, 1997, 239-247

$A^q(E) = A^0(\Lambda^q T^\star(X) \otimes E)$.
$\Lambda^q_X = \Lambda^q T^\star(X)$.
$\mathcal{A}(E) = \{a : A^0(T(X)) \times A^0(E) \longrightarrow A^0(E);$ a unitary connection in $E\}$.
For a given Lie group we shall denote by Ad the adjoint morphism from the Lie group to its group of automorphisms and by ad the adjoint representation of the Lie group in its Lie algebra.

Convention: $su(2)$ shall be considered endowed with the scalar product $< a, b >= -\frac{1}{2}Tr(ab)$, for any $a, b \in su(2)$.

The adjoint representation of the Lie group $SU(2)$ on its Lie algebra $su(2)$ gives the exact sequence

$$1 \longrightarrow \mathbf{Z}_2 \hookrightarrow SU(2) \xrightarrow{ad} SO(su(2)) \longrightarrow 1$$

which induces the exact sequence

$$1 \longrightarrow \mathbf{Z}_2 \hookrightarrow SU(2) \xrightarrow{p} SO(3) \longrightarrow 1$$

Since $SU(2)$ is connected and simply connected it follows that the group $Spin(3)$, defined as the universal cover of $SO(3)$, is $SU(2)$.

Definition 1.1 *Let (V, h) be a hermitian vector space of finite dimension. Let $G \subsetneq U(V)$ be a closed subgroup of the unitary group of V which contains the central involution $-Id_V$. Then*

$$Spin^G(3) := Spin(3) \times_{Z_2} G = SU(2) \times_{Z_2} G,$$

where Z_2 as subgroup of $SU(2) \times G$ is $\{\pm(Id_2, Id_V)\}$.

Notation: $L := Lie(G)$.

Remark 1.2 *With the above notations, $L \subset u(V)$.*

Notations: For $(u, g) \in SU(2) \times G$ we denote by $[u, g]$ the corresponding class modulo $\mathbf{Z}_2$ of (u, g) in $Spin^G(3)$.
When there is no danger of confusion we use the same notation for elements in fibre bundles associated with a given principal bundle and a left action of its structure Lie group on a given manifold. (For such fibre bundles we use the definition from [KN], pg. 52-53]).

We define now some important Lie group morphisms defined on $Spin^G(3)$.

Definition 1.3 *1) $ad_S : Spin^G(3) \longrightarrow SO(su(2))$ is the morphism canonically induced by the adjoint representation of $SU(2)$ in its Lie algebra: $ad_S([u, g]) = ad(u)$.*
2) $ad_G : Spin^G(3) \longrightarrow O(L)$ is the morphism canonically induced by the adjoint representation of G in its Lie algebra: $ad_G([u, g]) = ad(g)$.
3) $Ad_S : Spin^G(3) \longrightarrow Aut(SU(2))$ is the morphism canonically induced by the adjoint morphism from $SU(2)$ to $Aut(SU(2))$: $Ad_S([u, g]) = Ad(u)$.
4) $Ad_G : Spin^G(3) \longrightarrow Aut(G)$ is the morphism canonically induced by the adjoint

morphism from G to $Aut(G)$: $Ad_G([u,g]) = Ad(g)$.
5) $\lambda : Spin^G(3) \longrightarrow U(C^2 \otimes_C V)$ is the morphism defined by $\lambda([u,g])(z \otimes v) = uz \otimes gv$ for any $u \in SU(2)$, $g \in G$, $z \in C^2$ and $v \in V$.
6) $\pi : Spin^G(3) \longrightarrow SO(3)$ is the morphism defined by $\pi([u,g]) = p(u)$ for any $u \in SU(2)$ and $g \in G$.
7) $\delta : Spin^G(3) \longrightarrow G/Z_2$ is the canonical projection $\delta([u,g]) = [g] (mod\, Z_2)$.

Remark 1.4 *The following sequences are exact:*

$$1 \longrightarrow SU(2) \hookrightarrow Spin^G(3) \xrightarrow{\delta} G/Z_2 \longrightarrow 1\,,$$
$$1 \longrightarrow G \hookrightarrow Spin^G(3) \xrightarrow{\pi} SO(3) \longrightarrow 1\,,$$
$$1 \longrightarrow Z_2 \hookrightarrow Spin^G(3) \xrightarrow{(\pi,\delta)} SO(3) \times G/Z_2 \longrightarrow 1\,.$$

Proof The proof is a direct consequence of the general theorems of group isomorphisms.

Convention: From now on we shall always assume a Riemannian 3-manifold to be connected, compact and oriented.

Convention: If P is the $SO(3)$-principal bundle of the oriented and ρ-orthonormal coframes of (X, ρ) then $P \times_{SO(3)} \mathbf{R}^3$ shall be considered automatically endowed with the euclidian structure given by

$$< [p, (u^1, u^2, u^3)], [p, (v^1, v^2, v^3)] >= u^1 v^1 + u^2 v^2 + u^3 v^3$$

for any $[p, (u^1, u^2, u^3)], [p, (v^1, v^2, v^3)] \in P \times_{SO(3)} \mathbf{R}^3$.

Remark 1.5 *If P is the $SO(3)$-principal bundle of the oriented and ρ-orthonormal coframes of (X, ρ), then $P \times_{SO(3)} \mathbf{R}^3$ is canonically isometric in an orientation preserving way with Λ^1_X, where the cotangent bundle Λ^1_X is endowed with the metric induced by ρ.*

Proof One can check very easily that the morphism given by

$$P \times_{SO(3)} \mathbf{R}^3 \ni [p, (u_1, u_2, u_3)] \mapsto u^1 p_1 + u^2 p_2 + u^3 p_3 \in \Lambda^1_X$$

for any $p = (p_1, p_2, p_3) \in P$ and $(u_1, u_2, u_3) \in \mathbf{R}^3$ is the required isometry.

Definition 1.6 *A $Spin^G(3)$-structure on a Riemannian 3-manifold (X, ρ) is a pair $(P^G, \gamma : P \times_{SO(3)} \mathbf{R}^3 \xrightarrow{\simeq} P^G \times_\pi \mathbf{R}^3)$ where P^G is a $Spin^G(3)$-principal bundle over X, P is the $SO(3)$-principal bundle of the oriented and ρ-orthonormal coframes of (X, ρ) and γ is an orientation preserving linear isometry.*

Remark 1.7 *If $(P^G, \gamma : P \times_{SO(3)} \mathbf{R}^3 \longrightarrow P^G \times_\pi \mathbf{R}^3)$ is a $Spin^G(3)$-structure on Riemannian 3-manifold (X, ρ) then $P^G \times_\pi \mathbf{R}^3$ can be isometrically identified with*

$P^G \times_{ad_S} su(2)$. Furthermore, $P \times_{SO(3)} \mathbf{R}^3$ can be identified with Λ^1_X as seen in remark 1.5 and γ can be considered as an orientation preserving isometry

$$\gamma : \Lambda^1_X \longrightarrow P^G \times_{ad_S} su(2).$$

Definition 1.8 *If (P^G, γ) is a $Spin^G(3)$-structure in a Riemannian 3-manifold (X, ρ) then the isometry γ is called the* ***Clifford map*** *of the given structure.*

Convention: From now on we shall consider a fixed hermitian vector space (V, h) and an associated Lie group $G \subset U(V)$ as in definition 1.1. We shall consider a fixed Riemannian 3-manifold (X, ρ) endowed with a given $Spin^G(3)$-structure (P^G, γ).

We shall define now some fibre bundles associated to (P^G, γ).

Definition 1.9 *$ad(P^G) := P^G \times_{ad_S} su(2)$,*
$\mathcal{L}(P^G) := P^G \times_{ad_G} L$,
$\Sigma(P^G) := P^G \times_\lambda (\mathbf{C}^2 \otimes_{\mathbf{C}} V)$,
$\delta(P^G) := P^G \times_\delta (G/\mathbf{Z}_2)$.

Definition 1.10 *The vector bundle $\Sigma(P^G)$ is called the* ***spinor bundle*** *and its associated space of sections, $A^0(\Sigma(P^G))$, the* ***space of spinors***. *The elements of $A^0(\Sigma(P^G))$ are called* ***spinors***.

Convention: By composing with the respective isomorphims between the Lie algebras we shall consider
1) the space of connections in $\delta(P^G)$ as an affine space over $A^1(\mathcal{L}(P^G))$,
2) the connections in P^G defined by $su(2) \oplus L$ valued 1-forms.

Proposition 1.11 *The real vector bundles $ad(P^G)$ and $\mathcal{L}(P^G)$ are isomorphically included in $End(\Sigma(P^G))$ as real subbundles via the morphisms:*
$\alpha : ad(P^G) \longrightarrow End(\Sigma(P^G))$, $\alpha([p, u]) = [p, u \otimes Id_V]$ for any $p \in P^G$ and $u \in su(2)$ and
$\beta : \mathcal{L}(P^G) \longrightarrow End(\Sigma(P^G))$, $\beta([p, g]) = [p, Id_2 \otimes g]$ for any $p \in P^G$ and $g \in L$, respectively. Considering a convenient scalar product for complex 2×2-matrices, β is an isometry onto its image and by rescaling the scalar product in $End(V)$ such that Id_V has norm 1, α becomes an isometry onto its image, too.

Proof Direct easy computations on the fibres. •

2 The Dirac Operators Associated to a $Spin^G(3)$ -Structure

Let (X, ρ) be the given Riemannian 3-manifold and (P^G, γ) its fixed $Spin^G(3)$-structure.

Convention: Using remark 1.7, we shall consider from now on

$$\gamma : \Lambda^1_X \longrightarrow ad(P^G)$$

as well as

$$_{SO(3)}\mathbf{R}^3 \longrightarrow ad(P^G).$$

Lemma 2.1 *The Clifford map γ defines a structure of $A^1(X)$-Clifford module in $A^0(\Sigma(P^G))$. Furthermore, $\Sigma(P^G)$ is a complex vector bundle of Clifford-modules over the corresponding fibres of Λ^1_X .*

Proof According to proposition 1.11 and to the propositions 1.1 and 1.3 from [LM] we only have to check the Clifford identity for γ, i.e. that

$$\gamma(\omega)\gamma(\theta) + \gamma(\theta)\gamma(\omega) = -2\rho(\omega,\theta)Id_{\Sigma(P^G)} \ \ \forall\omega,\theta \in \Lambda^1_X.$$

The checking can be done fibrewise. With convenient choices of trivialisations we may assume that $\omega, \theta \in \{f_1, f_2, f_3\}$ where (f_1, f_2, f_3) is an oriented orthonormal frame of $\Lambda^1(\mathbf{R}^3)$ which is transformed via γ into the oriented orthonormal frame (τ_1, τ_2, τ_3) in $su(2)$, where

$$\tau_0 := Id_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tau_1 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \tau_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tau_3 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

For this choice of ω and θ the Clifford identity is a direct consequence of the relations

$$\tau_1^2 = \tau_2^2 = \tau_3^2 = -\tau_0,$$
$$\tau_1\tau_2 = -\tau_2\tau_1 = \tau_3,$$
$$\tau_2\tau_3 = -\tau_3\tau_2 = \tau_1,$$
$$\tau_3\tau_1 = -\tau_1\tau_3 = \tau_2.$$

and it extends in general by linearity.

Notations:

1) We shall also denote by γ the morphism $A^1(\Sigma(P^G)) \longrightarrow A^0(\Sigma(P^G))$ naturally induced by the Clifford map, $\gamma(\omega \otimes \psi) = \alpha(\gamma(\omega))(\psi)$ for any $\omega \in A^1(X)$ and $\psi \in A^0(\Sigma(P^G))$ and extented to $A^1(\Sigma(P^G))$ by linearity where α is the morphism given in proposition 1.11.

2) We shall denote by Γ the morphism $\Lambda^2_X \longrightarrow End(\Sigma(P^G))$ induced by the Clifford map γ and given fibrewise by $\Gamma(\omega \wedge \theta) := \frac{1}{2}[\alpha(\gamma(\omega))\alpha(\gamma(\theta)) - \alpha(\gamma(\theta))\alpha(\gamma(\omega))]$ for any $\omega, \theta \in \Lambda^1_X$ and then extended to Λ^2_X by linearity.

3) The same notation Γ shall be used for the multiplication with 2-forms $A^2(X) \longrightarrow End(A^0(\Sigma(P^G)))$ induced by γ, namely $[\Gamma(\omega(\varphi))](x) := \Gamma(\omega_x(\varphi_x))$ for any $\omega \in A^2(X)$, $\varphi \in A^0(\Sigma(P^G))$ and $x \in X$ and then extended to $A^2(X)$ by linearity.

4) The same notations shall be used for the canonical extensions to $\mathbf{C}$-valued forms of γ and Γ, respectively: $\gamma(i\theta) := i\gamma(\theta)$ for any 1-form θ and $\Gamma(i\omega) := i\Gamma(\omega)$ for any 2-form ω.

Remark 2.2 *Γ extends naturally to a morphism (which shall be denoted by Γ as well)*

$$\Gamma : A^2(\mathcal{L}(P^G)) \longrightarrow A^0(ad(P^G) \otimes \mathcal{L}(P^G)),$$

defined by $\Gamma(\omega \otimes f) = \Gamma(\omega) \otimes f$ and then extended by linearity.

Remark 2.3 *The multiplication with forms induced by γ acts locally (via trivialisations) as identity on the V-component of a section in $\Sigma(P^G)$.*

Proof Direct consequence of proposition 1.11. •

Definition 2.4 *Let P be the $SO(3)$-principal bundle of the oriented and ρ-orthonormal coframes of (X,ρ). Let $A \in \mathcal{A}(\delta(P^G))$ be a unitary connection in $\delta(P^G)$ and $\hat{A}$ the unitary connection induced in $\Sigma(P^G)$ by A and the Levi-Civita connection in P. The* ***Dirac operator*** *defined by A is*

$$\not{D}_A := \gamma \circ \nabla_{\hat{A}} : A^0(\Sigma(P^G)) \longrightarrow A^0(\Sigma(P^G))$$

where $\gamma : A^1(\Sigma(P^G)) \longrightarrow A^0(\Sigma(P^G))$ and $\nabla_{\hat{A}} : A^0(\Sigma(P^G)) \longrightarrow A^1(\Sigma(P^G))$.

Definition 2.5 *Let $A \in \mathcal{A}(\delta(P^G))$. A spinor $\psi \in A^0(\Sigma(P^G))$ is called* ***$\not{D}_A$-harmonic*** *iff $\not{D}_A(\psi) = 0$.*

Now we shall give some properties of the Dirac operators.

Remark 2.6 *For any $A \in \mathcal{A}(\delta(P^G))$, $\not{D}_A$ has symbol $\sigma(\not{D}_A) = \gamma$.*
Consequently, the symbol of $\not{D}_A$ does not depend on A.

Proof Direct easy computations using the general definition of symbols. •

Lemma 2.7 *For any $A \in \mathcal{A}(\delta(P^G))$ the associated Dirac operator $\not{D}_A$ is an elliptic self-adjoint differential operator of order 1.*

Proof See [LM, Lemma II.5.1 and Example III.1.5.] for ellipticity, [LM, Proposition II.5.3.] for self-adjointness.
In order to use the proofs given in [LM] it is necessary to add the following remarks:
for " Ellipticity" : The symbol of $\not{D}_A$ is γ and for any $\xi \in \Lambda^1_X$ $det(\gamma(\xi)) = ||\xi||^2$. It follows that $\sigma_\xi(\not{D}_A)$ is an isomorphism for any $\xi \in \Lambda^1_X \setminus \{0\}$.
for "Self-adjointness" : In the proof of II.5.3., when related to spinors we have to use $\nabla_{\hat{A}}$ instead of ∇, which is the Levi-Civita connection in Λ^1_X. Then we use that $\nabla_{\hat{A}}$ is a unitary connection in $\Sigma(P^G)$:
$d < \varphi, \psi > = < \nabla_{\hat{A}}\varphi, \psi > + < \varphi, \nabla_{\hat{A}}\psi >$ for any $\varphi, \psi \in \Sigma(P^G)$.
for "Order of $\not{D}_A$": Since γ is an isomorphism (has order 0) and $\nabla_{\hat{A}}$ has order 1 it follows that $\not{D}_A := \gamma \circ \nabla_{\hat{A}}$ has order 1. •

3 The Weitzenböck Formula for a $Spin^G(3)$ -Structure

Let (X,ρ) be the given Riemannian 3-manifold and (P^G,γ) its fixed $Spin^G(3)$-structure.
The relation we shall give in the following theorem is to be considered subject to the conventions stated after definition 1.10.

Theorem 3.1 *Let P be the $SO(3)$-principal bundle of the oriented and ρ-orthonormal coframes of (X,ρ) and s the scalar curvature of (X,ρ). Let $A \in \mathcal{A}(\delta(P^G))$, F_A its curvature, $\nabla_{\hat{A}}$ the connection induced in $\Sigma(P^G)$ by A and the Levi-Civita connection in P. Let $\nabla^*_{\hat{A}}$ be the adjoint operator of $\nabla_{\hat{A}}$ with respect to the L_2-scalar product. Then the following relation holds:*

$$\not{D}_A^2 = \nabla^*_{\hat{A}}\nabla_{\hat{A}} + \frac{1}{4}s + \Gamma(F_A).$$

Proof The links between the Lie groups we use can be illustrated in the following diagram, the closed polygons being commutative:

$$\begin{array}{ccc} & & SU(2) \\ & \overset{p_1}{\nearrow} & \\ SU(2)\times G & \xrightarrow{p_2} & G \\ \downarrow q & \searrow \nu & \downarrow \mu \\ Spin^G(3) & \xrightarrow{\delta} & G/\mathbb{Z}_2\,, \end{array}$$

where each morphism is the respective canonical projection.

Let $r : SU(2)\times G \longrightarrow U(\mathbf{C}^2 \otimes_{\mathbf{C}} V)$ be the canonical representation of $SU(2)\times G$ in $\mathbf{C}^2 \otimes_{\mathbf{C}} V$. Then the following diagram is commutative:

$$\begin{array}{ccc} SU(2)\times G & & \\ \downarrow q & \searrow r & \\ Spin^G(3) & \xrightarrow{\lambda} & U(\mathbf{C}^2 \otimes_{\mathbf{C}} V). \end{array}$$

The Weitzenböck formula can be proven locally since all the operators involved are local. Thus we can suppose that all the bundles we consider here are trivial and that P^G admits a lift $\tilde{P} \xrightarrow{f} P^G$ of type q. We need to introduce some Notations: $P_1 := \tilde{P} \times_{p_1} SU(2)$, $P_2 := \tilde{P} \times_{p_2} G$, $S := \tilde{P} \times_{p_1} \mathbf{C}^2$, $E := \tilde{P} \times_{p_2} V$.

Using the above two diagrams one can check very easily that:
1) $S = P_1 \times_{p_1} \mathbf{C}^2$ and is a $Spin(3)$-spinorial vector bundle,
2) $E = P_2 \times_{p_2} V$ and is an unitary complex vector bundle,
3) P_2 is a lift $P_2 \xrightarrow{g} \delta(P^G)$ of type μ,
4) $\Sigma(P^G) = S \otimes E$.

Let ω be the connection form in $\delta(P^G)$ associated to A and ω_G the connection form in P^G associated to ω and the Levi-Civita connection in P. Furthermore, let $\tilde{\omega}$ be the connection form induced in P_2 by ω and $\tilde{\omega_G}$ the connection form induced in $\tilde{P}$ by

ω_G. The two following diagrams:

$$\begin{array}{ccc} \mathcal{T}(P_2) & \xrightarrow{dg} & \mathcal{T}(\delta(P^G)) \\ \downarrow \tilde{\omega} & & \downarrow \omega \\ L & \xrightarrow{\mu_*} & Lie(G/\mathbf{Z}_2) \end{array}$$

and

$$\begin{array}{ccc} \mathcal{T}(\tilde{P}) & \xrightarrow{df} & \mathcal{T}(P^G) \\ \downarrow \tilde{\omega_G} & & \downarrow \omega_G \\ su(2) \oplus L & \xrightarrow{q_*} & Lie(Spin^G(3)) \end{array}$$

are exact.

Moreover, the Lie algebra morphisms μ_* and q_* are isomorphisms. With the convention given after definition 1.10 we shall consider these diagrams as: $\tilde{\omega} = \omega \circ dg$ and $\tilde{\omega_G} = \omega_G \circ df$, respectively. It follows that the curvature form of the connection induced by $\tilde{\omega}$ in E is equal to F_A and that the connection form induced by $\tilde{\omega_G}$ in $S \otimes E$ is equal to the connection form of $\nabla_{\hat{A}}$. The claim follows now from [LM, Theorem 2.8.17.] applied to $S \otimes E$ with the connection induced in E by $\tilde{\omega}$. •

Corollary 3.2 *Under the conditions of theorem 3.1, if $s = 0$ and if A is flat then every $\not{D}_A$-harmonic spinor is globally parallel with respect to $\nabla_{\hat{A}}$.*

Proof Let $\psi \in A^0(\Sigma(P^G))$ be a $\not{D}_A$-harmonic spinor, namely $\not{D}_A(\psi) = 0$. From theorem 3.1 it follows that $\nabla^*_{\hat{A}}\nabla_{\hat{A}}(\psi) = 0$. We now take the L_2-scalar product with ψ and get $< \nabla^*_{\hat{A}}\nabla_{\hat{A}}(\psi), \psi >_{L^2} = 0$ which implies $||\nabla_{\hat{A}}(\psi)||_{L^2} = 0$. It follows that $\nabla_{\hat{A}}(\psi) = 0$, i.e. that ψ is $\nabla_{\hat{A}}$-parallel. •

Bibliography

[DK] Donaldson, S.; Kronheimer, P.B.: *The Geometry of Four-Manifolds*, Oxford Science Publications, 1990.

[FU] Freed D. S.; Uhlenbeck, K.: *Instantons and Four-Manifolds.* Springer-Verlag, 1984.

[K] Kobayashi, S.: *Differential Geometry of Complex Vector Bundles*, Princeton University Press, 1987.

[KN] Kobayashi, S.; Nomizu, K.: *Foundations of Differential Geometry*, Volume I, Interscience Publishers, New-York-London, 1963.

[LM] Lawson, H. B. Jr.; Michelson, M. L.: *Spin Geometry*, Princeton University Press, New Jersey, 1989.

[LT] Lübke, M.; Teleman, Andrei: *The Kobayashi-Hitchin Correspondence*, World Scientific Publishing Co. 1995.

[OT1] Okonek, Ch.; Teleman, Andrei: *The Coupled Seiberg-Witten Equations, Vortices and Moduli Spaces of Stable Pairs*, Int. J. Math. Vol. 6, No. 6, 893-910 (1995).

[OT2] Okonek, Ch.; Teleman, Andrei: *Seiberg-Witten Invariants and Rationality of Complex Surfaces*, Math. Z., (1996).

[OT3] Okonek, Ch.; Teleman, Andrei: *Quaternionic Monopoles*, Comptes Rendus Acad. Sci. Paris, t. 321, Série I, 601-606 (1995).

[OT4] Ch, Okonek.; Teleman, Andrei: *Quaternionic Monopoles*, Commun. Math. Phys., Vol 180, Nr. 2, 363-388, (1996).

[T1] Teleman, Andrei:*Non-Abelian Seiberg-Witten Theory*, Habilitationsschrift, Universität Zürich, 1996.

[T2] Teleman, Andrei:*Non-Abelian Seiberg-Witten Theory and Projectively Stable Pairs*, Preprint, Universität Zürich, 1996.

[T3] Teleman, Andrei:*Non-Abelian Seiberg-Witten Theory and Stable Oriented Pairs*, Int. J. Math., Vol. 8, No. 4 (1997).

[T4] Teleman, Andrei: *Seiberg- Witten Theorie und Komplexe Geometrie*, Vorlesung, Philosophsche Fakultät II , Universität Zürich, 1996-1997.

[T] Teleman, Kostake: *Teorie Seiberg-Witten*, Curs-Anul 4, Universitatea Bucuresti, 1994-1995, 1995-1996 (Romanian).

[We] Wells, R.O.: *Differential Analysis on Complex Manifolds*, Springer-Verlag, (1973).

[W] Witten, E.: *Monopoles and Four-Manifolds*, Math. Res. Letters 1, 769-796 (1994).

Manuscript received at March 15, 2001.

Author's address :

Ana-Maria Teleman, Universitatea Bucureşti, Facultatea de Matematică, Catedra de Geometrie, Str. Academiei 14.