

ISOTOPIC AND GENOTOPIC LIE-ALGEBRAS AND THE NEW DEFORMED HEISENBERG QUANTUM MECHANICS

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Abstract

In this paper we study the general Isobosonic Algebras, the general Lie-admissible algebra with real elements T and R , $T \neq R$. Hermitian operators, and the general Lie-admissible algebra with complex elements T and R , for the case where the operators T and R are (well behaved) functions $(n+1)$, where n is the conventional occupation number. By means of these concepts we formulate the new deformed Heisenberg quantum mechanics.

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1 Introduction

According to Santilli [1], the greatest number of contributions on Bosonic algebras of Lie-isotopic and Lie-admissible type has been done by Jannussis and his associates [2]. Their axiomatic formulation has been reached in the recent draft [3]. The emerging formulations are here called Jannussis' isotopic and genotopic algebras.

The only deformed bosonic algebra known at this time which possess the above axiomatic structure are given by the Lie-isotopic algebra (Jannussis' isobosonic algebra)

$$\hat{a}T(q, \dots)\hat{a}^+ - \hat{a}^+T(q, \dots)\hat{a} = (T(q\dots))^{-1} = \hat{I}, T = T^+ \quad (1.1)$$

and by the more general Lie-admissible algebras (Jannussis genobosonic algebras) for motion forward to future time:

$$\hat{a}^<T(q, \dots)\hat{a}^+ - \hat{a}^+T^>(q, \dots)\hat{a} = [T^>(q\dots)]^{-1} = \hat{I}^>, \quad (1.2)$$

or motion forward from past time:

$$\hat{a}^{\langle T(q, \dots) \rangle} \hat{a}^+ - \hat{a}^+ T^{\rangle}(q, \dots) \hat{a} == [T^{\rangle}(q, \dots)]^{-1} =^{\langle} \hat{I}, \quad (1.3)$$

under the condition

$$\langle T(q, \dots) = [T^{\rangle}(q, \dots)]^+, \quad (1.4)$$

which is necessary for the preservation of Hermiticity-observability, of a Lie-admissible group structure with consequential preservation of causality, etc., where T , $\langle T$ and T^{\rangle} are in general operators verifying the needed conditions of regularity, boundedness, etc., but otherwise possess an unrestricted functional dependence on the parameter q as well as any other quantity.

From few years ago Lopez [4] has studied the Santilli's axiomatization of q -deformations via isotopies and genotopies of classical and quantum mechanics [5]. In general, the genotopic deformations [6] include a great variety of deformations of fundamental commutation rules

$$xTp - pRx = i\hbar\Omega(x, p, t), \quad (1.5)$$

where x and p are position and momentum operators, $\Omega(x, p, t)$ is the operational form of the Lie-admissible tensor and T , R a re suitable operators (it is supposed to represent, in general, non-linear, non-local-integral and non-canonical, non-hamiltonian interactions). The case $T = R$ and $HT \neq TH$ corresponds to the so called Lie-isotopic case, where H is the usual Hamilton operator.

The time evolution of the operators is ruled by Santilli's [1] generalization of Heisenberg's equation of motion, introduced in 1978 as:

$$i\hbar \frac{\partial A}{\partial t} = (A, H) = ATH - HRA. \quad (1.6)$$

For T , R independent of time we have:

$$A(t) = e^{\frac{iH R t}{\hbar}} A(0) e^{-\frac{iT H t}{\hbar}}, \quad (1.7)$$

which implies a non-unitary time evolution.

Some time ago Caldi [7], Chodos and Caldi [8] and Ubriaco [9] have discussed two formulations of q -quantum mechanics, based on quantum deformations of the Heisenberg equation of motion.

According to Jannussis [10, 11] the above formulations are particular cases of Santilli's Hadronic mechanics [1].

An independent development of the q -quantum mechanics has been presented many years ago by Jannussis [12], based on the q -algebra of Kuryskhin [13] and on its Fock representation. Also Jannussis [14] in a recent paper has studied the genotopic and isotopic structure of q -quantum groups. Especially he has introduced a generalized Lie-deformed Heisenberg algebra [15] for $T = 1 + i\lambda$, $R = T^* = 1 - i\lambda$ and $\Omega = 1$. Recently Hui-yun Pan and Zu Sen Zhao [16] have shown that the non-Hermitian realization of the above algebra is closely related with the q -Heisenberg-Weyl algebra of Biedecharn [17] and Macfarlane [18] with q being a phase ($q = e^{i\theta}$

with θ real). The above algebra according to Jannussis formed a new Lie-deformed Heisenberg quantum mechanics (non-canonical quantum mechanics) and construct the new modified Lie-admissible statistics for open quantum systems.

In the present paper we will study the general isobosonic and genobosonic algebras, when the operators T and R are (well behaved) functions of $n + 1$, where n is the conventional occupation number. In Section 2 we study the general Isobosonic algebra. In Section 3 we study the general Lie-admissible algebra with real elements T and R and, as example, we present the (μ, v) mutation algebra. In Section 4 we study the general Lie-admissible algebra with complex elements T and R . In Section 5 we formulate the genobosonic algebra with the elements T, T^+ and, as example, we give the Lie-deformed Heisenberg algebra. In Section 6 we formulate the Lie-deformed Heisenberg quantum mechanics. Section 7 is devoted to concluding remarks.

2 General Isobosonic Algebra

According to Santilli [1, 3] we consider now the general isotopic case, in which we do expect a deformation of the algebra as well as of the eigenvalues.

For this purpose, we still assume as the fundamental isounit of the theory, the quantity $\hat{I} = T^{-1}$ with related isofields, isohilbert space, isobasis $|N\rangle = T^{-\frac{1}{2}} |n\rangle$, etc., but we assume also that $a \neq \hat{a}T$ holds in general. This case requires the solution of general isotopic structure

$$\hat{a}T\hat{a}^+ - \hat{a}^+T\hat{a} = \hat{I} = \frac{1}{T}. \quad (2.1)$$

We consider now the case where T is a (well behaved) function of $(n + 1)$.

$$aT(n+1)\hat{a}^+ - \hat{a}^+T(n+1)\hat{a} = \frac{1}{T(n+1)} = \hat{I}(n+1) = \hat{I}^+(n+1), \quad (2.2)$$

where n is the conventional occupation number.

The use of the bosonization method [1,2] implies the following expression for the unknown real function $f(n+1)$:

$$(n+1)T(n+2)f^2(n+1) - nT(n)f^2(n) = \frac{1}{T(n+1)}. \quad (2.3)$$

The solution of the above equation, after some algebras, takes the form:

$$f(n+1) = \frac{1}{\sqrt{T(n+2)T(n+1)}} \quad (2.4)$$

and the desired general solution for the isobosonic creation and annihilation operators is the following:

$$\hat{a}(n+1) = \frac{1}{\sqrt{T(n+1)T(n+2)}}a, \hat{a}^+(n+1) = a^+ \frac{1}{\sqrt{T(n+1)T(n+2)}}, \quad (2.5)$$

with isoeigenvalues on an isonormalized isobasis

$$|N\rangle = \frac{1}{\sqrt{T(n+1)}} |n\rangle, \langle n|n\rangle = 1, \langle N|T(n+1)|N\rangle = 1 \quad (2.6)$$

From (2.5) and (2.6) we obtain:

$$\begin{aligned} \hat{a} * |N\rangle &= \hat{a}T(n+1)|N\rangle = \hat{a}\sqrt{T(n+1)}|n\rangle = \\ &= \frac{1}{\sqrt{T(n+1)T(n+2)}} a \sqrt{T(n+1)}|n\rangle = \frac{1}{\sqrt{T(n+1)}} a |n\rangle = \\ &= \sqrt{n} \frac{1}{\sqrt{T(n)}} |n-1\rangle = \sqrt{n}|N-1\rangle. \end{aligned} \quad (2.7)$$

In the same way we obtain:

$$\hat{a}^+ * |N\rangle = \hat{a}^+T(n+1)|N\rangle = \sqrt{n+1}|N+1\rangle \quad (2.8)$$

$$\hat{a}^+ * \hat{a} * |N\rangle = n|N\rangle \quad (2.9)$$

$$\hat{a} * \hat{a}^+ * |N\rangle = (n+1)|N\rangle \quad (2.10)$$

and the Hamilton operator H has the form:

$$H^> = \hbar\omega(n + \frac{1}{2})\hat{I}, \quad (2.11)$$

$$H^> * |N\rangle = H^>T(n+1)|N\rangle = \hbar\omega(n + \frac{1}{2})|N\rangle. \quad (2.12)$$

From (2.10) and (2.12) we obtain:

$$(\hat{a} * \hat{a}^+ - \hat{a}^+ * \hat{a}) * |N\rangle = |N\rangle. \quad (2.13)$$

The above results were expected according to the Lie-isotopic axiomatization.

The same we expect for the original commutation relation (2.2), so the following commutation relation are also valid

$$(\hat{N}, \hat{a}) = -\hat{a}, \quad (\hat{N}, \hat{a}^+) = \hat{a}^+, \quad (2.14)$$

where: $\hat{N} = \hat{a}^+T(n+1)\hat{a}$ is a isotopic number operator and $(\hat{N}, \hat{a}) = \hat{N}T\hat{a} - \hat{a}T\hat{N}$.

The commutation relations (2.14) are necessary for the quantum groups.

3 General Lie-Admissible Algebra with Real Elements T and R

The formulation of the Lie-admissible algebra, according to Santilli [1], satisfies the commutation relations:

$$A * A^+ - A^+ * A = ATA^+ - A^+RA = \frac{1}{T} = I^>, \quad (3.1)$$

$$B * B^+ - B^+ * B = BTB^+ - B^+RB = \frac{1}{R} =^< I, \quad (3.2)$$

where the elements T and R characterize the motion forward and backward.

In the following, we study the case when the elements T and R are functions of the conventional occupation number n ,

$$n|n \rangle = n|n \rangle. \quad (3.3)$$

By using the Bosonization method [19], we obtain the following expressions for the operators $A \cdot A^+$ and $B \cdot B^+$:

$$A = f(n+1)a, \quad A^+ = a^+f(n+1), \quad (3.4)$$

$$B = g(n+1)a, \quad B^+ = a^+g(n+1), \quad (3.5)$$

where the real structure functions $f(n+1)$, $g(n+1)$ satisfies the equations:

$$(n+1)T(n+2)f^2(n+1) - nR(n)f^2(n) = \frac{1}{T(n+1)} = 1^>, \quad (3.6)$$

$$(n+1)T(n+2)g^2(n+1) - nR(n)g^2(n) = \frac{1}{R(n+1)} =^< 1. \quad (3.7)$$

The solutions of the above equations, according to [20], have the following forms:

$$f(n+1) = \frac{1}{\sqrt{(n+1)T(n+1)T(n+2)}} \left(1 + \frac{R(n)}{T(n)} + \frac{R(n)R(n-1)}{T(n)T(n-1)} + \dots \right. \\ \left. + \frac{R(n)R(n-1) \dots R(1)}{T(n)T(n-1) \dots T(1)} \right)^{1/2}, \quad (3.8)$$

$$g(n+1) = \frac{1}{\sqrt{(n+1)R(n+1)T(n+2)}} \left(1 + \frac{R(n+1)}{T(n+1)} + \frac{R(n+1)R(n)}{T(n+1)T(n)} + \dots \right. \\ \left. + \frac{R(n+1)R(n) \dots R(2)}{T(n+1)T(n) \dots T(2)} \right)^{1/2}. \quad (3.9)$$

From the above results we see that for the Lie-admissible algebra with real elements T and R , we have two different pairs of creation and annihilation operators for forward and backward motion. It is very simple now to construct the corresponding number operators $N^>$, $^<N$, the Hamilton operators $H^>$, $^<H$ and other physical entities in the Lie-admissible formulation, i.e.:

$$N^> = A^+ * A = A^+R(n+1)A, \quad ^<N = B^+ * B = B^+R(n+1)B, \quad (3.10)$$

$$H^> = \frac{\hbar\omega}{2} (A * A^+ + A^+A) = \frac{\hbar\omega}{2} (AT(n+1)A^+ + A^+R(n+1)A), \quad (3.11)$$

$$^<H = \frac{\hbar\omega}{2} (B * B^+ + B^+B) = \frac{\hbar\omega}{2} (BT(n+1)B^+ + B^+R(n+1)B), \quad (3.12)$$

and because of the commutation relations (2.1) and (2.2) we obtain:

$$H^> = \hbar\omega \left(N^> + \frac{1}{2} I^> \right), \quad <H = \hbar\omega \left(<N + \frac{1}{2} I^< \right). \quad (3.13)$$

From (3.8) and (3.9) we have:

$$N^> = A^+ R(n+1) A = \left\{ \frac{R(n)}{T(n)} + \frac{R(n)R(n-1)}{T(n)T(n-1)} + \cdots + \frac{R(n)R(n-1)\cdots R(1)}{T(n)T(n-1)\cdots T(1)} \right\} I^> \quad (3.14)$$

$$<N = B^+ R(n+1) B = <I \left\{ \frac{R(n+1)}{T(n+1)} + \frac{R(n+1)R(n)}{T(n+1)T(n)} + \cdots + \frac{R(n+1)R(n)\cdots R(2)}{T(n+1)T(n)\cdots T(2)} \right\} \quad (3.15)$$

$$H^> = \hbar\omega \left(\frac{R(n)}{T(n)} + \frac{R(n)R(n-1)}{T(n)T(n-1)} + \cdots + \frac{R(n)R(n-1)\cdots R(1)}{T(n)T(n-1)\cdots T(1)} + \frac{1}{2} \right) I^> \quad (3.16)$$

$$<H = \hbar\omega <I \left(\frac{R(n+1)}{T(n+1)} + \frac{R(n+1)R(n)}{T(n+1)T(n)} + \cdots + \frac{R(n+1)R(n)\cdots R(2)}{T(n+1)T(n)\cdots T(2)} + \frac{1}{2} \right). \quad (3.17)$$

In the following we can construct the Lie-admissible common normalized basis:

$$|N\rangle = \frac{1}{4\sqrt{TR}} |n\rangle = \frac{1}{4\sqrt{T(n+1)R(n+1)}} |n\rangle, \quad (3.18)$$

$$\langle n | n\rangle = 1, \quad \langle N | \sqrt{TR} |N\rangle = 1$$

and we obtain:

$$N^> * |N\rangle = \left(\frac{R(n)}{T(n)} + \cdots + \frac{R(n)\cdots R(1)}{T(n)\cdots T(1)} \right) |N\rangle, \quad (3.19)$$

$$\langle N | * \langle N = \langle N | \left(\frac{R(n+1)}{T(n+1)} + \cdots + \frac{R(n+1)\cdots R(2)}{T(n+1)\cdots T(2)} \right), \quad (3.20)$$

$$\begin{aligned} A * |N\rangle &= AT(n+1) |N\rangle = \\ &= \sqrt[4]{\frac{R(n)}{R(n+1)} \frac{T(n+1)}{T(n)}} \left\{ 1 + \frac{R(n-1)}{T(n-1)} + \cdots + \frac{R(n-1)\cdots R(1)}{T(n-1)\cdots T(1)} \right\}^{1/2} |N-1\rangle, \end{aligned} \quad (3.21)$$

$$\begin{aligned} A^+ * |N\rangle &= A^+ R(n+1) |N\rangle = \\ &= \sqrt[4]{\frac{R(n+2)T(n+1)}{R(n+1)T(n+2)}} \left\{ \frac{R(n+1)}{T(n+1)} + \cdots + \frac{R(n+1)\cdots R(1)}{T(n+1)\cdots T(1)} \right\}^{1/2} |N+1\rangle, \end{aligned} \quad (3.22)$$

$$\begin{aligned} B * |N\rangle &= BT(n+1) |N\rangle = \\ &= \sqrt[4]{\frac{T(n)T(n+1)}{R(n)R(n+1)}} \left\{ 1 + \frac{R(n)}{T(n)} + \cdots + \frac{R(n)\cdots R(2)}{T(n)\cdots T(2)} \right\}^{1/2} |N-1\rangle, \end{aligned} \quad (3.23)$$

$$B^+ * |N\rangle = B^+ R(n+1) |N\rangle = \sqrt[4]{\frac{R(n+1)R(n+2)}{T(n+1)T(n+2)}} \cdot \left\{ 1 + \frac{R(n+1)}{T(n+1)} + \cdots + \frac{R(n+1) \cdots R(2)}{T(n+1) \cdots T(2)} \right\}^{1/2} |N+1\rangle, \quad (3.24)$$

$$A * A^+ * |N\rangle = AT(n+1)A^+ * |N\rangle, \quad A^+ * A * |N\rangle = A^+ R(n+1)A * |N\rangle, \quad (3.25)$$

$$(A * A^+ - A^+ * A) * |N\rangle = I^> * |N\rangle = |N\rangle, \quad (3.26)$$

$$(B * B^+ - B^+ * B) * |N\rangle = {}^< I * |N\rangle = |N\rangle, \quad (3.27)$$

$$\begin{aligned} H^> * |N\rangle &= \hbar\omega \left(\frac{R(n)}{T(n)} + \cdots + \frac{R(n) \cdots R(1)}{T(n) \cdots T(1)} + \frac{1}{2} \right) I^> * |N\rangle = \\ &= \hbar\omega \left(\frac{R(n)}{T(n)} + \cdots + \frac{R(n) \cdots R(1)}{T(n) \cdots T(1)} + \frac{1}{2} \right) |N\rangle, \end{aligned} \quad (3.28)$$

$$\begin{aligned} < N | * {}^< H &= \\ &= \hbar\omega < N | {}^< I R(n+1) \left(\frac{R(n+1)}{T(n+1)} + \cdots + \frac{R(n+1) \cdots R(2)}{T(n+1) \cdots T(2)} + \frac{1}{2} \right) = \\ &= \hbar\omega < N | \left(\frac{R(n+1)}{T(n+1)} + \cdots + \frac{R(n+1) \cdots R(2)}{T(n+1) \cdots T(2)} + \frac{1}{2} \right). \end{aligned} \quad (3.29)$$

Finally, from the relations (3.10), we obtain the following commutation relations:

$$(A, N^>) = I^> RA, \quad (N^>, A^+) = A^+ RI^>, \quad (3.30)$$

$$(B, {}^< N) = B, \quad ({}^< N, B^+) = B^+ \quad (3.31)$$

and for $T = R$ we obtain exactly the formulae (2.14).

Example α . For $T = \mu$, $R = v$ we obtain:

$$A = \frac{1}{\mu} \sqrt{\frac{[n+1]}{n+1}} a, \quad A^+ = \frac{1}{\mu} a^+ \sqrt{\frac{[n+1]}{n+1}}, \quad (\alpha(1))$$

$$B = \frac{1}{\sqrt{v\mu}} \sqrt{\frac{[n+1]}{n+1}} a, \quad B^+ = \frac{1}{\sqrt{v\mu}} a^+ \sqrt{\frac{[n+1]}{n+1}}, \quad (\alpha(2))$$

where $Q = \frac{v}{\mu}$ and $[x] = \frac{Q^x - 1}{Q - 1}$

$$\begin{aligned} N^> &= A^+ * A = A^+ v A = Q \frac{Q^n - 1}{Q - 1} I^>, \quad {}^< N = B^+ * B = \\ &= B^+ * v A = a {}^< I \frac{Q^n - 1}{Q - 1}. \end{aligned} \quad (\alpha(3))$$

Common basis:

$$|N\rangle = \frac{1}{\sqrt{\mu\nu}} |n\rangle, \quad \langle n | n \rangle = 1, \quad \langle N | \sqrt{\mu\nu} |N\rangle = 1 \quad (\alpha(4))$$

$$H^> = \frac{\hbar\omega}{2}(A * A^+ + A^+ * A) = \frac{\hbar\omega}{2}([n+1] + Q[n])I^> \quad (\alpha(5))$$

$$\langle H = \frac{\hbar\omega}{2}(B * B^+ + B^+ * B) = \frac{\hbar\omega}{2} I([n+1] + Q[n]) \quad (\alpha(6))$$

$$A * |N\rangle = A\mu |N\rangle = \sqrt{[n]} |N-1\rangle \quad (\alpha(7))$$

$$A^+ * |N\rangle = A^+v |N\rangle = Q\sqrt{[n+1]} |N+1\rangle \quad (\alpha(8))$$

$$B * |N\rangle = B\mu |N\rangle = Q^{-1/2}\sqrt{[n]} |N-1\rangle \quad (\alpha(9))$$

$$B^+ * |N\rangle = B^+v |N\rangle = Q^{1/2}\sqrt{[n+1]} |N+1\rangle \quad (\alpha(10))$$

$$N^> * |N\rangle = [n] |N\rangle \quad (\alpha(11))$$

$$\langle N | * \langle N = \langle N | Q[n] \quad (\alpha(12))$$

and

$$H^> * |N\rangle = \hbar\omega([n+1] - \frac{1}{2}) |N\rangle, \quad \langle N | * \langle H = \hbar\omega \langle N | ([n+1] - \frac{1}{2}). \quad (\alpha(13))$$

4 General Lie-admissible Algebra with Complex elements T and R

In the following we consider the general case where the elements $T(n+1)$ and $R(n+1)$ are complex. For simplicity we take the two pairs of operators A, B and A', B' and according to Lie-admissible algebra we have the following commutation relations:

$$A * B - B * A = AT(n+1)B - BR(n+1)A = \frac{1}{T(n+1)} = I_1^>, \quad (4.1)$$

$$B^+ * A^+ - A^+ * B^+ = B^+T^+(n+1)A^+ - A^+R^+(n+1)B^+ = \frac{1}{T^+(n+1)} = {}^< I_1, \quad (4.2)$$

$$\dot{A} * \dot{B} - \dot{B} * \dot{A} = \dot{A}T(n+1)\dot{B} - \dot{B}R(n+1)\dot{A} = \frac{1}{R(n+1)} = I_2^>, \quad (4.3)$$

$$\dot{B}^+ * \dot{A}^+ - \dot{A}^+ * \dot{B}^+ = \dot{B}^+T^+(n+1)\dot{A}^+ - \dot{A}^+R^+(n+1)\dot{B}^+ = \frac{1}{R^+(n+1)} = {}^< I_2, \quad (4.4)$$

where $(I_1^<)^+ = {}^< I_1$ and $(I_2^>)^+ = {}^< I_2$ are corresponding units.

According to [20] and by using the well known bosonization method [19], we can put:

$$A = f(n+1)a, \quad B = a^+f(n+1), \quad (4.5)$$

$$\dot{A} = g(n+1)a, \dot{B} = a^+g(n+1), \quad (4.6)$$

where the structures functions $f(n+1)$ and $g(n+1)$ are complex now.

By setting (4.5) in (4.1)-(4.2) and (4.5) in (4.3)-(4.4) we obtain the following equations for the functions $f(n+1)$ and $g(n+1)$:

$$(n+1)T(n+2)f^2(n+1) - nR(n)f^2(n) = \frac{1}{T(n+1)}, \quad (4.7)$$

$$(n+1)T^+(n+2)f^{+2}(n+1) - nR^+(n)f^{+2}(n) = \frac{1}{T^+(n+1)}, \quad (4.8)$$

$$(n+1)T(n+2)g^2(n+1) - nR(n)g^2(n) = \frac{1}{R(n+1)}, \quad (4.9)$$

$$(n+1)T^+(n+2)g^{+2}(n+1) - nR^+(n)g^{+2}(n) = \frac{1}{R^+(n+1)} \quad (4.10)$$

and, according to [20], we obtain:

$$f(n+1) = \frac{1}{\sqrt{(n+1)T(n+1)T(n+2)}} \left\{ 1 + \frac{R(n)}{T(n)} + \cdots + \frac{R(n)R(n-1)\cdots R(1)}{T(n)T(n-1)\cdots T(1)} \right\}^{1/2}, \quad (4.11)$$

$$f^+(n+1) = \frac{1}{\sqrt{(n+1)T^+(n+1)T^+(n+2)}} \cdot \left\{ 1 + \frac{R^+(n)}{T^+(n)} + \cdots + \frac{R^+(n)R^+(n-1)\cdots R^+(1)}{T^+(n)T^+(n-1)\cdots T^+(1)} \right\}^{1/2}, \quad (4.12)$$

$$g(n+1) = \frac{1}{\sqrt{(n+1)R(n+1)T(n+2)}} \cdot \left\{ 1 + \frac{R(n+1)}{T(n+1)} + \cdots + \frac{R(n+1)R(n)\cdots R(2)}{T(n+1)T(n)\cdots T(2)} \right\}^{1/2}, \quad (4.13)$$

$$g^+(n+1) = \frac{1}{\sqrt{(n+1)R^+(n+1)T^+(n+2)}} \cdot \left\{ 1 + \frac{R^+(n+1)}{T^+(n+1)} + \cdots + \frac{R^+(n+1)R^+(n)\cdots R^+(2)}{T^+(n+1)T^+(n)\cdots T^+(2)} \right\}^{1/2}. \quad (4.14)$$

Furthermore it is very simple to define the corresponding number operators and the Hamilton operators, i.e.

$$N_1^> = B * A = BR(n+1)A, < N_1 = (N_1^>)^+ = A^+R^+(n+1)B^+, \quad (4.15)$$

$$N_2^> = \dot{B} * \dot{A} = \dot{B} R(n+1)\dot{A}, < N_2 = (N_2^>)^+ = \dot{A}^+R^+(n+1)\dot{B}^+, \quad (4.16)$$

$$\begin{aligned} H_1^{\rangle} &= \frac{\hbar\omega}{2}(A * B + B * A) = \frac{\hbar\omega}{2}(AT(n+1)B + BR(n+1)A) = \quad (4.17) \\ &= \frac{\hbar\omega}{2}(I_1^{\rangle} + 2N_1^{\rangle}TI_1^{\rangle}) = \hbar\omega \left(N_1^{\rangle}T + \frac{1}{2} \right) I_1^{\rangle}, \end{aligned}$$

$$\langle H_1 = (H_1^{\rangle})^+ = \hbar\omega \langle I_1 \left(T^+ \langle N_1 + \frac{1}{2} \right), \quad (4.18)$$

$$H_2^{\rangle} = \hbar\omega \left(N_2^{\rangle}T + \frac{1}{2} \right) I_2^{\rangle}, \langle H_2 = \hbar\omega \langle I_2 \left(T^+ \langle N_2 + \frac{1}{2} \right), \quad (4.19)$$

and, performing some computations, we obtain:

$$N_1^{\rangle} = \left(\frac{R(n)}{T(n)} + \dots + \frac{R(n)R(n-1)\dots R(1)}{T(n)T(n-1)\dots T(1)} \right) I_1^{\rangle}, \quad (4.20)$$

$$\langle N_1 = \langle I_1 \left(\frac{R^+(n)}{T^+(n)} + \dots + \frac{R^+(n)R^+(n-1)\dots R^+(1)}{T^+(n)T^+(n-1)\dots T^+(1)} \right), \quad (4.21)$$

$$N_2^{\rangle} = \left(\frac{R(n+1)}{T(n+1)} + \dots + \frac{R(n+1)R(n)\dots R(2)}{T(n+1)T(n)\dots T(2)} \right) I_2^{\rangle}, \quad (4.22)$$

$$\langle N_2 = \langle I_2 \left(\frac{R^+(n+1)}{T^+(n+1)} + \dots + \frac{R^+(n+1)R^+(n)\dots R^+(2)}{T^+(n+1)T^+(n)\dots T^+(2)} \right), \quad (4.23)$$

$$H_1^{\rangle} = \hbar\omega \left(\frac{R(n)}{T(n)} + \dots + \frac{R(n)R(n-1)\dots R(1)}{T(n)T(n-1)\dots T(1)} + \frac{1}{2} \right) I_1^{\rangle}, \quad (4.24)$$

$$\langle H_1 = \hbar\omega \langle I_1 \left(\frac{R^+(n)}{T^+(n)} + \dots + \frac{R^+(n)R^+(n-1)\dots R^+(1)}{T^+(n)T^+(n-1)\dots T^+(1)} + \frac{1}{2} \right), \quad (4.25)$$

$$H_2^{\rangle} = \hbar\omega \left(\frac{R(n+1)}{T(n+1)} + \dots + \frac{R(n+1)R(n)\dots R(2)}{T(n+1)T(n)\dots T(2)} + \frac{1}{2} \right) I_2^{\rangle}, \quad (4.26)$$

$$\langle H_2 = \hbar\omega \langle I_2 \left(\frac{R^+(n+1)}{T^+(n+1)} + \dots + \frac{R^+(n+1)R^+(n)\dots R^+(2)}{T^+(n+1)T^+(n)\dots T^+(2)} + \frac{1}{2} \right). \quad (4.27)$$

It is very simple now to construct genobosonic normalized basis, i.e.

$$\begin{aligned} |N\rangle &= \frac{1}{\sqrt[4]{|T(n+1)| |R(n+1)|}} |n\rangle, \langle n, n\rangle = 1, \\ \langle N | \sqrt{|T| |R|} |N\rangle &= 1 \end{aligned} \quad (4.28)$$

and from (4.20)-(4.28) we obtain:

$$N_1^{\rangle} * |N\rangle = \left(\frac{R(n)}{T(n)} + \dots + \frac{R(n)R(n-1)\dots R(1)}{T(n)T(n-1)\dots T(1)} \right) |N\rangle, \quad (4.29)$$

$$\langle N | * N_1 = \langle N | \left(\frac{R^+(n)}{T^+(n)} + \dots + \frac{R^+(n)R^+(n-1)\dots R^+(1)}{T^+(n)T^+(n-1)\dots T^+(1)} \right), \quad (4.30)$$

$$N_2^> * |N\rangle = \left(\frac{R(n+1)}{T(n+1)} + \cdots + \frac{R(n+1)R(n)\cdots R(2)}{T(n+1)T(n)\cdots T(2)} \right) |N\rangle, \quad (4.31)$$

$$\langle N | *^< N_2 = \langle N | \left(\frac{R^+(n+1)}{T^+(n+1)} + \cdots + \frac{R^+(n+1)R^+(n)\cdots R^+(2)}{T^+(n+1)T^+(n)\cdots T^+(2)} \right), \quad (4.32)$$

$$H_1^> * |N\rangle = \hbar\omega \left(\frac{R(n)}{T(n)} + \cdots + \frac{R(n)R(n-1)\cdots R(1)}{T(n)T(n-1)\cdots T(1)} + \frac{1}{2} \right) |N\rangle, \quad (4.33)$$

$$\langle N | *^< H_1 = \hbar\omega \langle N | \left(\frac{R^+(n)}{T^+(n)} + \cdots + \frac{R^+(n)R^+(n-1)\cdots R^+(1)}{T^+(n)T^+(n-1)\cdots T^+(1)} + \frac{1}{2} \right), \quad (4.34)$$

$$H_2^> * |N\rangle = \hbar\omega \left(\frac{R(n+1)}{T(n+1)} + \cdots + \frac{R(n+1)R(n)\cdots R(2)}{T(n+1)T(n)\cdots T(2)} + \frac{1}{2} \right) |N\rangle, \quad (4.35)$$

$$\langle N | *^< H_2 = \hbar\omega \langle N | \left(\frac{R^+(n+1)}{T^+(n+1)} + \cdots + \frac{R^+(n+1)R^+(n)\cdots R^+(2)}{T^+(n+1)T^+(n)\cdots T^+(2)} + \frac{1}{2} \right). \quad (4.36)$$

Summarizing, the above equations take the form:

$$N_1^> * |N\rangle = N_1^>(n) |N\rangle, \langle N | *^< N_1 = \langle N |^< N_1(n), \quad (4.37)$$

$$N_2^> * |N\rangle = N_2^>(n) |N\rangle, \langle N | *^< N_2 = \langle N |^< N_2(n), \quad (4.38)$$

$$H_1^> * |N\rangle = \hbar\omega \left(N_1^>(n) + \frac{1}{2} \right) |N\rangle, \langle N | *^< H_1 = \hbar\omega \langle N | \left(\langle N_1(n) + \frac{1}{2} \right), \quad (4.39)$$

$$H_2^> * |N\rangle = \hbar\omega \left(N_2^>(n) + \frac{1}{2} \right) |N\rangle, \langle N | *^< H_2 = \hbar\omega \langle N | \left(\langle N_2(n) + \frac{1}{2} \right), \quad (4.40)$$

where $N_1^>(n), \dots, \langle N_2(n)$ are the eigenvalues of the number operators.

Also we have:

$$\begin{aligned} A * |N\rangle &= AT(n+1) |N\rangle = \\ &= \sqrt[4]{\frac{|T(n)| |R(n)|}{|T(n+1)| |R(n+1)|}} \sqrt{\frac{T(n+1)}{T(n)}} \\ &\quad \left\{ 1 + \frac{R(n-1)}{T(n-1)} + \cdots + \frac{R(n-1)\cdots R(1)}{T(n-1)\cdots T(1)} \right\}^{1/2} |N-1\rangle, \end{aligned} \quad (4.41)$$

$$\begin{aligned} B * |N\rangle &= BR(n+1) |N\rangle = \\ &= \sqrt[4]{\frac{|T(n+2)| |R(n+2)|}{|T(n+1)| |R(n+1)|}} \sqrt{\frac{R(n+1)}{T(n+2)}} \\ &\quad \left\{ \frac{R(n+1)}{T(n+1)} + \cdots + \frac{R(n+1)\cdots R(1)}{T(n+1)\cdots T(1)} \right\}^{1/2} |N+1\rangle, \end{aligned} \quad (4.42)$$

$$\begin{aligned}
\dot{A} * |N\rangle &= \dot{A} T(n+1) |N\rangle = & (4.43) \\
&= \sqrt[4]{\frac{|T(n)| |R(n)|}{|T(n+1)| |R(n+1)|}} \sqrt{\frac{T(n+1)}{T(n)}} \\
&\quad \left\{ 1 + \frac{R(n)}{T(n)} + \cdots + \frac{R(n) \cdots R(2)}{T(n) \cdots T(2)} \right\}^{1/2} |N-1\rangle,
\end{aligned}$$

$$\begin{aligned}
\dot{B} * |N\rangle &= \dot{B} R(n+1) |N\rangle = & (4.44) \\
&= \sqrt[4]{\frac{|T(n+2)| |R(n+2)|}{|T(n+1)| |R(n+1)|}} \sqrt{\frac{R(n+1)}{T(n+2)}} \\
&\quad \left\{ 1 + \frac{R(n+1)}{T(n+1)} + \cdots + \frac{R(n+1) \cdots R(2)}{T(n+1) \cdots T(2)} \right\}^{1/2} |N+1\rangle.
\end{aligned}$$

In the same way, the conjugation relations of the above four equations, are valid.

In the following, we study the special case $R = T^+$ which form the general genobosonic algebra.

5 General Genobosonic Algebra.

For the general genobosonic algebra we have only two commutation relations, i.e.:

$$A * B - B * A = AT(n+1)B - BT^+(n+1)A = I^> = \frac{1}{T(n+1)}, \quad (5.1)$$

$$B^+ * A^+ - A^+ * B^+ = B^+ T^+(n+1)A^+ - A^+ T^+(n+1)B^+ = I^< = \frac{1}{T^+(n+1)} \quad (5.2)$$

and the expressions of A and B , according to (4.5), are:

$$A = f(n+1)a, B = a^+ f(n+1), \quad (5.3a)$$

$$A^+ = a^+ f^+(n+1), B^+ = f^+(n+1)a, \quad (5.3b)$$

where:

$$\begin{aligned}
f(n+1) &= \frac{1}{\sqrt{(n+1)T(n+1)T(n+2)}} & (5.4) \\
&\quad \left\{ 1 + \frac{T^+(n)}{T(n)} + \cdots + \frac{T^+(n)T^+(n-1) \cdots T^+(1)}{T(n)T(n-1) \cdots T(1)} \right\}^{1/2},
\end{aligned}$$

$$\begin{aligned}
f^+(n+1) &= \frac{1}{\sqrt{(n+1)T^+(n+1)T^+(n+2)}} & (5.5) \\
&\quad \left\{ 1 + \frac{T(n)}{T^+(n)} + \cdots + \frac{T(n)T(n-1) \cdots T(1)}{T^+(n)T^+(n-1) \cdots T^+(1)} \right\}^{1/2}
\end{aligned}$$

and the genobosonic number operators and genobosonic Hamiltonian, according to (4.15) and (4.18), are given by the expressions:

$$N^> = B * A = BT^+(n+1)A, <N = (N^>)^+ = A^+T(n+1)B^+ \quad (5.6)$$

$$H^> = \frac{\hbar\omega}{2} \left(N^>T + \frac{1}{2} \right) I^>, <H = (H^>)^+ = \hbar\omega <I \left(T^+<N + \frac{1}{2} \right) \quad (5.7)$$

and, after a straightforward computation, we obtain:

$$N^> = \left(\frac{T^+(n)}{T(n)} + \dots + \frac{T^+(n)T^+(n-1)\dots T^+(1)}{T(n)T(n-1)\dots T(1)} \right) I^>, \quad (5.8)$$

$$<N = <I \left(\frac{T(n)}{T^+(n)} + \dots + \frac{T(n)T(n-1)\dots T(1)}{T^+(n)T^+(n-1)\dots T^+(1)} \right), \quad (5.9)$$

$$H^> = \hbar\omega \left(\frac{T^+(n)}{T(n)} + \dots + \frac{T^+(n)T^+(n-1)\dots T^+(1)}{T(n)T(n-1)\dots T(1)} + \frac{1}{2} \right) I^>, \quad (5.10)$$

$$<H = \hbar\omega <I \left(\frac{T(n)}{T^+(n)} + \dots + \frac{T(n)T(n-1)\dots T(1)}{T^+(n)T^+(n-1)\dots T^+(1)} + \frac{1}{2} \right). \quad (5.11)$$

The common genobosonic normalized basis according to (4.28) yield:

$$|N\rangle = \frac{1}{\sqrt{|T(n+1)|}} |n\rangle, \langle n | n\rangle = 1, \langle N || T(n+1) | |N\rangle = 1. \quad (5.12)$$

From (4.37) and (4.39) we obtain:

$$N^> * |N\rangle = N^>(n) |N\rangle, <N | * <N = <N | <N(n), \quad (5.13)$$

$$H^> * |N\rangle = \hbar\omega \left(N^>(n) + \frac{1}{2} \right) |N\rangle, <N | * <H = \hbar\omega <N | \left(<N(n) + \frac{1}{2} \right), \quad (5.14)$$

where:

$$N^>(n) = \frac{T^+(n)}{T(n)} + \dots + \frac{T^+(n)T^+(n-1)\dots T^+(1)}{T(n)T(n-1)\dots T(1)} \quad (5.15)$$

are the eigenvalues of the number operator $N^>$.

Finally, for (4.41) and (4.42), we obtain:

$$\begin{aligned} A * |N\rangle &= AT(n+1) |N\rangle = \\ &= \sqrt{\frac{|T(n)|}{|T(n+1)|}} \sqrt{\frac{T(n+1)}{T(n)}} \\ &\quad \left\{ 1 + \frac{T^+(n-1)}{T(n-1)} + \dots + \frac{T^+(n-1)\dots T^+(1)}{T(n-1)\dots T(1)} \right\}^{1/2} |N-1\rangle \end{aligned} \quad (5.16)$$

$$\begin{aligned}
B * |N\rangle &= BT(n+1) |N\rangle = \\
&= \sqrt{\frac{|T(n+2)|}{|T(n+1)|}} \sqrt{\frac{T^+(n+1)}{T(n+2)}} \\
&\quad \left\{ \frac{T^+(n+1)}{T(n+1)} + \dots + \frac{T^+(n+1) \cdots T^+(1)}{T(n+1) \cdots T(1)} \right\}^{1/2} |N-1\rangle
\end{aligned} \tag{5.17}$$

Example β .

For $T = T_1 + iT_2$, $R = T^+ = T_1 - iT_2$, $Q = \frac{T_1 - iT_2}{T_1 + iT_2}$, where T_1, T_2 are constant and real, we obtain:

$$f(n+1) = \frac{1}{T_1 + iT_2} \sqrt{\frac{[n+1]}{n+1}}, f^+(n+1) = \frac{1}{T_1 - iT_2} \sqrt{\frac{[n+1]^+}{n+1}}, \tag{\beta(1)}$$

where $[n] = \frac{Q^n - 1}{Q - 1}$,

$$A = \frac{1}{T_1 + iT_2} \sqrt{\frac{[n+1]}{n+1}} a, B = \frac{1}{T_1 + iT_2} a^+ \sqrt{\frac{[n+1]}{n+1}}, \tag{\beta(2)}$$

$$N^> = Q[n+1]I^>, <N = < IQ^+[n+1]^+, \tag{\beta(3)}$$

$$H^> = \hbar\omega \left(Q[n+1] + \frac{1}{2} \right) I^>, <H = \hbar\omega < I \left(Q^+[n+1]^+ + \frac{1}{2} \right), \tag{\beta(4)}$$

$$N^> * |N\rangle = Q[n] |N\rangle, <N | * <N = <N | Q^+[n]^+, \tag{\beta(5)}$$

$$H^> * |N\rangle = \hbar\omega \left(Q \frac{Q^n - 1}{Q - 1} + \frac{1}{2} \right) |N\rangle = \hbar\omega \left(\frac{Q^{n+1} - 1}{Q - 1} + \frac{1}{2} \right) |N\rangle, \tag{\beta(6)}$$

$$<N | * <H = \hbar\omega <N | \left(Q^* \frac{Q^{*n} - 1}{Q^* - 1} + \frac{1}{2} \right) = \hbar\omega <N | \left(\frac{Q^{*n+1} - 1}{Q^* - 1} - \frac{1}{2} \right), \tag{\beta(7)}$$

$$A * |N\rangle = \sqrt{\frac{Q^n - 1}{Q - 1}} |N-1\rangle, \tag{\beta(8)}$$

$$B * |N\rangle = Q \sqrt{\frac{Q^{n+1} - 1}{Q - 1}} |N+1\rangle, \tag{\beta(9)}$$

$$(A * B - B * A) * |N\rangle = |N\rangle. \tag{\beta(10)}$$

Because the above examples describes the non-hermitian realization of the new Lie-deformed Heisenberg algebra [15, 16], in the following section we investigate some new properties for the deformed Heisenberg quantum mechanics.

6 The Deformed Heisenberg Quantum Mechanics.

According to Jannussis and Skaltsas [21], we can define the Lie-admissible von Neumann equation for the density operator ρ , i.e.:

$$i\hbar \frac{d\rho}{dt} = (H, \rho) = HT\rho - \rho T^+ H, \quad (6.1)$$

with the commutation relation:

$$xT\rho - \rho T^+ x = i\hbar \hat{I}, \quad (6.2)$$

where $H(x, p)$ is the usual Hamilton operator of the system, $T \neq T^+$ and \hat{I} is the new unity. At this point we can exploit the theory of Santilli's Hadronic mechanics [1] and define two units, i.e.:

$$\hat{I} = I^>, \hat{I} = I^<. \quad (6.3)$$

In the framework of the general Lie-admissible theory the commutation relation (6.2) is separated in two relations of the following form:

$$xT\rho - \rho T^+ x = i\hbar I^>, \quad (6.4)$$

$$xT\rho - \rho T^+ x = i\hbar I^<, \quad (6.5)$$

where:

$$I^< = T^{-1}, I^> = (T^+)^{-1}. \quad (6.6)$$

Due to Hermitian character of density operator $\rho(x, p, t)$ and the non unitary character of the time evolution, a kind of contradiction becomes evident between (6.1) and (6.2) inasmuch as the operators x, p are not anymore hermitian, while according to the genotopic Heisenberg equations of motion [1, 15], i.e.:

$$i\hbar \frac{\partial x}{\partial t} = xTH - HT^+ x, \quad (6.7)$$

$$i\hbar \frac{\partial p}{\partial t} = pTH - HT^+ p, \quad (6.8)$$

they are still hermitian. We can overcome this discrepancy only when the commutation relations (6.4), (6.5) take the form:

$$xT\rho = \rho T^+ x = i\hbar \frac{1}{\sqrt{|TT^+|}} = i\hbar \frac{1}{|T|}. \quad (6.9)$$

This fact is implied from the physical reality, which characterizes the physics of the system. Based on the above commutation relation, we initiate the operator T_{jk} and we form the commutation relations:

$$x_j T_{jk} p_k - p_k T_{jk}^+ x_j = i\hbar \frac{\delta_{jk}}{|T_{jk}|}, \quad (6.10)$$

$$x_j x_k - x_k x_j = 0, \quad p_j p_k - p_k p_j = 0, \quad (6.11)$$

where T_{jk} are fixed elements.

For the special case

$$T_{jk} = 1 + i\lambda_{jk}, T_{jk}^+ = 1 - i\lambda_{jk}, \lambda_{jk} = \lambda_j \delta_{jk}, \quad (6.12)$$

λ_{jk} real, we obtain the new Heisenberg ring [15]:

$$x_j(1 + i\lambda_{jk}\delta_{jk})p_k - p_k(1 - i\lambda_{jk}\delta_{jk})x_j = i\hbar\delta_{jk}(1 + i\lambda_{jk}^2\delta_{jk})^{-1/2}, \quad (6.13)$$

$$[x_j, p_k] = 0, [x_j, x_k] = 0, [p_j, p_k] = 0, j \neq k. \quad (6.14)$$

For $\lambda_{jk} = 0$ the above relations are reduced to the usual Heisenberg ring of the canonical quantum mechanics. The new operators x_j and p_k of the above ring take the following form and representations, respectively:

$$x_j \rightarrow x_j, p_k = \frac{\hbar}{2\lambda_k\sqrt{1+\lambda_k^2}} \cdot \frac{1}{x_k} \left(1 - e^{2i\theta_k x_k \frac{\partial}{\partial x_k}}\right), \quad (6.15)$$

$$p_k \rightarrow p_k, x_j = \frac{\hbar}{2\lambda_j\sqrt{1+\lambda_j^2}} \cdot \frac{1}{p_j} \left(1 - e^{2i\theta_j p_j \frac{\partial}{\partial p_j}}\right), \quad (6.16)$$

where $\theta_k = \arctan \lambda_k$ and the operators x_j, p_k are hermitian. For $x_1 = x, p_1 = p$ we obtain the commutation relation:

$$x(1 + i\lambda)p - p(1 - i\lambda)x = \frac{i\hbar}{\sqrt{1+\lambda^2}} \quad (6.17)$$

and then corresponding representations are:

$$x \rightarrow x, p = \frac{\hbar}{2\lambda\sqrt{1+\lambda^2}} \cdot \frac{1}{x} \left(1 - e^{2i\theta x \frac{\partial}{\partial x}}\right), \quad (6.18)$$

$$p \rightarrow p, x = \frac{\hbar}{2\lambda\sqrt{1+\lambda^2}} \cdot \frac{1}{p} \left(1 - e^{2i\theta p \frac{\partial}{\partial p}}\right), \quad (6.19)$$

where $\theta = \arctan \lambda$. The Hamilton operator:

$$H(x, p) = \frac{p^2}{2m} + v(x) \quad (6.20)$$

in x and p representations, according to relations (6.18), (6.19), takes the forms:

$$H\left(x, \frac{\partial}{\partial x}, \lambda\right) = \frac{\hbar^2}{2m\lambda^2(1+\lambda^2)} \left(\frac{1}{x}(1 - e^{2i\theta x \frac{\partial}{\partial x}})\right)^2 + v(x), \quad (6.21)$$

$$H\left(p, \frac{\partial}{\partial p}, \lambda\right) = \frac{p^2}{2m} + v\left(\frac{\hbar}{2\lambda\sqrt{1+\lambda^2}} \frac{1}{p}(1 - e^{-2i\theta p \frac{\partial}{\partial p}})\right) \quad (6.22)$$

and they remain hermitian.

The fact that the operators x and p remain Hermitian has an important physical meaning, since in the opposite case the hermiticity of new deformed Hamiltonian is violated. Consequently, the new deformed Heisenberg quantum mechanics satisfies the commutation relations (6.10), (6.11) and the genotopic Heisenberg equations of motion (6.7), (6.8). Based on the above results, the problem of quantum measurement has been confronted in [22], where we have formulated the new modified Lie-admissible statistics.

7 Conclusion

In the present paper we have studied the general Lie-admissible algebras according to Santilli's axiomatization of q -deformations via isotopies and genotopies of quantum mechanics. With the help of these algebras we established a new deformed Heisenberg quantum mechanics, in which the genotopic Heisenberg equations of motion (6.7), (6.8) with the commutation relations (6.10), (6.11), are valid. Based on the above results, we have formulated the new modified Lie-admissible statistics [22] and the corresponding density operator ρ satisfying the deformed Lie-admissible von Neumann equation (6.1).

As it is well known, the dynamics of a quantum system is fully determined by its Hamilton operator and the Heisenberg algebra. A similar situation occurs for the deformed Hamilton operator (6.21) or (6.22) and the corresponding deformed Heisenberg algebra (6.17). The main difference between usual quantum mechanics and deformed quantum mechanics is based on the fact that in the first case we have unitary time evolution, but in the second case the time evolution is non-unitary.

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