A shock wave propagation into a transonic flow of a compressible fluid

Elena Corina Cipu

Abstract. The interaction of weak pressure waves with a transonic flow due to a small perturbation of a second order compressible fluid is studied. It is first described the nondimensional problem and a multiscale asymptotic development. In the hypothesis of small variations of the vorticity motion the equation of the shock surface caused by a pressure discontinuity and shock jumps conditions are presented. The transonic model equations predict behaviour at the wavefront. The model could be used to analyse the propagation of the sonic boom shock wave through the turbulent atmospheric boundary layer.

Mathematics Subject Classification 2000: 76N15, 58K55, 76L05.

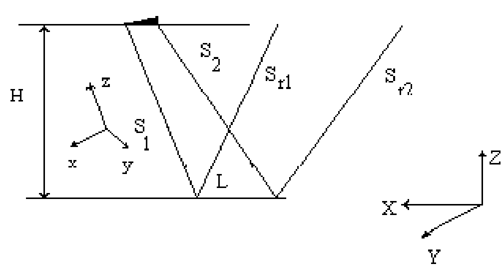
Key words: Compressible fluid, transonic flow, asymptotic development, weak shock wave propagation, discontinuity of pressure.

1. Introduction

The problem is to explain the propagation phenomena of a weak shock wave into a fluid with small heterogeneities.

A weak shock wave is defined as a discontinuity surface for the fields that describe the fluid, the jump of the pressure field being small compared to p_∞ (pressure in the unperturbed state). Such a problem appears in case of the sonic boom. A supersonic aircraft creates a variation of the pressure that at the soil level produce two weak shock weaves (leading and trailing waves).

With a coordinate transformation, the problem fits into a more basic framework with stationary planar shock wave and uniform flow moving perpendicular to the wavefront (see Fig.1).



Schematic sonic boom wave system

Proceedings of The 2-nd International Colloquium of Mathematics in Engineering and Numerical Physics (MENP-2), April 22-27, 2002, University Politehnica of Bucharest, Romania.
BSG Proceedings 8, pp. 31-39, Geometry Balkan Press, 2003.

With the index "∞" for the fields that describe a perturbed atmosphere by an horizontal motion $\vec{v}_\infty = U_\infty \vec{e}_x$ the sound speed is $a_\infty = \sqrt{\gamma R T_\infty(0)}$. This atmosphere is considered unperturbed according to the transonic motion owed to the sonic boom (weak jump of the pressure).

We consider that the atmosphere is described by a linear viscous isotrop fluid, for which the heat flux is obtained by the Fourier law: $\vec{q} = -k\text{grad}\, T$ and the state equation is given by $p = \rho R T$; we have $R = c_p - c_v$, where c_p and c_v are the specific heats at constant pressure and volume. We suppose also that initial thermodynamic state is explained by: $\rho_\infty(z)$; $p_\infty(z)$; $T_\infty(z)$; $\vec{v}_\infty = U_\infty \vec{e_x}$,

$$\begin{aligned} \rho_\infty(z) &= \rho_\infty(0)\left[1 - \frac{\Gamma_\infty}{T_\infty(0)} z\right]^{(\frac{g}{\Gamma_\infty R}-1)} \\ T_\infty(z) &= T_\infty(0) - \Gamma_\infty z \\ p_\infty(z) &= p_\infty(0)\left[1 - \frac{\Gamma_\infty}{T_\infty(0)} z\right]^{\frac{g}{\Gamma_\infty R}} \end{aligned} \tag{1.1}$$

where $p_\infty(z) = R\rho_\infty(z)T_\infty(z)$; $a_\infty = \sqrt{\gamma R T_\infty(0)}$ is the sound speed and $M_\infty = \frac{U_\infty}{a_\infty} > 1$ the Mach number. The motion is supposed transonic with $M_\infty > 1$, $M_\infty \in \mathcal{O}(1)$ and $M_\infty^2 = 1 + \alpha\bar{\varepsilon}^2$; $\alpha \in \mathcal{O}(1)$ and $\varepsilon = \bar{\varepsilon}^2$, $\bar{\varepsilon} = \sqrt{\varepsilon} << 1$.

We shall consider the basic equations of the air flow given by

$$\begin{aligned} &\dot{\rho} + \rho \text{div}\vec{v} = 0, \\ &\rho\dot{\vec{v}} = -\text{grad}p + (\lambda+\mu)\text{grad}(\text{div}\vec{v}) + \mu\Delta\vec{v} + \rho\vec{g}, \\ &\dot{T} - \tfrac{\gamma-1}{\gamma}\cdot\tfrac{T}{p}\dot{p} = \tfrac{\gamma-1}{\gamma}k\tfrac{T}{p}\Delta T + \tfrac{2(\gamma-1)}{\gamma}\mu\tfrac{T}{p}\text{tr}(\mathbf{D^2}) + \tfrac{\gamma-\mathbf{1}}{\gamma}\mu\tfrac{\mathbf{T}}{\mathbf{p}}(\mathbf{trD})^{\mathbf{2}}, \\ &p = \rho R T. \end{aligned} \tag{1.2}$$

2. Dimensional analysis of the flow problem

The characteristic fields used to obtain the nondimensional fields and equations were considered such that the dimension order be that of the receding waves which appear within an ideal fluid.

With $M_\infty^2 = 1 + \alpha\varepsilon$, $\alpha \in \mathcal{O}(1)$ the nondimensional fields are

$$\begin{aligned} &\vec{v} = U_\infty \vec{v}^*,\ \rho = \rho_\infty\rho^*,\ p = p_\infty p^*,\ T = T_\infty T^*,\ \vec{v}^* = (u^*, v^*, w^*), \\ &x^* = x/l_x,\ y^* = y/l_y,\ z^* = z/l_z;\ L = l_y = l_z,\ l_x = L\sqrt{\varepsilon} = L\bar{\varepsilon}, \\ &t^* = t/t_0,\ t_0 = (L/U_\infty)\cdot(1/\sqrt{\varepsilon} = (L/U_\infty)\cdot(1/\bar{\varepsilon}). \end{aligned} \tag{1.3}$$

Nondimensional equations are:

$$\bar{\varepsilon}\frac{\partial\rho^*}{\partial t^*} + \frac{1}{\bar{\varepsilon}}\frac{\partial}{\partial x^*}(\rho^* u^*) + \frac{\partial}{\partial y^*}(\rho^* v^*) + \frac{\partial}{\partial z^*}(\rho^* w^*) + Ld_\infty(\rho^* w^*) = 0 \tag{1.4}$$

$$\overline{\varepsilon}\rho^* \frac{\partial u^*}{\partial t^*} + \frac{1}{\overline{\varepsilon}}\rho^* u^* \frac{\partial u^*}{\partial x^*} + \rho^* v^* \frac{\partial u^*}{\partial y^*} + \rho^* w^* \frac{\partial u^*}{\partial z^*} = -\frac{1}{\gamma M_\infty^2}\frac{1}{\overline{\varepsilon}}\frac{\partial p^*}{\partial x^*} + \frac{1}{Re}\left\{\left(\frac{\lambda}{\mu}+1\right)\left[\frac{1}{\varepsilon}\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{1}{\overline{\varepsilon}}\frac{\partial^2 v^*}{\partial x^* \partial y^*} + \frac{1}{\overline{\varepsilon}}\frac{\partial^2 w^*}{\partial y^* \partial z^*}\right] + \frac{1}{\varepsilon}\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right\} \tag{1.5}$$

$$\overline{\varepsilon}\rho^* \frac{\partial v^*}{\partial t^*} + \frac{1}{\overline{\varepsilon}}\rho^* u^* \frac{\partial v^*}{\partial x^*} + \rho^* v^* \frac{\partial v^*}{\partial y^*} + \rho^* w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\gamma M_\infty^2}\frac{\partial p^*}{\partial y^*} + \frac{1}{Re}\left\{\left(\frac{\lambda}{\mu}+1\right)\left[\frac{1}{\overline{\varepsilon}}\frac{\partial^2 u^*}{\partial x^* \partial y^*} + \frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial y^* \partial z^*}\right] + \frac{1}{\varepsilon}\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right\} \tag{1.6}$$

$$\overline{\varepsilon}\rho^* \frac{\partial w^*}{\partial t^*} + \frac{1}{\overline{\varepsilon}}\rho^* u^* \frac{\partial w^*}{\partial x^*} + \rho^* v^* \frac{\partial w^*}{\partial y^*} + \rho^* w^* \frac{\partial w^*}{\partial z^*} = \frac{-1}{\gamma M_\infty^2}\frac{\partial p^*}{\partial z^*} + \frac{Lg(p^*-\rho^*)}{U_\infty^2} + \frac{1}{Re}\left\{\left(\frac{\lambda}{\mu}+1\right)\left[\frac{1}{\overline{\varepsilon}}\frac{\partial^2 u^*}{\partial x^* \partial z^*} + \frac{\partial^2 v^*}{\partial y^* \partial z^*} + \frac{\partial^2 w^*}{\partial z^{*2}}\right] + \frac{1}{\varepsilon}\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right\} \tag{1.7}$$

$$\begin{aligned} &\overline{\varepsilon}\left(p^*\frac{\partial T^*}{\partial t^*} - \frac{(\gamma-1)T^*}{\gamma}\frac{\partial p^*}{\partial t^*}\right) + \frac{1}{\overline{\varepsilon}}u^*\left(p^*\frac{\partial T^*}{\partial x^*} - \frac{(\gamma-1)T^*}{\gamma}\frac{\partial p^*}{\partial x^*}\right) \\ &+v^*\left(p^*\frac{\partial T^*}{\partial y^*} - \frac{(\gamma-1)T^*}{\gamma}\frac{\partial p^*}{\partial y^*}\right) + w^*\left(p^*\frac{\partial T^*}{\partial z^*} - \frac{(\gamma-1)T^*}{\gamma}\frac{\partial p^*}{\partial z^*}\right) = \\ &\frac{k(\gamma-1)t^*}{\mu\gamma Re}\left[\frac{1}{\varepsilon}\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right] + \frac{(\gamma-1)M_\infty^2 T^*}{Re}(2d_2^* + d_1^*/\mu) \end{aligned} \tag{1.8}$$

$$p^* = \rho^* T^*. \tag{1.9}$$

We have made the notations: $Re = \frac{\rho_\infty U_\infty L}{\mu}$ is the Reynolds number (that depends on z^* through ρ_∞), $d_\infty = \frac{R\Gamma_\infty - g}{RT_\infty}$, depending on z^* through T_∞ and d_1^*, d_2^* expressing nondimensional nonlinear terms obtained from $\mathrm{tr}(\mathbf{D}^2)$ and $(\mathrm{tr}\mathbf{D})^2$ in (1.2_3):

$$d_1^* = \left(\frac{1}{\varepsilon}\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*}\right)^2,$$

$$\begin{aligned} d_2^* = &\frac{1}{\varepsilon}\left[\left(\frac{\partial u^*}{\partial x^*}\right)^2 + \frac{1}{2}\left(\frac{\partial v^*}{\partial x^*}\right)^2 + \frac{1}{2}\left(\frac{\partial w^*}{\partial z^*}\right)^2\right] + \frac{1}{\overline{\varepsilon}}\left[\frac{\partial u^*}{\partial y^*}\cdot\frac{\partial v^*}{\partial x^*} + \frac{\partial u^*}{\partial z^*}\cdot\frac{\partial w^*}{\partial x^*}\right] \\ &+\frac{1}{2}\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial u^*}{\partial z^*}\right)^2 + \left(\frac{\partial v^*}{\partial z^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right] + \left(\frac{\partial v^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial z^*}\right)^2 + \frac{\partial v^*}{\partial z^*}\cdot\frac{\partial w^*}{\partial y^*}. \end{aligned}$$

We used also that: $p_\infty/\rho_\infty U_\infty^2 = 1/\gamma M_\infty$, necessary for the nondimensionality of the pressure gradient, and $U_\infty/p_\infty L = \gamma M_\infty^2/\mu Re$, which is necessary for the nondimensionality of the equation (1.8) of the temperature propagation.

3. Asymptotic development. Approximation of the solution

The nondimensional system (1.4)-(1.9) is strongly nonlinear. In order to simplify the equations we use an asymptotic study on a double scale according with parameter $\varepsilon << 1$ and parameter $\overline{\varepsilon} = \sqrt{\varepsilon} << 1$, with the assumption that the perturbation of speed in direction x induces an horizontal speed of order one in direction x and of order ε in the other two directions y and z.

Depending on the parameter ε, the asymptotic development is:

$$\begin{array}{lllll} u^* & = 1+ & \varepsilon u_1 + \varepsilon^2 u_2 & + \ldots + \varepsilon^k u_k + \ldots \\ v^* & = & \varepsilon v_1 + \varepsilon^2 v_2 & + \ldots + \varepsilon^k v_k + \ldots \\ w^* & = & \varepsilon w_1 + \varepsilon^2 w_2 & + \ldots + \varepsilon^k w_k + \ldots \\ \rho^* & = 1+ & \varepsilon \rho_1 + \varepsilon^2 \rho_2 & + \ldots + \varepsilon^k \rho_k + \ldots \\ p^* & = 1+ & \varepsilon p_1 + \varepsilon^2 p_2 & + \ldots + \varepsilon^k p_k + \ldots \\ T^* & = 1+ & \varepsilon T_1 + \varepsilon^2 T_2 & + \ldots + \varepsilon^k T_k + \ldots \end{array} \tag{1.10}$$

Taking into account that the receding waves damp slowly in time with fast fluctuations of speed we shall consider the second development comparison with parameter $\overline{\varepsilon}$:

$$\begin{aligned} u_k &= u_{k,0} + \overline{\varepsilon} u_{k,1} \\ v_k &= v_{k,0} + \overline{\varepsilon} v_{k,1} \\ w_k &= w_{k,0} + \overline{\varepsilon} w_{k,1} \end{aligned} \tag{1.11}$$

where k expresses the order of the development comparison with ε, and the index 0 or 1 the development comparison with $\overline{\varepsilon}$.

The final development becomes:

$$\begin{array}{llllll} u^* & = 1+ & \overline{\varepsilon}^2 u_{1,0} + \overline{\varepsilon}^3 u_{1,1} & + \overline{\varepsilon}^4 u_{2,0} + \overline{\varepsilon}^5 u_{2,1} & + \overline{\varepsilon}^6 u_{3,0} + \overline{\varepsilon}^7 u_{3,1} & + \ldots \\ v^* & = & \overline{\varepsilon}^2 v_{1,0} + \overline{\varepsilon}^3 v_{1,1} & + \overline{\varepsilon}^4 v_{2,0} + \overline{\varepsilon}^5 v_{2,1} & + \overline{\varepsilon}^6 v_{3,0} + \overline{\varepsilon}^7 v_{3,1} & + \ldots \\ w^* & = & \overline{\varepsilon}^2 w_{1,0} + \overline{\varepsilon}^3 w_{1,1} & + \overline{\varepsilon}^4 w_{2,0} + \overline{\varepsilon}^5 w_{2,1} & + \overline{\varepsilon}^6 w_{3,0} + \overline{\varepsilon}^7 w_{3,1} & + \ldots \\ \rho^* & = 1+ & \overline{\varepsilon}^2 \rho_1 & + \overline{\varepsilon}^4 \rho_2 & + \overline{\varepsilon}^6 \rho_3 & + \ldots \\ p^* & = 1+ & \overline{\varepsilon}^2 p_1 & + \overline{\varepsilon}^4 p_2 & + \overline{\varepsilon}^6 p_3 & + \ldots \\ T^* & = 1+ & \overline{\varepsilon}^2 T_1 & + \overline{\varepsilon}^4 T_2 & + \overline{\varepsilon}^6 T_3 & + \ldots . \end{array} \tag{1.12}$$

We shall obtain the equation system for first four orders according with parameter $\overline{\varepsilon}$ where x, y, z, t represent in fact the transonic coordinates x^*, y^*, z^*, t^*.

At order zero in $\overline{\varepsilon}$, the equations are

$$\frac{\partial^2 u_{1,0}}{\partial x^2} = 0, \frac{\partial^2 v_{1,0}}{\partial x^2} = 0, \frac{\partial^2 w_{1,0}}{\partial x^2} = 0, \frac{\partial^2 T_1}{\partial x^2}. \tag{1.13}$$

At order one in $\overline{\varepsilon}$ we shall obtain:

$$\begin{aligned}
&\frac{\partial}{\partial x}(\rho_{1,0}+u_{1,0})=0\\
&\frac{\partial u_{1,0}}{\partial x}=-\frac{1}{\gamma}\frac{\partial p_1}{\partial x}+\frac{1}{Re}\left\{\left(\frac{\lambda}{\mu}+1\right)\left(\frac{\partial^2 v_{1,0}}{\partial x\partial y}+\frac{\partial^2 w_{1,0}}{\partial y^*\partial z^*}\right)+\left(\frac{\lambda}{\mu}+2\right)\frac{\partial^2 u_{1,1}}{\partial x^2}\right\}\\
&\frac{\partial v_{1,0}}{\partial x}=\frac{1}{Re}\left(\frac{\lambda}{\mu}+1\right)\frac{\partial^2 u_{1,0}}{\partial x\partial y}+\frac{1}{Re}\frac{\partial^2 v_{1,1}}{\partial x^2}\\
&\frac{\partial w_{1,0}}{\partial x}=\frac{1}{Re}\left(\frac{\lambda}{\mu}+1\right)\frac{\partial^2 u_{1,0}}{\partial x\partial z}+\frac{1}{Re}\frac{\partial^2 w_{1,1}}{\partial x^2}\\
&\frac{\partial T_1}{\partial x}-\frac{\gamma-1}{\gamma}\frac{\partial p_1}{\partial x}=0
\end{aligned}\tag{1.14}$$

At order two according with $\overline{\varepsilon}$ we find

$$\begin{aligned}
&\frac{\partial u_{1,1}}{\partial x}+\frac{\partial v_{1,0}}{\partial y}+\frac{\partial w_{1,0}}{\partial z}+Ld_\infty w_{1,0}=0\\
&\frac{\partial u_{1,1}}{\partial x}=\frac{1}{Re}\left(\frac{\lambda}{\mu}+1\right)\left[\frac{\partial^2 u_{2,0}}{\partial x^2}+\frac{\partial^2 v_{1,1}}{\partial x\partial y}+\frac{\partial^2 w_{1,1}}{\partial x\partial z}\right]\\
&\qquad+\frac{1}{Re}\left[\frac{\partial^2 u_{2,0}}{\partial x^2}+\frac{\partial^2 u_{1,0}}{\partial y^2}+\frac{\partial^2 u_{1,0}}{\partial z^2}\right]\\
&\frac{\partial v_{1,1}}{\partial x}=-\frac{1}{\gamma}\frac{\partial p_1}{\partial y}+\frac{1}{Re}\left(\frac{\lambda}{\mu}+1\right)\left[\frac{\partial^2 u_{1,1}}{\partial x\partial y}+\frac{\partial^2 v_{1,0}}{\partial y^2}+\frac{\partial^2 w_{1,0}}{\partial y\partial z}\right]\\
&\qquad+\frac{1}{Re}\left[\frac{\partial^2 v_{2,0}}{\partial x^2}+\frac{\partial^2 v_{1,0}}{\partial y^2}+\frac{\partial^2 v_{1,0}}{\partial z^2}\right]\\
&\frac{\partial w_{1,1}}{\partial x}=-\frac{1}{\gamma}\frac{\partial p_1}{\partial z}+\frac{1}{Re}\left(\frac{\lambda}{\mu}+1\right)\left[\frac{\partial^2 u_{1,1}}{\partial x\partial z}+\frac{\partial^2 v_{1,0}}{\partial y\partial z}+\frac{\partial^2 w_{1,0}}{\partial z^2}\right]\\
&\qquad+\frac{1}{Re}\left[\frac{\partial^2 w_{2,0}}{\partial x^2}+\frac{\partial^2 w_{1,0}}{\partial y^2}+\frac{\partial^2 w_{1,0}}{\partial z^2}\right]+\frac{Lg}{U_\infty^2}(p_1-\rho_1)\\
&\frac{k(\gamma-1)}{\gamma\mu Re}\left[\frac{\partial^2 T_2}{\partial x^2}+\frac{\partial^2 T_1}{\partial y^2}+\frac{\partial^2 T_1}{\partial z^2}\right]\\
&\quad+\frac{\gamma-1}{Re}\left[2\left(\frac{\partial u_{1,0}}{\partial x}\right)^2+\left(\frac{\partial v_{1,0}}{\partial x}\right)^2+\left(\frac{\partial w_{1,0}}{\partial x}\right)^2\right]+\frac{\gamma-1}{\mu Re}\left(\frac{\partial u_{1,0}}{\partial x}\right)^2=0\\
&p_1=\rho_1+T_1.
\end{aligned}\tag{1.15}$$

A similar calculus leads us to the system corresponding to the third and fourth order according with $\overline{\varepsilon}$. We shall present here only the first two equations at order $\overline{\varepsilon}^3$

obtained from equations (1.4) and (1.5):

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial}{\partial x}\left(\rho_1 u_{1,0} + \rho_2 + u_{2,0}\right) + \frac{\partial v_{1,1}}{\partial y} + \frac{\partial w_{1,1}}{\partial z} + Ld_\infty w_{1,1} = 0$$

$$\frac{\partial u_{1,0}}{\partial t} + (\rho_1 + u_{1,0})\frac{\partial u_{1,0}}{\partial x} + \frac{\partial u_{2,0}}{\partial x} = -\frac{1}{\gamma}\left(\frac{\partial p_2}{\partial x} - \alpha\frac{\partial p_1}{\partial x}\right)$$
$$+\frac{1}{Re}\left(\frac{\lambda}{\mu}+1\right)\left[\frac{\partial^2 u_{2,1}}{\partial x^2} + \frac{\partial^2 v_{2,0}}{\partial x\partial y} + \frac{\partial^2 w_{2,0}}{\partial x\partial z}\right] + \frac{1}{Re}\left[\frac{\partial^2 u_{2,1}}{\partial x^2} + \frac{\partial^2 u_{1,1}}{\partial y^2} + \frac{\partial^2 u_{1,1}}{\partial z^2}\right]$$

A first study of these equations which have unknowns that explain the first approximation of the solution yields to

$$\rho_1 + u_{1,0} = F,\ u_{1,0} + \frac{1}{\gamma}p_1 = G,\ \frac{\partial v_{1,1}}{\partial y} + \frac{\partial w_{1,1}}{\partial z} + Ld_\infty\left(\frac{\lambda}{\mu}+2\right)w_{1,0} = f, \quad (1.16)$$

where F, G and f are unknown functions depending on initial and boundary conditions independently of the direction of the motion x.

Using these equations, we obtain the speed equation

$$2\frac{\partial u_{1,0}}{\partial t} + \left[\alpha + F - \gamma G + (1+\gamma u_{1,0}\right]\frac{\partial u_{1,0}}{\partial x} - \frac{\partial v_{1,1}}{\partial y} - \frac{\partial w_{1,1}}{\partial z} - Ld_\infty w_{1,1} =$$
$$= \frac{1}{Re}\left(\frac{\lambda}{\mu} - 2 - 2\gamma\right)\frac{\partial u_{1,0}}{\partial x}\cdot\frac{\partial u_{1,1}}{\partial x} - \frac{1}{Re}\left(\frac{\partial^2 v_{2,0}}{\partial x^2} + \frac{\partial^2 w_{2,0}}{\partial x\partial z}\right) - \quad (1.17)$$
$$-\frac{Ld_\infty}{Re}\left(\frac{\lambda}{\mu}+2\right)\frac{\partial w_{2,0}}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u_{1,1}}{\partial y^2} + \frac{\partial^2 u_{1,1}}{\partial y^2}\right) + \frac{\gamma-1}{\mu Re}Ld_\infty w_{1,0} + \frac{\partial G}{\partial t}.$$

We shall consider the hypothesis of having a second order vorticity. We shall suppose that the transonic flow produce a vorticity of order two according with ε, with $\lambda + 2\mu \neq 0$. We shall use the supplementary relations in the speed equation (1.17) that becomes

$$2\frac{\partial u_{1,0}}{\partial t} - \frac{\partial v_{1,1}}{\partial y} - \frac{\partial w_{1,1}}{\partial z} - Ld_\infty w_{1,1} = \frac{\partial G}{\partial t} -$$
$$-\frac{Ld_\infty}{Re}\left(\frac{\lambda}{\mu}+2\right)\frac{\partial u_{1,1}}{\partial z} + \frac{\gamma-1}{\mu Re}Ld_\infty w_{1,0}. \quad (1.18)$$

4. Existence and local unicity of a weak shock wave due to a discontinuity of the pressure

It was supposed that due to a small discontinuity of the pressure, appears a weak shock wave. The equation of the surface is: $\mathcal{S} : x^* = h(y^*, z^*, t^*)$ or $\mathcal{F} : x^* - h(y^*, z^*, t^*) = 0$. The jump on the surface of a function φ is $[\varphi] = \varphi^+ - \varphi^-$ where $\varphi^+ = \varphi\,|_{(x,y,z)\in S_t^+}$ and $\varphi^- = \varphi\,|_{(x,y,z)\in S_t^-}$.

We shall write the jump conditions from the equations (1.17) and we shall use the second order vorticity hypothesis to find:

$$[F]=[G]=0; \quad [v_{1,0}]=[w_{1,0}]=0; \quad [w_{1,1}]=-[u_{1,0}]h,_z\,; \quad [v_{1,1}]=-[u_{1,0}]h,_y\,;$$
$$[w_{1,1}]h,_y=[v_{1,1}]h,_z\,; \quad [w_{2,0}]=-[u_{1,1}]h,_z\,; \quad [v_{2,0}]=-[u_{1,1}]h,_y$$

and from the speed equation (1.18) one could obtain

$$[u_{1,0}]\left(2h,_t+(h,_y)^2+(h,_z)^2\right)=\frac{Ld_\infty}{Re}\left(\frac{\lambda}{\mu}+2\right)[u_{1,1}]h,_z\,, \tag{1.19}$$

where $h,_t$, $h,_y$, $h,_z$ are the derivatives of h with respect to t, y and z. We obtain also $[u_{1,1}]=0$ and, as long as $[u_{1,0}]\neq 0$, we find the equation of propagation of the shock wave,

$$2h,_t+(h,_y)^2+(h,_z)^2=0. \tag{1.20}$$

With the initial conditions:

$$h(0,y,z)=g(y,z),\ g(y_0,z_0)=x_0,\ x_0>0,\ (y,z)\in D,\ \ x_0>0,$$

we determine the solution of problem for the surface wave,

$$\begin{aligned}&\mathcal{F}: x-h(t,r,\theta)=0, t>0, r>0, \theta\in[0,\pi],\\ &h(t,r,\theta)=\frac{r^2}{4(t+C_0)}\left(1+\cos(2\theta+C)\right),\ \cos C\neq -1,\end{aligned} \tag{1.21}$$

uniquely determined by the constant C.

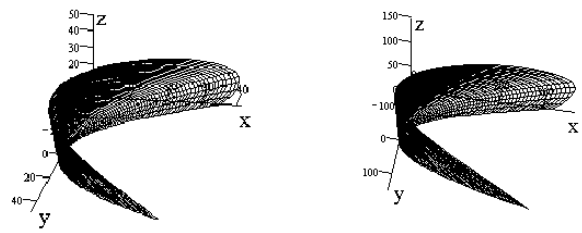
5. Representations of the shock surface

For $t=0$, $\mathcal{S}\equiv\mathcal{S}_0$ will contain the point $(x_0,y_0,0)$, with the conditions: $x_0>0$, $\cos C\neq -1$, $y_0\neq 0$ and $H(1-\cos C)>2\sin C\, y_0$, where H is the height where flies the object which creates the pressure discontinuity. The equation of $\mathcal{S}_0$ with $C_0=\frac{y_0^2(1+\cos C)}{4x_0}$ is

$$x=\left[y^2+z^2+(y^2-z^2)\cos C-2yz\sin C\right]/4C_0.$$

Note that $\mathcal{S}\equiv\mathcal{S}_1$ will contain the point at the time t_1, which will be assumed initial moment,

$$t_1=\left[H-\frac{y_0(1+\cos C)}{\sin C}\right]^2\cdot\frac{1-\cos C}{4x_0}-C_0.$$



Discontinuity surface at initial time, $H = 50$, respectively $H = 150$

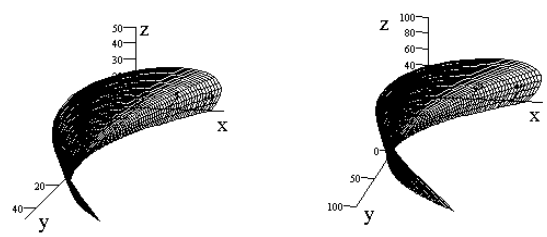
For graphical representations we shall consider the particular case

$$C_0 = \pi/4,\ x_0 = H,\ y_0 = H(\sqrt{2}-1)/4.$$

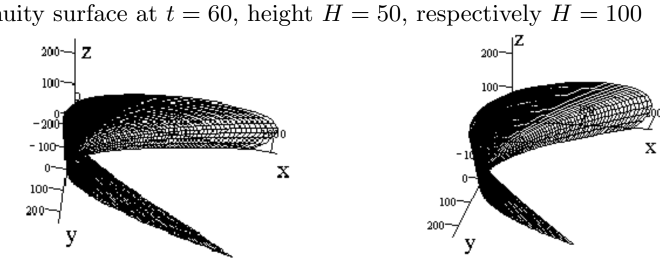
With the notation $\delta = t_1 + C_0$, the equation of the shock surface at time t is

$$\mathcal{S}_t : x - \frac{1}{4(\delta + t)}\left[y^2(1+\sqrt{2}/2) + z^2(1-\sqrt{2}/2) - yz\sqrt{2}/2\right] = 0. \qquad (1.22)$$

At the moment t_1 with $H = 50$, respectively $H = 150$ we find the representations illustrated in Fig.2. At $t = 60$; $H = 50$, and $H = 250$ respectively, we represent the shock surface in Fig.3. In Fig.4 we shall present the shape of the shock surface at height $H = 250$ for initial moment and the evolution of the surface at $t = 120$.



Discontinuity surface at $t = 60$, height $H = 50$, respectively $H = 100$



Time evolution of the discontinuity surface at height $H = 250$

References

[1] T.E. Giddings, J.Fish, *A transonic small-disturbance model for the propagation of weak shock waves in heterogeneous gases*, J.Fluid Mech. 429 (2001), 255-280.

[2] G.B. Witham, *On the propagation of weak shock waves*, J.Fluid Mech. 1 (1956), 290-318.

[3] P.L.Sachdev, R. Seebas, *Propagation of spherical and cylindrical N-waves*, J.Fluid Mech. 58 (1973), 1, 197-205.

[4] I.V.Pescaru, V. Ţigoiu, *Elements of Thermodynamics and Atmospherical Dynamics* (in Romanian), University of Bucharest Editors, 1997.

[5] I. Roşca, *Partial Differential Equations* (in Romanian), University of Bucharest Editors, 1997.

Elena Corina Cipu
University Politehnica of Bucharest, Department Mathematics III
Splaiul Independenţei 313, RO-77206 Bucharest, Romania
Email address: corinac@math.math.unibuc.ro