Optimal receiver of solar power station

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Abstract. This paper applies the geometric dynamics to describe optimal shape of structures used to couple a transmitter or receiver to a medium in which waves can propagate.
Section 1 recalls the laws of regular reflection. Section 2 analyses the optimal shape receiver for classical reflection on a spherical mirror. Section 3 describes the optic geometric dynamics using an optic vector field and an adapted Fermat principle. Section 4 shows that a mirror of revolution and a nonholonomic mirror (collection of wires) accept holonomic optimal receivers. Section 5 shows that Tzitzeica mirror and canonical nonholonomic mirror (collection of wires) accept only nonholonomic optimal receivers (collection of wires). Section 6 visualizes the abstract objects and processes that are connected directly with the theory developed in Sections 2-5, with emphasis on optic geometric dynamics, using MAPLE 6-7 routines.

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Key words: optic geometric dynamics, wave-guides, optimal receiver.

1 Preliminaries

By reflection we understand a deviation of the direction of radiant flux taking place entirely within or at the surface (mirror) of a single optical medium. Interfaces, between two media, belonging to class C^1 produce regular reflections. In order to apply geometric dynamics to optical problems we need class C^2.

Laws of Regular Reflection. The incident and reflected rays are:
(1) in a normal plane to the surface;
(2) on the same side to the surface;
(3) at congruent angles with the normal.

If we denote by $\vec{s}$ the incident ray (versor), by $\vec{n}$ the unit normal vector to the mirror and by $\vec{r}$ the reflected ray (versor), then

$$\vec{r} = \vec{s} - 2(\vec{s}, \vec{n})\vec{n}.$$

Of course, $\vec{r}$ is a versor field on the mirror. Traditionally, we accept that the paths of the reflected light rays are semi-striaght lines starting on the mirror in the direction $\vec{r}$. This point of view separates the mirrors into two classes: planar mirror who produces parallel reflected rays and non-planar mirrors who produce non-parallel reflected rays. Generally, to build an optimal shape receiver associated to a non-planar mirror is

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strongly dependent on the shape of the mirror. This problem has simple solutions only for particular mirrors (example, parabolic mirror).

To avoid such problems, we reformulate the Fermat principle saying that *the light rays are field lines, starting on the mirror, of a vector field who extend $\vec{r}$ to a domain in space*. These field lines did not intersect. Also, some practical mirrors produce practical optimal shape receivers.

All our theory can be applied to any structure (antenna) used to couple a transmitter or receiver to a medium in which electromagnetic waves can propagate. These structures range in form from simple straight lines, used either singly or in arrays, to quasioptical devices employing mettalic mirrors or dielectric lenses. Particularly, optimal multiple wire antennas must be part of a nonholonomic surface.

2 Optimal Receiver for Classical Reflection on a Spherical Mirror

A spherical mirror has an axial symmetry. For that reason it is enough to judge in the plane xOy. Then the mirror is a part of the circle

$$x^2 + y^2 = R^2, \quad x \geq \frac{R}{\sqrt{2}}.$$

and the family of reflected rays (straight lines) is

$$\frac{x - R\cos t}{\cos 2t} = \frac{y - R\sin t}{\sin 2t}.$$

Reasons regarding the cosinus effect on the intensity of solar radiation show that the optimal shape receiver must be selected using as profiles the orthogonal trajectories of this family of straight lines. Such curves are described by the algebraic differential system

$$\begin{cases} \dfrac{x - R\cos t}{\cos 2t} = \dfrac{y - R\sin t}{\sin 2t} \\ \dfrac{1}{\cos 2t} = -\dfrac{1}{y'(x)\sin 2t} \end{cases}$$

We look for solutions of the type

$$\begin{cases} x(t) = \varphi(t)\cos 2t + R\cos t \\ y(t) = \varphi(t)\sin 2t + R\sin t \\ z'(t)\cos 2t + y'(t)\sin 2t = 0, \end{cases}$$

with the unknown function φ. We find,

$$\varphi'(t) + R\sin t = 0$$

and hence

$$\varphi(t) = R(C + \cos t).$$

Finally, the orthogonal trajectories is the family of curves

$$\left.\begin{aligned} \frac{x(t)}{R} &= (c + \cos t)\cos 2t + \cos t \\ \frac{y(t)}{R} &= (c + \cos t)\sin 2t + \sin t \end{aligned}\right., \quad t \in \left[0, \frac{\pi}{4}\right]$$

Since

$$\begin{cases} x(-t) = x(t) \\ y(-t) = -y(t) \end{cases}$$

each curve in this family is symmetric with respect to Ox.

The significant points of these curves are

$$x(0) = R(c+2), \quad y(0) = 0$$

$$x\left(\pm\frac{\pi}{4}\right) = \frac{R}{\sqrt{2}}, \quad y\left(\pm\frac{\pi}{4}\right) = \pm R(c + \sqrt{2})$$

To obtain the shape of orthogonal trajectories we need the derivatives

$$\left.\begin{aligned} \frac{x'(t)}{R} &= -(2c + 3\cos t)\sin 2t \\ \frac{y'(t)}{R} &= (2c + 3\cos t)\cos 2t \end{aligned}\right., \quad t \in \left[0, \frac{\pi}{4}\right].$$

If $|c| \le \frac{3}{2}$, then $x'(t) = y'(t) = 0$ for $t_* = \pm \arccos\left(-\frac{2c}{3}\right)$. Also $t_* \in \left(0, \frac{\pi}{4}\right)$ iff $c \in \left(-\frac{3}{2}, -\frac{3}{2\sqrt{2}}\right)$. Then is maximum point for y and minimum point for x. We have $\cos t = -\frac{2C}{3}$, whence $c + \cos t = -\frac{1}{2}\cos t = \frac{c}{3}$; $\cos 2t = \frac{8c^2}{9} - 1$; $\sin t = \pm\sqrt{1 - \frac{4c^2}{9}}$; $\sin 2t = \pm\frac{4c}{3}\sqrt{1 - \frac{4c^2}{9}}$. We obtain

$$x(t_*) = \left(\frac{8c^3}{27} - c\right)R; \quad y(t_*) = \pm\left(1 - \frac{4c^2}{9}\right)^{\frac{3}{2}} R$$

and the table of variation

t	0		t_*	$()$	$\pi/4$
$x'(t)$	0	$-$	0	$+$	
$x(t)$	$R(c+2)$	$\searrow$	$x(t_*)$	$\nearrow$	$R/52$
$y'(t)$		$+$	0	$-$	0
$y(t)$	0	$\nearrow$	$y(t_*)$	$\searrow$	$R(c+\sqrt{2})$

The length of the receiver curve as function of $c \in \left(-\frac{3}{2}, -\frac{3}{2\sqrt{2}}\right)$ is given by

$$l(c) = R\int_0^{\pi/4} |2c + 3\cos t| dt.$$

Since

$$l'(c) = 4\arccos\left(-\frac{2c}{3}\right) - \frac{\pi}{2} = 0,$$

has the solution

$$c_1 = -\frac{3}{4}\sqrt{2+\sqrt{2}} = -1,385819299,$$

with $l''(c_1) > 0$, the value c is a minimum point for the length.

Implicitly, it appears another idea to select the optimal profile as the curve corresponding to the value c_1.

On the other hand, for a spatial spherical mirror, the surfaces that are orthogonal to relected rays are obtained by the rotation around Ox of the previous curves. The best receiver is the surface of minimum area,

$$S(c) = 2\pi R^2 \int_0^{\pi/4} [(c + \cos t)\sin 2t + \sin t]|2c + 3\cos t| dt.$$

Using MAPLE 7 routines we find:
int(((c+cos(t))*sin(2*t)+sin(t))*(2*c+3*cos(t)), t);
int(((c+cos(t))*sin(2*t)+sin(t))*(2*c+3*cos(t)) , t=0..Pi/4);
$minimize(-11/6 * c * sqrt(2) + 15/8 + c^2 + 16/3 * c, c = -2..2, location = true)$;
$-889/72 - 11/6 * (11/12 * sqrt(2) - 8/3) * sqrt(2) + (11/12 * sqrt(2) - 8/3)^2 + 44/9 * sqrt(2), \{[\{c = 11/12 * sqrt(2) - 8/3\}, -889/72 - 11/6 * (11/12 * sqrt(2) - 8/3) * sqrt(2) + (11/12 * sqrt(2) - 8/3)^2 + 44/9 * sqrt(2)]\}$;
evalf(11/12*sqrt(2)-8/3);
c1 = -1.370304235.

3 Optic Geometric Dynamics

Let us consider a mirror σ like a part of a holonomic surface in R^3 of class C^2 whose unit normal vector field is $\vec{n}$. If $\vec{s}$ is the versor of incident ray, then

$$\vec{r} = \vec{s} - 2(\vec{s}, \vec{n})\vec{n}$$

is the versor of reflected ray. Then $\vec{r}$ appears like a vector field on σ. Any vector field $\vec{U}(X, Y, Z)$ of class C^1 extending $\vec{r}$ to a domain of R^3, in the sense $\vec{U}|_\sigma = \lambda_\sigma \vec{r}$, is called *optic vector field*. The field lines of $\vec{U}$ are described by the first order differential system

$$\frac{dx}{dt} = X(x, y, z), \quad \frac{dy}{dt} = Y(x, y, z), \quad \frac{dz}{dt} = Z(x, y, z).$$

This system and the Riemannian (Euclidean) structure δ_{ij} of the space produce the Lagrangian energy density

$$2L = \left(\frac{dx}{dt} - X(x,y,z)\right)^2 + \left(\frac{dy}{dt} - Y(x,y,z)\right)^2 + \left(\frac{dz}{dt} - Z(x,y,z)\right)^2,$$

of least squares type (see [10],[11]). Let us accept that the path of a light ray moving in an inhomogeneous 3-dimensional medium is an extremal of the functional (*adapted Fermat principle*)

$$\int_{t_1}^{t_2} L(x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t))dt.$$

These extremals are solutions of Euler-Lagrange equations

$$\begin{aligned}
\frac{d^2x}{dt^2} &= \left(\frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x}\right)\frac{dy}{dt} + \left(\frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x}\right)\frac{dz}{dt} + \frac{\partial f}{\partial x}\\
\frac{d^2y}{dt^2} &= \left(\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y}\right)\frac{dx}{dt} + \left(\frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}\right)\frac{dz}{dt} + \frac{\partial f}{\partial y}\\
\frac{d^2z}{dt^2} &= \left(\frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z}\right)\frac{dx}{dt} + \left(\frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z}\right)\frac{dy}{dt} + \frac{\partial f}{\partial z},
\end{aligned}$$

where

$$2f = X^2 + Y^2 + Z^2$$

is the *optic density of energy*.

Every nonconstant trajectory of this dynamical system which has the constant total energy

$$2\mathcal{H} = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 - f$$

is a reparametrized geodesic of the Riemann-Jacobi-Lagrange manifold

$$(D \setminus \mathcal{E},\ g_{ij} = (\mathcal{H} + f)\delta_{ij},\ N^i_j = \Gamma^i_{jk}y^k - (\mathrm{rot}\vec{U})^i_j)$$

$$(\mathrm{rot}\vec{U})^i_j = \delta^{ih}(\mathrm{rot}\vec{U})_{jh}, \quad i,j,h = 1,2,3$$

$$\mathcal{E} = \text{the set of zeros of } \vec{U}.$$

The set of all trajctories splits in three parts: optic lines (for $\mathcal{H} = 0$), trajectories with positive energy (for $\mathcal{H} = \text{const} > 0$), trajectories with negative energy (for $\mathcal{H} = \text{const} < 0$).

A similar theory can be developed when the mirror σ is a part of a nonholonomic surface (collection of curves) in R^3 whose unit normal vector field

$$\vec{n} = (n_1(x,y,z), n_2(x,y,z), n_3(x,y,z))$$

is of class C^1 and determines the Pfaff equation

$$n_1(x,y,z)dx + n_2(x,y,z)dy + n_3(x,y,z)dz = 0.$$

In this case

$$\vec{U}(x,y,z) = \lambda(x,y,z)\vec{r}(x,y,z),\ \vec{r} = \vec{s} - 2(\vec{s},\vec{n})\vec{n}.$$

The previous extremals are trajectories of a geodesic motion in a gyroscopic field [10], [11], i.e., a motion produced by ∇f and rot$\vec{U}$.

4 Holonomic Optimal Receiver Mirror of Revolution Nonholonomic Ox-Symmetric Mirror

We accept that the paths of light rays are field lines of the optic vector field $\vec{U}(X,Y,Z)$ that start on the (holonomic or nonholonomic) reflectant surface (mirror). As field lines starting from different points, these paths did not intersect (compare with the situation of a plane mirror when the reflected rays are semi-straight lines).

The cosinus effect on the intensity of solar radiation imposes that the optimal shape of the receiver in a Solar Power Station is a (holonomic) surface where the reflected rays (field lines of optic vector field $\vec{U}$) fall down perpendicular (see [2]). According to known theory [10], such a surface (orthogonal to field lines) exists iff the optic vector field is biscalar, i.e., $(\vec{U}, \text{rot}\vec{U}) = 0$ or $\vec{U} = \lambda \text{grad}\, F$. In this case the Pfaff equation

$$X(x,y,z)dx + Y(x,y,z)dy + Z(x,y,z)dz = 0$$

has a general solution

$$F(x,y,z) = c$$

consisting from a family with one parameter of (holonomic) surfaces orthogonal to field lines of $\vec{U}$.

Let us show that any reflectant surface of revolution produces a biscalar optic vector field. Any surface of revolution has an implicit Cartesian equation of the form

$$y^2 + z^2 - 2h(x) = 0,$$

with h of class C^2. Consequently

$$\vec{n} = \frac{(-h', y, z)}{\sqrt{h'^2 + 2h}},$$

and, without loss of generality, we can accept $\vec{s} = (1,0,0)$. These produce the reflected ray

$$\vec{r} = \left(1 - \frac{2h'^2}{h'^2 + 2h}, \frac{2h'y}{h'^2 + 2h}, \frac{2h'z}{h'^2 + 2h}\right)$$

and by extension to a domain of R^3 we obtain the optic vector field

$$\vec{U} = \left(1 - \frac{2h'^2}{h'^2 + 2h}, \frac{2h'y}{h'^2 + 2h}, \frac{2h'z}{h'^2 + 2h}\right)$$

Since two collinear vector fields have the same orbits (images of field lines) and the same family of surfaces or family of curves orthogonal to orbits, we replace $\vec{U}$ by the potential vector field

$$X(x, y, z) = \frac{2h(x) - h'^2(x)}{2h'(x)},\ Y(x, y, z) = y,\ Z(x, y, z) = z.$$

In this case the differential system

$$\frac{2h'(x)dx}{2h(x) - h'^2(x)} = \frac{dy}{y} = \frac{dz}{z}$$

has the general solution (field lines, reflected rays)

$$y^2 = C_1 \exp\left(\int \frac{h'(x)dx}{2h(x) - h'^2(x)}\right), \quad z^2 = C_2 \exp\left(\int \frac{h'(x)dx}{2h(x) - h'^2(x)}\right).$$

Also the Pfaff equation

$$\frac{2h(x) - h'^2(x)}{2h'^2(x)} dx + ydy + zdz = 0$$

has the general solution (family of surfaces of revolution)

$$y^2 + z^2 + \int \frac{2h(x) - h'^2(x)}{h'^2(x)} dx = c.$$

Giving a suitable point, we find the equation of the optimal shape receiver.

Particularly, for a sphere of radius R with the center at origin, we have:

- function h,

$$h(x) = \frac{R^2 - x^2}{2};$$

- reflected vector field,

$$\vec{r} = \left(1 - \frac{2x^2}{R^2}, \frac{-2xy}{R^2}, \frac{-2xz}{R^2}\right);$$

- optic vector field,

$$X(x, y, z) = \frac{R^2 - 2x^2}{-2x},\ Y(x, y, z) = y,\ Z(x, y, z) = z;$$

- paths of reflected light rays (field lines),

$$2x^2 - c_1 y^2 = R^2,\ 2x^2 - c_2 z^2 = R^2;$$

- family of optimal shape receivers,

$$R^2 \ln \frac{x}{R} - x^2 - y^2 - z^2 = c_3.$$

Using the canonical parametrization of the sphere, and imposing the values of angles to obtain a true mirror, we find $c_1 < 0$, $c_2 < 0$. Consequently the path of reflected light rays are intersections of two elliptical cylinders.

The optimal shape receiver is a surface of revolution obtained by revolving the curve

$$y^2 + x^2 - R^2 \ln \frac{x}{R} = c$$

around Ox. This surface can be seen like a "cylindrical" surface with axis Ox and variable radius

$$R_c = \sqrt{y^2 + z^2} = \sqrt{c + R^2 \ln \frac{x}{R} - x^2}$$

The constant c is determined by the condition [2]

$$R_c(x_0) = 8.10^{-3} R,$$

where $x_0 = R/\sqrt{2}$ is the critical point of R_c (maximum point of R_c). It follows

$$\frac{R_c}{R} = \sqrt{\ln \frac{x}{R} - \frac{x^2}{R^2} + 1.653564}$$

Taken $\frac{x}{R} \in [0.55, 0.80]$, Fig.2 gives the graph of the function $\frac{R_c}{R}$ and Fig.3 describes the shape of the Receiver.

Let us show that the nonholonomic mirror $ydz - zdy + \sqrt{y^2 + z^2}dx = 0$ accepts holonomic optimal receiver. Of course this mirror is symmetric with respect to Ox since the symmetry $y \to -y$, $z \to -z$ does not modify the previous Pfaff equation. We find

$$\vec{s} = (1, 0, 0), \quad \vec{n} = \left(\frac{1}{\sqrt{2}}, \frac{-z}{\sqrt{2(y^2 + z^2)}}, \frac{y}{\sqrt{2(y^2 + z^2)}} \right)$$

$$\vec{r} = \left(0, \frac{z}{y^2 + z^2}, \frac{-y}{y^2 + z^2} \right)$$

$$X(x, y, z) = 0,\ Y(x, y, z) = z,\ Z(x, y, z) = -y.$$

Consequently:

- paths of reflected light rays (field lines),

$$x = c_1,\ y^2 + z^2 = c_2;$$

- family of optimal receivers,

$$zdy - ydz = 0 \quad \text{or} \quad y = c_3 z \quad \text{(family of planes)}$$

5 Nonholonomic Optimal Receiver Tzitzeica Mirror Canonical Nonholonomic Mirror

Suppose the optical vector field $\vec{U}$ is not a biscalar field, i.e., $(\vec{U}, \mathrm{rot}\vec{U}) \neq 0$. Then the Pfaff equation

$$X(x,y,z)dx + Y(x,y,z)dy + Z(x,y,z)dz = 0$$

defines a nonholonomic surface (collection of curves orthogonal to field lines). Consequently, there are reflectant (holonomic or nonholonomic) surfaces whose associated optimal receiver is a nonholonomic surface.

An example is the *Tzitzeica mirror* of equation $xyz = 1$. Indeed, in this case

$$\vec{s} = (1,0,0),\ \vec{n} = \frac{(yz, xz, xy)}{\sqrt{y^2z^2 + x^2z^2 + x^2y^2}},$$

$$\vec{r} = \left(1 - \frac{2y^2z^2}{y^2z^2 + x^2z^2 + x^2y^2}, \frac{-2z}{y^2z^2 + x^2z^2 + x^2y^2}, \frac{-2y}{y^2z^2 + x^2z^2 + x^2y^2}\right),$$

$$X(x,y,z) = x^2(y^2 + z^2) - \frac{1}{x^2},\ Y(x,y,z) = -2z,\ Z(x,y,z) = -2y.$$

$$\mathrm{rot}(X,Y,Z) = \left(0, \frac{2}{y} + \frac{2x}{z^3}, \frac{-2}{y} - \frac{2x^2}{y^3}\right).$$

The Pfaff equation

$$\left(x^2(y^2 + z^2) - \frac{1}{x^2}\right)dx - 2zdy - 2ydz = 0$$

has as solutions only curves, and their family (nonholonomic surface) is the optimal receiver.

Let us show that the canonical nonholonomic mirror $zdx - dy = 0$ produces a non-biscalar optic vector field. It follows

$$\vec{s} = (1,0,0), \quad \vec{n} = \left(\frac{z}{\sqrt{1+z^2}}, \frac{-1}{\sqrt{1+z^2}}, 0\right)$$

$$\vec{r} = \left(\frac{1-z^2}{1+z^2}, \frac{2}{1+z^2}, 0\right)$$

$$X(x,y,z) = 1 - z^2,\ Y(x,y,z) = 2,\ Z(x,y,z) = 0$$

$$\mathrm{rot}(X,Y,Z) = (0, -2z, 0).$$

Consequently:

- paths of reflected light rays (field lines),

$$z = c_1,\ 2x + (1 - z^2)y = c_2;$$

- optimal shape receiver, $(1 - z^2)dx + 2dy = 0$ (nonholonomic surface).

6 MAPLE 6 and MAPLE 7 Simulations

$T := plot([(-1.385819299+cos(t))*cos(2*t)+cos(t), (-1.385819299+cos(t))*sin(2*t) + sin(t), t = -Pi/4..Pi/4]);$

$C := implicitplot(x^2 + y^2 = 1, x = 0.4..1, y = -1..1);$

$a6 := implicitplot(x * sin(Pi/3) - y * cos(Pi/3) - sin(Pi/6) = 0, x = 0.4..1, y = -1..1,$
title=‘REFLECTED STRAIGHT LINE‘,color=blue);

$a7 := implicitplot(x * sin(-Pi/3) - y * cos(-Pi/3) - sin(-Pi/6) = 0, x = 0.4..1, y = -1..1,$
title=‘REFLECTED STRAIGHT LINE‘,color=blue);

$a8 := implicitplot(x * sin(Pi/5) - y * cos(Pi/5) - sin(Pi/10) = 0, x = 0.4..1, y = -1..1,$
title=‘REFLECTED STRAIGHT LINE‘,color=blue);

$a9 := implicitplot(x * sin(-Pi/5) - y * cos(-Pi/5) - sin(-Pi/10) = 0, x = 0.4..1, y = -1..1,$
title=‘REFLECTED STRAIGHT LINE‘,color=blue);

$a10 := implicitplot(y - sin(-Pi/10) = 0, x = 0.4..1, y = -1..1,$
title=‘INCIDENT STRAIGHT LINE‘,color=black);

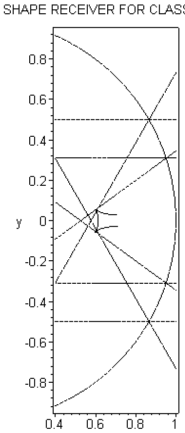
$a11 := implicitplot(y + sin(-Pi/10) = 0, x = 0.4..1, y = -1..1,$
title=‘INCIDENT STRAIGHT LINE‘,color=black);

$a12 := implicitplot(y - sin(-Pi/6) = 0, x = 0.4..1, y = -1..1,$
title=‘INCIDENT STRAIGHT LINE‘,color=black);

$a13 := implicitplot(y - sin(Pi/6) = 0, x = 0.4..1, y = -1..1,$
title=‘INCIDENT STRAIGHT LINE‘,color=black);

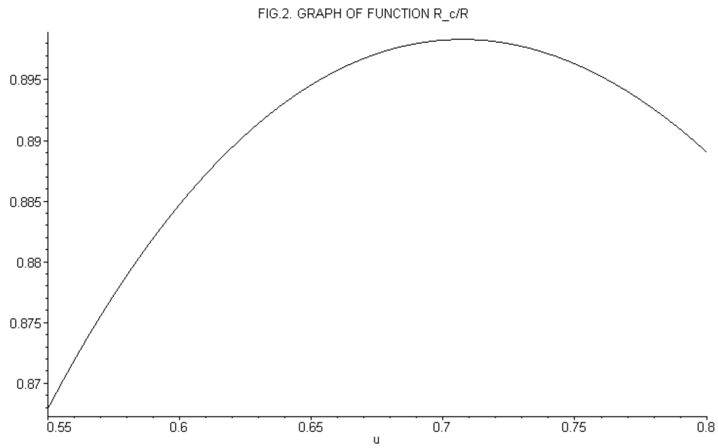
display({T,C,a6,a7,a8,a9,a10,a11,a12,a13},axes=boxed,scaling=constrained,
title=‘FIG.1.OPTIMAL SHAPE RECEIVER FOR CLASSICAL REFLECTION‘);

FIG.1.OPTIMAL SHAPE RECEIVER FOR CLASSICAL REFLECTION

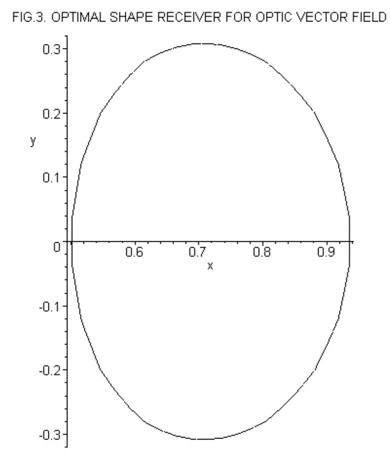


$plot(sqrt(ln(u) - u^2 + 1.653564), u = 0.55..0.80,$

$title = `FIG.2.GRAPHOFFUNCTIONR_c/R`);$



$implicitplot(ln(x) - x^2 - y^2 + ln(2) + 1/4 = 0, x = 0.4..1, y = -1..1,$
title=‘FIG.3. OPTIMAL SHAPE RECEIVER FOR OPTIC VECTOR FIELD‘);



implicitplot3d(x*sin(2*t)-y*cos(2*t)-sin(t)=0,x=0..1,
y=-1/sqrt(2)..1/sqrt(2),t=-Pi/4..Pi/4,grid=[30,30,30],
title=‘FIG.4. SURFACE SWEPT OUT FAMILY OF STRAIGHT LINES‘);
$implicitplot3d(2 * (x^2) * (sin(t))^2 - (2 * (cos(t))^2 - 1) * y^2 - (sin(t))^2 = 0, x = 0..1, y = -1/sqrt(2)..1/sqrt(2), t = -Pi/4..Pi/4, grid = [30, 30, 30],$
title=‘FIG.5. SURFACE SWEPT OUT BY FAMILY OF ELLIPSES‘);
$a1 := implicitplot(ln(x) - x^2 - y^2 + ln(2) + 1/4 = 0, x = 0.4..1, y = -1..1,$
title=‘RECEIVER‘,color=red);
$a2 := implicitplot(2*(x^2)*(sin(Pi/6))^2 - (2*(cos(Pi/6))^2 - 1)*y^2 - (sin(Pi/6))^2 = 0, x = 0.4..1, y = -1..1,$

FIG.4. SURFACE SWEPT OUT FAMILY OF STRAIGHT LINES

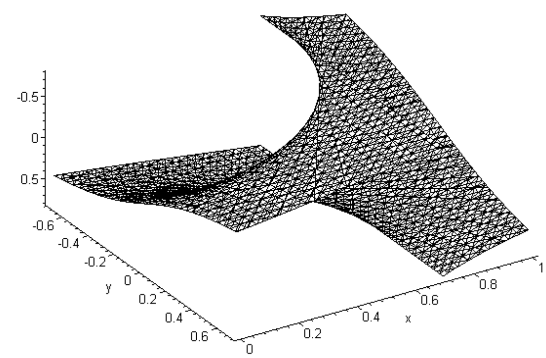
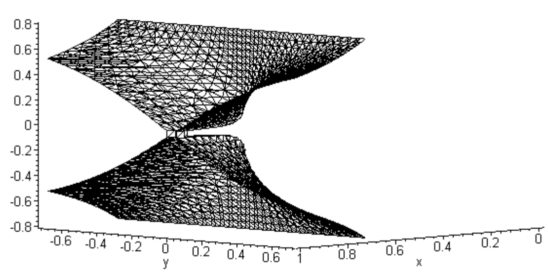


FIG.5. SURFACE SWEPT OUT BY FAMILY OF ELLIPSES



title=‘REFLECTED ELLIPSE‘,color=green);

$a3 := implicitplot(2*(x^2)*(sin(-Pi/6))^2 - (2*(cos(-Pi/6))^2 - 1)*y^2 - (sin(-Pi/6))^2 = 0, x = 0.4..1, y = -1..1,$

title=‘REFLECTED ELLIPSE‘,color=green);

$a4 := implicitplot(2*(x^2)*(sin(Pi/10))^2 - (2*(cos(Pi/10))^2 - 1)*y^2 - (sin(Pi/10))^2 = 0, x = 0.4..1, y = -1..1,$

title=‘REFLECTED ELLIPSE‘,color=green);

$a5 := implicitplot(2*(x^2)*(sin(-Pi/10))^2 - (2*(cos(-Pi/10))^2 - 1)*y^2 - (sin(-Pi/10))^2 = 0, x = 0.4..1, y = -1..1,$

```
title=‘REFLECTED ELLIPSE‘,color=green);
a6:=implicitplot(x*sin(Pi/3)-y*cos(Pi/3)-sin(Pi/6)=0,x=0.4..1,y=-1..1,
title=‘REFLECTED STRAIGHT LINE‘,color=blue);
a7:=implicitplot(x*sin(-Pi/3)-y*cos(-Pi/3)-sin(-Pi/6)=0,x=0.4..1,y=-1 ..1,
title=‘REFLECTED STRAIGHT LINE‘,color=blue);
a8:=implicitplot(x*sin(Pi/5)-y*cos(Pi/5)-sin(Pi/10)=0,x=0.4..1,y=-1.. 1,
title=‘REFLECTED STRAIGHT LINE‘,color=blue);
a9:=implicitplot(x*sin(-Pi/5)-y*cos(-Pi/5)-sin(-Pi/10)=0,x=0.4..1,y=- 1..1,
title=‘REFLECTED STRAIGHT LINE‘,color=blue);
a10:=implicitplot(y-sin(-Pi/10)=0,x=0.4..1,y=-1..1,
title=‘INCID ENT STRAIGHT LINE‘,color=black);
a11:=implicitplot(y+sin(-Pi/10)=0,x=0.4..1,y=-1..1,
title=‘INCID ENT STRAIGHT LINE‘,color=black);
a12:=implicitplot(y-sin(-Pi/6)=0,x=0.4..1,y=-1..1,
title=‘INCIDE NT STRAIGHT LINE‘,color=black);
a13:=implicitplot(y-sin(Pi/6)=0,x=0.4..1,y=-1..1,
title=‘INCIDEN T STRAIGHT LINE‘,color=black);
```

$a14 := implicitplot(x^2 + y^2 = 1, x = 0.4..1, y = -1..1, title = ‘CIRCLE‘);$

```
display({a1,a2,a3,a4,a5,a6,a7,a8,a9,a10,a11,a12,a13,a14 },
axes=boxed,scaling=constrained,
title=‘FIG.6. PROJECTION OF REFLECTION ON SPHERICAL MIRROR
FOR OPTIC VECTOR FIELD‘);
```

$f := (1/2)*(R1^2 + R2^2);$

```
contourplot(f,x=0.4..1,y=-1..1,
title=‘FIG.7.CONSTANT LEVEL SETS OF OPTIC DENSITY OF ENERGY‘);
```

$f := (1/2)*(R1^2 + R2^2);$

```
diff(f,x);
diff(f,y);
with(DEtools):
```

DE1:={diff(x(t),t2) $= diff(f,x)(t), diff(y(t),t2)$=diff(f,y)(t)};

```
DEplot(DE1,[x(t),y(t)],t=-100..100,[[x(0)=cos(Pi/6),y(0)=sin(Pi/6),
D(x)(0)=-1,D(y)(0)=-1],
[x(0)=cos(Pi/6),y(0)=sin(Pi/6),D(x)(0)=-1,D(y)(0) =1]],x=0.1..1,y=-1..1,
```

FIG.6. PROJECTION OF REFLECTION ON SPHERICAL MIRROR FOR OPTIC VECTOR FIELD

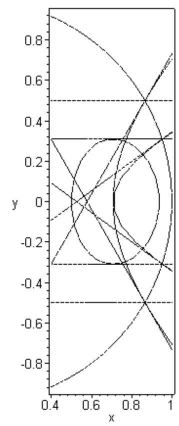
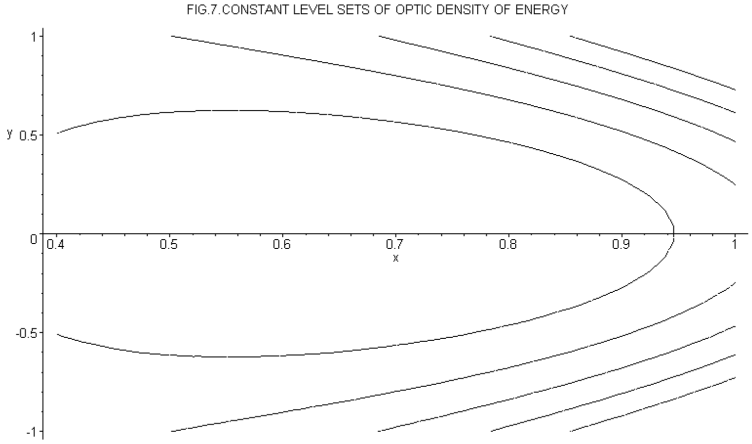
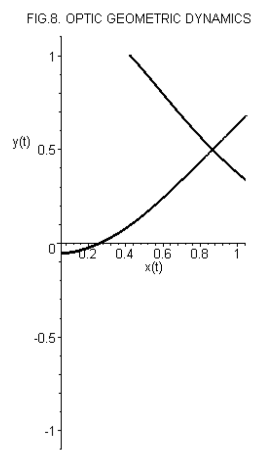


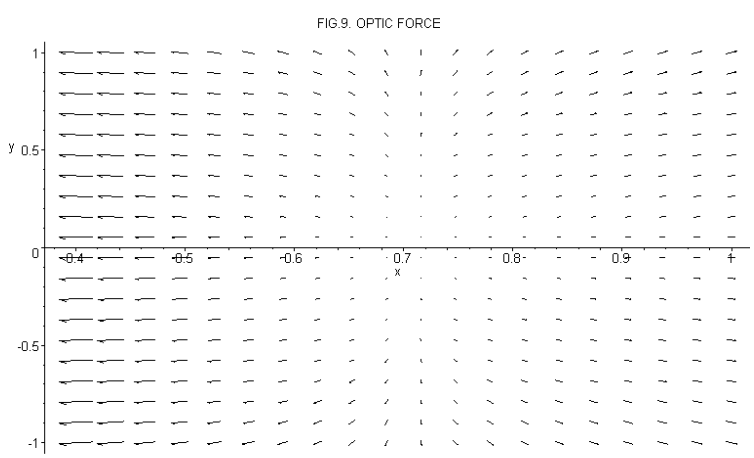
FIG.7.CONSTANT LEVEL SETS OF OPTIC DENSITY OF ENERGY



scene=[x(t),y(t)],obsrange=true,
title=‘FIG.8. OPTIC GEOMETRIC DYNAMICS‘,
linecolor=red,stepsize=.00001,iterations=1000);

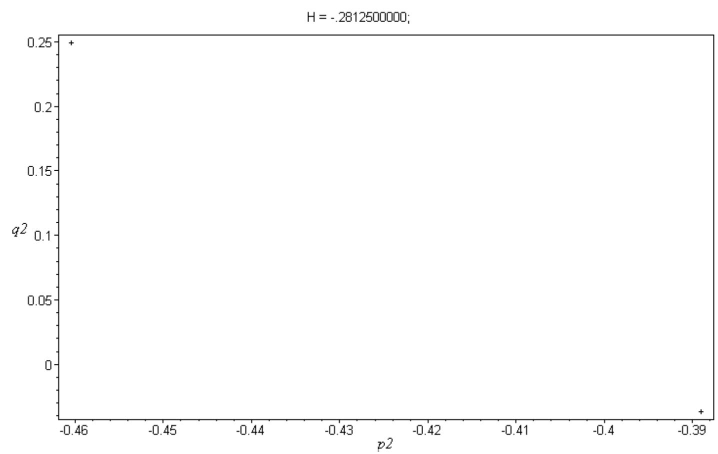


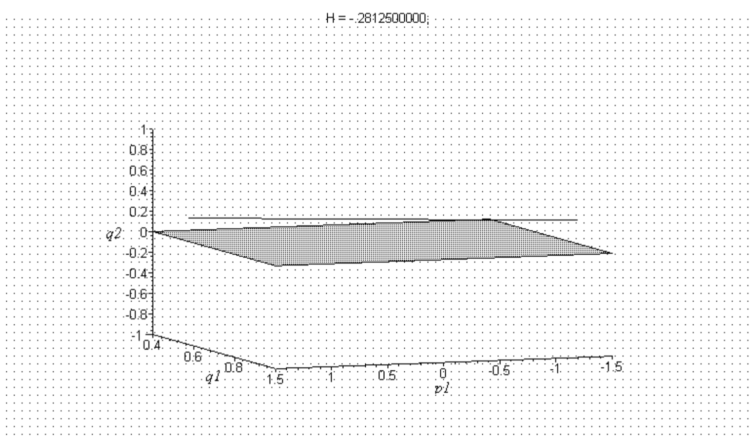
with(plots): fieldplot([diff(f,x),diff(f,y)],x=0.4..1,y=-1..1,
title=‘FIG.9. OPTIC FORCE‘);



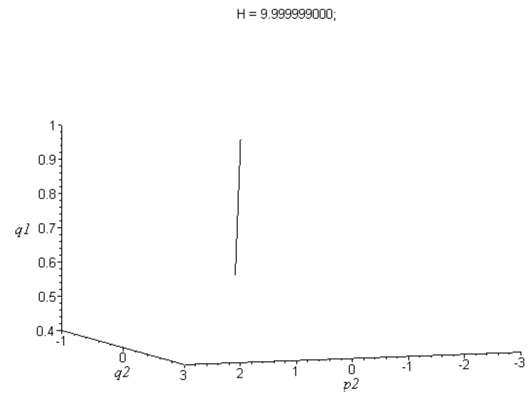
R1:=q1-0.5*(1/q1); R2:=q2; $f := (1/2) * (R1^2 + R2^2)$;
with(DEtools):
$H := 1/2 * (p1^2 + p2^2) - f$; $hamilton_e qs(H)$;
H, t=-100..100, {[-0.1,.1,-0.4,0.4,0.1]}:
poincare(%, stepsize=.05,iterations=5);

poincare(%%,stepsize=.05,iterations=5,scene=[p2,q2]);
F2 := poincare(H,t=-100..100, {[0,.1,-0.4,0.4,0.1]
},stepsize=.1,iterations=4,
scene=[p1=-1.5..1.5,q1=0.4..1,q2=-1..1],3):
F2;
$ics_10 := generate_ic(H, \{t = 0, p2 = 1, q1 = 0.6, q2 = 0, energy = 10\}, 1)$:
$F3b := poincare(H, t = 0..20, ics_10, stepsize = .01, iterations = 4,$
scene=[p2,q2,q1],3): (See Figs. 10, 11, 12)





Conclusions. 1) The reformulation of Fermat principle as optic geometric dynamics justifies and clarifies better the optic phenomena. Since this reformulation means a Lorentz-Udrişte World-Force Law 10], [11], this theory can be applied to any emission and capture of "signals".



2) The optimal structure used to couple a transmitter or receiver to a medium in which wawes can propagate can be realized as apart of a holonomic surface or as a part of a nonholonomic surface (collection of wires).

Aknowledgments

Much of our understanding of optimal receiver requirements is a result of learning the theory of nonholonomic manifolds, produced by Gh. Vrănceanu (1900-1979), and of collaborations with Traian Creţu (1937-1995). Timely pointers from colleagues in the Department of Mathematics and Department of Physics, are also grateffuly aknowledged.

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