Numeric simulation of some stationary electric dynamical systems

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Abstract. As is well known, the basic equation of electrostatics is the Laplace equation $\text{div}\,(\varphi\,\text{grad}\,V) = -\rho$. This determines the electrostatic potential V, from which the electric field is computed via $E = -\,\text{grad}\,V$. On the other hand, the stationary electric field together with a suitable geometrical structure generates an electric geometric dynamics [2]-[4].
This paper rises the following problem: classify the trajectories in the electric geometric dynamics associated to a rectangular plate or to a metallic disk and find their physical interpretation. To catch the readers into our problem, we use a modern view on two classical examples of stationary electric fields analysing phase portraits, geometric dynamics (both, Lagrangian and Hamiltonian versions), Maple and Mathematica simulations.

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Key words: Laplace equation, electric potential, electric field, geometric dynamics.

1 Rectangular Metallic Plate

1.1 Electric Potential

Let us start with the stationary electric field E in a rectangular metallic plate of dimensions a and b, thickness h, and conductivity γ. We suppose that the electric current has constant intensity I, and electrodes of height h and width k are placed symmetrically in the centers of two opposite faces of the plate.

We exhibit the solution of Valer Novacu [1] determining the electric potential. Inside the plate, the potential V is described by the Laplace equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0,$$

with the boundary conditions

$$\left.\frac{\partial V}{\partial y}\right|_{y=-b} = \left.\frac{\partial V}{\partial y}\right|_{y=b} = 0, \quad \left.\frac{\partial V}{\partial x}\right|_{x=-a} = -\left.\frac{\partial V}{\partial x}\right|_{x=a} = f(y) = \begin{cases} -\dfrac{I}{2hk\gamma}, & |y| < k \\ 0, & |y| > k. \end{cases}$$

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BSG Proceedings 8, pp. 165-186, Geometry Balkan Press, 2003.

We look for a Fourier-type solution of the form

$$V(x,y)=\sum_{n=0}^{\infty}u_n(x)\cos\left(\frac{n\pi}{b}y\right).$$

The boundary conditions determine the functions $u_n(x)$, and the solution has the form

$$V(x,y)=-\frac{I}{2hb\gamma}\left(x+\frac{2b}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi k}{b}\right)}{\frac{n\pi k}{b}}\times\frac{\sinh\left(\frac{n\pi}{b}x\right)\cdot\cos\left(\frac{n\pi}{b}y\right)}{n\cosh\left(\frac{n\pi}{b}a\right)}\right)+\text{const.}$$

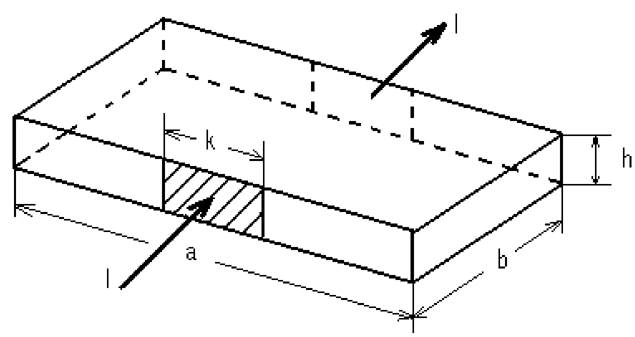

Fig. 1 Metallic plate

1.2 Electric Phase Portrait

with(plots): F:=fieldplot([1+2*(cosh(Pi*x))*cos(Pi*y)/cosh(2*Pi)+2*(cosh(2*Pi*x))*cos(2*Pi*y)/co sh(4*Pi)+2*(cosh(3*Pi*x))*cos(3*Pi*y)/cosh(6*Pi),-2*(sinh(Pi*x))*(sin(Pi*y))/cosh(2*Pi)-2*(sinh(2*Pi*x))*(sin(2*Pi*y))/cosh(4*Pi)-2*(sinh(3* Pi*x))*(sin(3*Pi*y))/cosh(6*Pi)],x=-2..2,y=-1..1);

G:=implicitplot({1+2*(cosh(Pi*x))*cos(Pi*y)/cosh(2*Pi)+2*(cosh(2*Pi*x))*cos(2*Pi*y)/c osh(4*Pi)+2*(cosh(3*Pi*x))*cos(3*Pi*y)/cosh(6*Pi),-2*(sinh(Pi*x))*(sin (Pi*y))/cosh(2*Pi)-2*(sinh(2*Pi*x))*(sin(2*Pi*y))/cosh(4*Pi)-2*(sinh(3 *Pi*x))*(sin(3*Pi*y))/cosh(6*Pi)},x=-2..2,y=-1..1);

plots[display](F,G,axes=none,title=‘FIG.2‘);

with(plots): F:=fieldplot([1+2*sum(’cosh(n*Pi*x)*cos(n*Pi*y)/(cosh(2*n*Pi))’, ’n’=1..10),-2*sum(’sinh(n*Pi*x)*sin(n*Pi*y)/(cosh(2*n*Pi))’,’n’=1..10)], x=-2..2,y=-1..1);

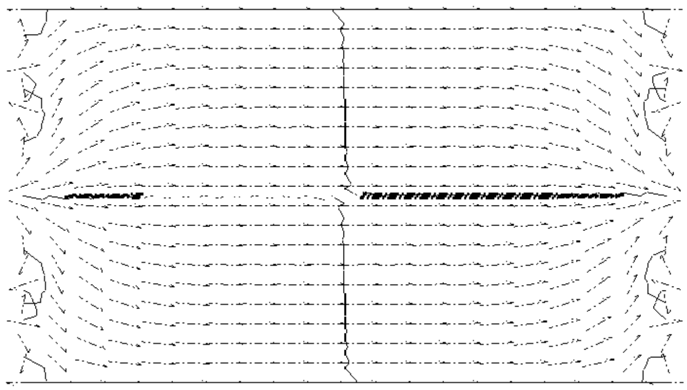

Fig. 2 Electric phase portrait of rectangle, nullclines, third approximation, $k \to 0$

G:=implicitplot({1+2*sum('cosh(n*Pi*x)*cos(n*Pi*y)/(cosh(2*n*Pi))','n'=1..10),-2*sum('sinh(n*Pi*x)*sin(n*Pi*y)/(cosh(2*n*Pi))','n'=1..10) },x=-2..2,y=-1..1);
plots[display](F,G,axes=none, title='FIG.3');

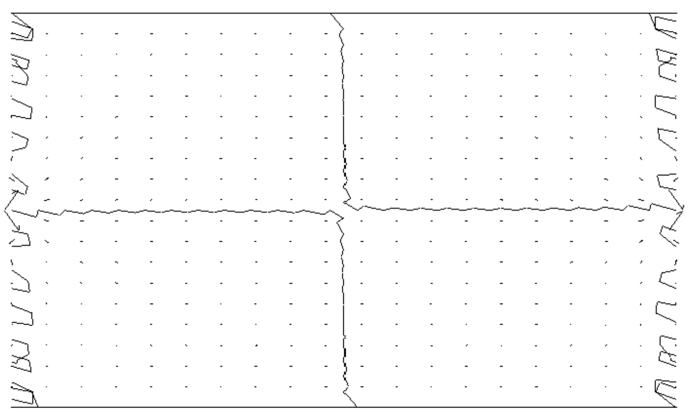

Fig. 3 Electric field, nullclines, 10th approximation, $k \to 0$

implicitplot((1+2*sum('cosh(n*Pi*x)*cos(n*Pi*y)/(cosh(2*n*Pi))','n'=1 ..10))^2+ $(-2*sum('sinh(n*Pi*x)*sin(n*Pi*y)/(cosh(2*n*Pi))','n'=1..10))^2$=1,x=-1..1,y=-1..1,color=blue,title='FIG.4');

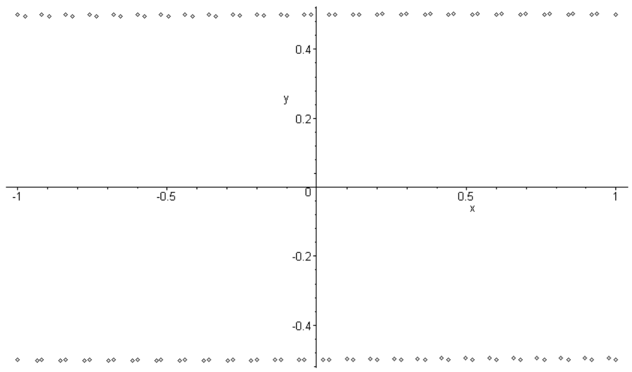

Fig. 4 1-Level set of electric energy density, 10th approximation, $k \to 0$

contourplot((1+2*sum('cosh(n*Pi*x)*cos(n*Pi*y)/(cosh(2*n*Pi))','n'=1. .10))^2+ $(-2 * sum('sinh(n * Pi * x) * sin(n * Pi * y)/(cosh(2 * n * Pi))',' n' = 1..10))^2$,x=-1..1,y=-1..1,contours=10,title=‘FIG.5‘);

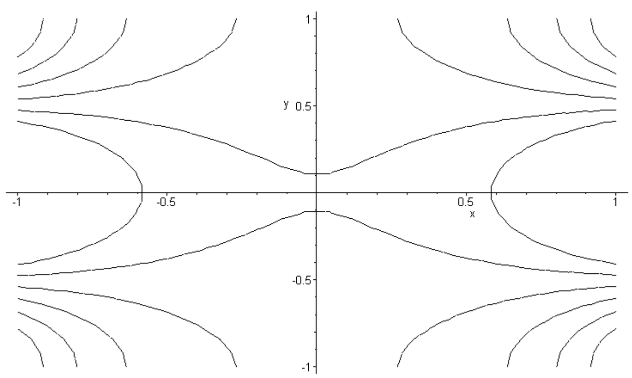

Fig. 5 Level sets of electric energy density, 10th approximation, $k \to 0$

2 Metallic Disk

2.1 Electric Potential

We solve a similar problem for a metallic disk of radius a. The electrodes are placed in diametral opposite points on the boundary of the disk.

with(plottools): a1:=arrow ([-3,0],[-2,0],.01,.1,.1,color=black): a2:=arrow ([2,0],[3,0],.01,.1,.1,color=black): c := disk([0,0],2, color=red): plots[display](c,a1,a2,title=‘FIG.6‘);

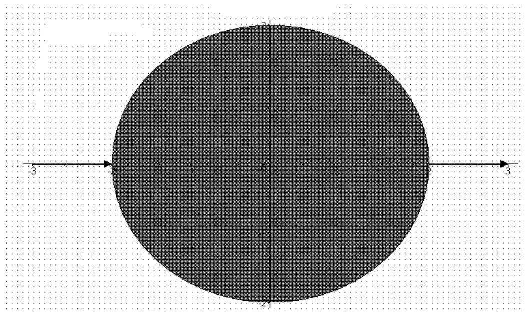

Fig. 6 Metallic disk

According to [1], the potential satisfies the Laplace equation (written in polar coordinates)

$$r\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right)+\frac{\partial^2 V}{\partial \varphi^2}=0.$$

The electrodes extend from 0 to δ and from π to $\pi+\delta$. The boundary conditions are

$$\left.\frac{\partial V}{\partial r}\right|_{r=a}=f(\varphi)=\begin{cases}-\dfrac{I}{ha\delta\gamma}, & 0<\varphi<\delta\\ \dfrac{I}{ha\delta\gamma}, & \pi<\varphi<\pi+\delta\\ 0, & \text{otherwise.}\end{cases}$$

The same Fourier analysis gives the solution

$$V(r,\varphi)=\frac{I}{\pi h\gamma}\sum_{n=1}^{\infty}\left[(-1)^n-1\right]\frac{\sin n\delta}{n\delta}\frac{1}{n}\left(\frac{r}{a}\right)^n+\text{const.}$$

In the particular case $\delta\to 0$, the sum of the above series can be performed. In this case, we find

$$V(r,\varphi)=\frac{I}{2\pi h\gamma}\ln\frac{r^2-2ar\cos\varphi+a^2}{r^2+2ar\cos\varphi+a^2}$$

or, in the Cartesian coordinates,

$$V(x,y)=\frac{I}{2\pi h\gamma}\ln\frac{x^2+y^2-2ax+a^2}{x^2+y^2+2ax+a^2}.$$

2.2 Electric Phase Portrait

```
EL:=DEplot( {D(rho)(t)=-2*((rho(t)-cos(theta(t)))/(rho(t)^2-2*rho(t)*cos(theta(t))+
1)) + 2 * ((rho(t) + cos(theta(t)))/(rho(t)^2 + 2 * rho(t) * cos(theta(t)) + 1)),
D (theta)(t)=-(1/rho(t)^2)*((2*rho(t)*sin(theta(t)))/(rho(t)^2-2*rho(t)*cos(theta(t))+
1)) + (2 * rho(t) * sin(theta(t))/(rho(t)^2 + 2 * rho(t) * cos(theta(t)) + 1)))},
```

{rho(t),theta(t)},t=-3..3,

[[rho(0)=0.1,theta(0)=Pi/4],[rho(0)=0.5,theta(0)=Pi/6]],rho=0..0.9,th eta=0..2*Pi, scene=[rho,theta], stepsize=.01,axes=BOXED,linestyle=4,linecolor=t, method=classical[rk4],startinit=true,maxfun=5000);

with(plots):

NU:=implicitplot($\{2*((rho-cos(theta))/(rho^2-2*rho*cos(theta)+1))-2*((rho+cos(theta))/(rho^2+2*rho*cos(theta)+1)), (1/rho^2)*((2*rho*sin(theta)/(rho^2-2*rho*cos(theta)+1))+(2*rho*sin(theta)/(rho^2+2*rho*cos(theta)+1)))\}, rho = 0..0.9, theta = 0..2*Pi, color = green)$;

plots[display](EL,NU,axes=none,title='FIG.7');

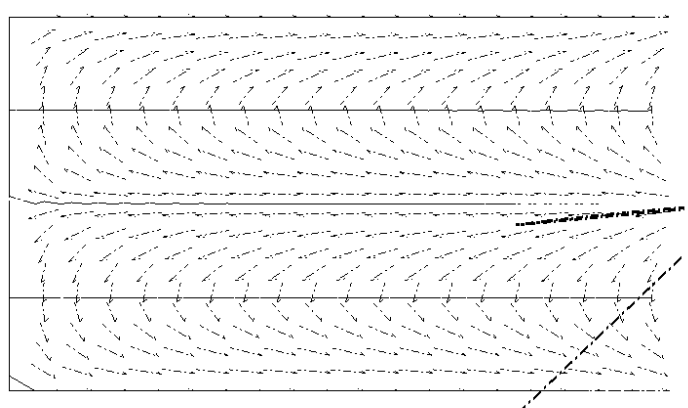

Fig. 7 Electric phase portrait of disk, nullclines, (rho, theta)-plane

E:=DEplot($\{D(x)(t) = -(2*(x-1)/((x-1)^2+y^2))(t)+(2*(x+1)/((x+1)^2+y^2))(t), D(y)(t) = -(2*y/((x-1)^2+y^2))(t)+(2*y/((x+1)^2+y^2))(t)\}$,

{x(t),y(t)},t=0..3,

$[[x(0) = 0.1*cos(Pi/4), y(0) = 0.1*sin(Pi/4)], [x(0) = 0.5*cos(Pi/6), y(0) = 0.5*sin(Pi/6)]], x = -0.9..0.9, y = -0.9..0.9, scene = [x, y], stepsize = .01, axes = BOXED, linestyle = 4, linecolor = t, method = classical[rk4], startinit = true$, maxfun=5000);

N:=implicitplot($\{(2*(x-1)/((x-1)^2+y^2))-(2*(x+1)/((x+1)^2+y^2)), (2*y/((x-1)^2+y^2))-(2*y/((x+1)^2+y^2))\}, x = -0.9..0.9, y = -0.9..0.9, color = green)$;

plots[display](E,N,axes=none,title='FIG.8');

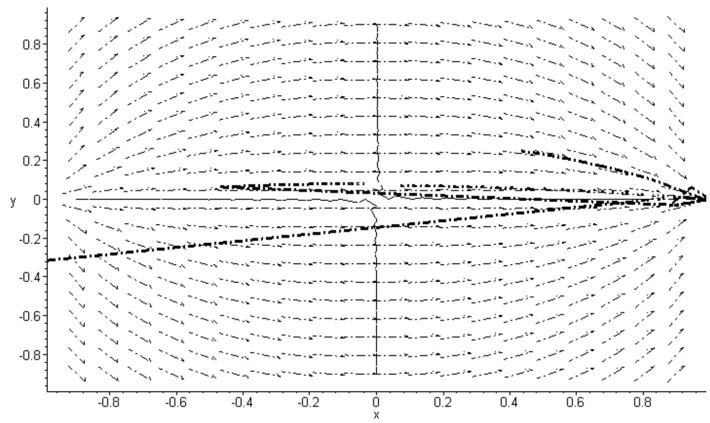

Fig. 8 Electric phase portrait of disk, nullclines, (x, y)-plane

3 Electric Geometric Dynamics

3.1 General Theory

Let $E\,(E_1, E_2)$ be the electric field. The electric lines are solutions of the first order differential system

$$\frac{dx}{dt} = E_1, \qquad \frac{dy}{dt} = E_2.$$

We introduce the electric density of energy

$$f = \frac{1}{2}\left(E_1^2 + E_2^2\right)$$

which is invariant with respect to the rotations of the electric field. Since $\operatorname{rot} E = 0$, the electric geometric dynamics is described by the second order gradient dynamical system [2]

$$\frac{d^2x}{dt^2} = \frac{\partial f}{\partial x}, \quad \frac{d^2y}{dt^2} = \frac{\partial f}{\partial y}.$$

This is a suitable prolongation of the system which describes the electric lines.

As it is seen, the electric geometric dynamics is created by the electrical force ∇f.

Every nonconstant trajectory of this dynamical system which has the constant total energy

$$\mathcal{H} = \frac{1}{2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right] - f(x, y)$$

is a reparametrized geodesic of the Riemann-Jacobi manifold

$$\left(D \setminus \mathcal{E}, \quad g_{ij} = (\mathcal{H} + f)\delta_{ij}, \quad i, j = 1, 2\right),$$

where $\mathcal{E}$ is the set of zeros of E. The set of all trajectories splits in three parts: electric lines (for $\mathcal{H} = 0$), trajectories with positive energy (for $\mathcal{H} = \text{const} > 0$), trajectories with negative energy (for $\mathcal{H} = \text{const} < 0$).

Open problem. Find the physical interpretation for the trajectories with $\mathcal{H} \neq 0$.

3.2 Case of Rectangular Plate. Lagrangian Version

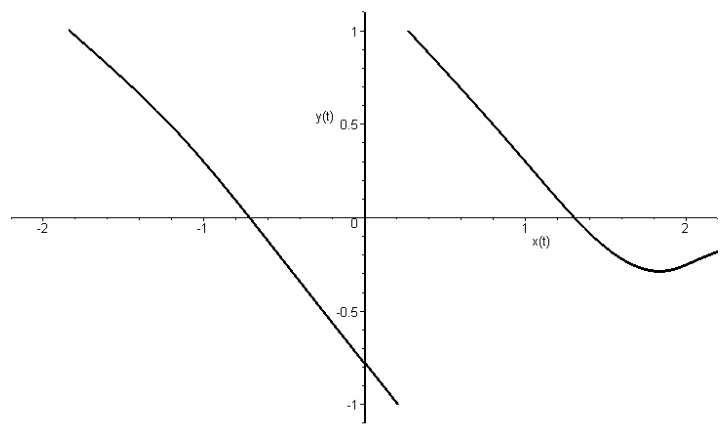

Fig. 9 Trajectories in electric geometric dynamics which are not electric lines

DE1:= {*diff*(*x*(*t*), *t*2)=(1+2*cosh(Pi*x(t))*cos(Pi*y(t))/cosh(2*Pi)+
2*cosh(2*P i*x(t))*cos(2*Pi*y(t))/cosh(4*Pi)+2*cosh(3*Pi*x(t))*cos(3*Pi*y(t))/
cos h(6*Pi))*(2*sinh(Pi*x(t))*Pi*cos(Pi*y(t))/cosh(2*Pi)+4*sinh(2*Pi*x(t)) *Pi*
cos(2*Pi*y(t))/cosh(4*Pi)+6*sinh(3*Pi*x(t))*Pi*cos(3*Pi*y(t))/ cosh(6*Pi))
+(-2*sinh(Pi*x(t))*sin(Pi*y(t))/cosh(2*Pi)-2*sinh(2*Pi*x(t))*si n(2*Pi*y(t))/
cosh(4*Pi)-2*sinh(3*Pi*x(t))*sin(3*Pi*y(t))/cosh(6*Pi))*(-2*cosh(Pi*x(t))*Pi*
sin(Pi*y(t))/cosh(2*Pi)-4*cosh(2*Pi*x(t))*Pi*sin(2 *Pi*y(t))/cosh(4*Pi)-6*
cosh(3*Pi*x(t))*Pi*sin(3*Pi*y(t))/cosh(6*Pi)),d iff(y(t),t2) = (1+2 ∗ *cosh*(*Pi* ∗ *x*(*t*)) ∗
cos(*Pi* ∗ *y*(*t*))/*cosh*(2 ∗ *Pi*) + 2 ∗ *cosh*(2 ∗ *Pi* ∗ *x*(*t*)) ∗ *cos*(2 ∗ *Pi* ∗ *y*(*t*))/*cosh*(4 ∗ *Pi*) +
2 ∗ *cosh*(3 ∗ *Pi* ∗ *x*(*t*)) ∗ *cos*(3 ∗ *Pi* ∗ *y*(*t*))/*cosh*(6 ∗ *Pi*)) ∗ (−2 ∗ *cosh*(*Pi* ∗ *x*(*t*)) ∗ *Pi* ∗
sin(*Pi* ∗ *y*(*t*))/*cosh*(2 ∗ *Pi*) − 4 ∗ *cosh*(2 ∗ *Pi* ∗ *x*(*t*)) ∗ *Pi* ∗ *sin*(2 ∗ *Pi* ∗ *y*(*t*))/*cosh*(4 ∗
Pi) − 6 ∗ *cosh*(3 ∗ *Pi* ∗ *x*(*t*)) ∗ *Pi* ∗ *sin*(3 ∗ *Pi* ∗ *y*(*t*))/*cosh*(6 ∗ *Pi*)) + (−2 ∗ *sinh*(*Pi* ∗
x(*t*)) ∗ *sin*(*Pi* ∗ *y*(*t*))/*cosh*(2 ∗ *Pi*) − 2 ∗ *sinh*(2 ∗ *Pi* ∗ *x*(*t*)) ∗ *sin*(2 ∗ *Pi* ∗ *y*(*t*))/*cosh*(4 ∗
Pi) − 2 ∗ *sinh*(3 ∗ *Pi* ∗ *x*(*t*)) ∗ *sin*(3 ∗ *Pi* ∗ *y*(*t*))/*cosh*(6 ∗ *Pi*)) ∗ (−2 ∗ *sinh*(*Pi* ∗ *x*(*t*)) ∗
Pi ∗ *cos*(*Pi* ∗ *y*(*t*))/*cosh*(2 ∗ *Pi*) − 4 ∗ *sinh*(2 ∗ *Pi* ∗ *x*(*t*)) ∗ *Pi* ∗ *cos*(2 ∗ *Pi* ∗ *y*(*t*))/*cosh*(4 ∗
Pi) − 6 ∗ *sinh*(3 ∗ *Pi* ∗ *x*(*t*)) ∗ *Pi* ∗ *cos*(3 ∗ *Pi* ∗ *y*(*t*))/*cosh*(6 ∗ *Pi*))};
DEplot(DE1,[x(t),y(t)],t=-15..15,[[x(0.9)=1,y(0.9)=0.3,D(x)(0.9)=-1,D (y)(0.9)=1],
[x(-0.9)=-1,y(-0.9)=0.3,D(x)(-0.9)=1,D(y)(-0.9)=-1]],x=-2. .2,y=-1..1,scene=
[x(t),y(t)], obsrange=true,title='FIG.9', linecolor=red,stepsize=.0001,iterations=100);

3.3 Case of Rectangular Plate. Hamiltonian Version

E1:=1+2*sum('cosh(n*Pi*q1)*cos(n*Pi*q2)/(cosh(2*n*Pi))','n'=1..10);
E2:=-2*sum('sinh(n*Pi*q1)*sin(n*Pi*q2)/(cosh(2*n*Pi))','n'=1..10);
f:=(1/2)*($E1^2 + E2^2$);
with(DEtools):
H := 1/2*($p1^2 + p2^2$) -f;
$hamilton_eqs(H)$;
H, t=-10..10, {[-0.1,.1,-0.4,0.1,0]}:
poincare(%, stepsize=.05,iterations=5);
poincare(%%,stepsize=.05,iterations=5,scene=[p2,q2]);
F2 := poincare(H,t=-100..100, {[0,.1,-0.4,0.1,0] },stepsize=.1,iterations=4,scene=[p1=-1.5..1.5,q1=-1..1,q2=-1..1],3):
F2;

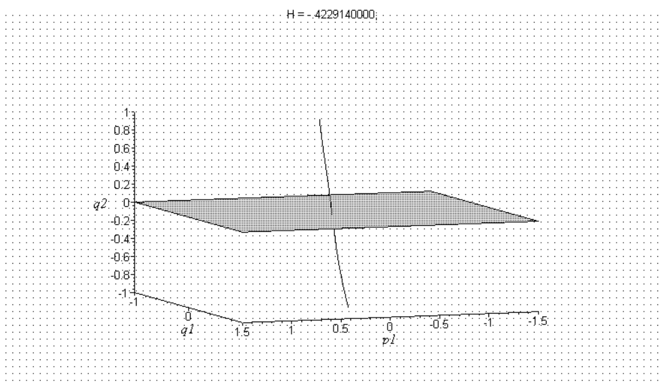

Fig. 10 Poincaré scene

3.4 Case of Metallic Disk. Lagrangian Version

f:=(1/2)*($E1^2 + E2^2$); diff(f,x);diff(f,y);
with(DEtools):
DE1:={diff(x(t),t2) = $diff(f,x)(t), diff(y(t),t2)$=diff(f,y)(t)}; DEplot(DE1, $[x(t),y(t)], t = -100..100, [[x(0) = 0.8, y(0) = 0.3, D(x)(0) = 1, D(y)(0) = 1]$,
[x(0)=0.1,y(0)=0.3,D(x)(0)=-1,D(y)(0)=1]],x=-1..1,y=-0.6..0.6,scene=[x(t),y(t)],

obsrange=true, title=‘FIG.11. Two Trajectories in Electric Geometric Dynamics which are not Electri Lines‘,linecolor=red,stepsize=.0001,iterations=100);

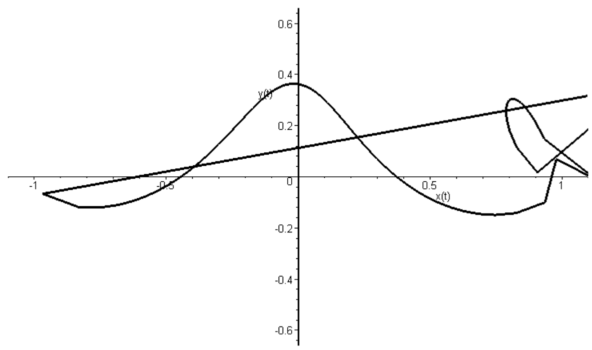

Fig. 11 Electric geometric dynamics (two curves)

with(plots): A:=fieldplot([diff(f,x),diff(f,y)],x=-0.5..0.5,y=-0.5..0.5,title=‘FIG.18‘);

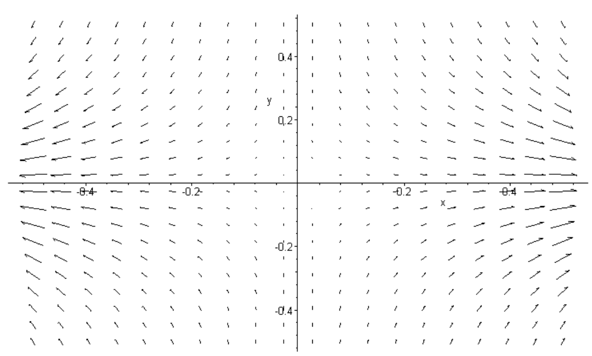

Fig. 12 Electric force

$implicitplot((x+0.2*diff(f,x))^2+(y+0.2*diff(f,y))^2=1, x=-1..1, y=-1..1, color=blue, title=`FIG.12`);$

3.5 Case of Metallic Disk. Hamiltonian Version

$E1 := -2*(q1-1)/((q1-1)^2+q2^2)+(2*(q1+1)/((q1+1)^2+q2^2));$

$E2 := -2*q2/((q1-1)^2+q2^2)+(2*q2/((q1+1)^2+q2^2));$

f:= $(1/2)*(E1^2+E2^2)$;

with(DEtools):

H := 1/2*$(p1^2+p2^2)$ -f;

$hamilton_eqs(H);$

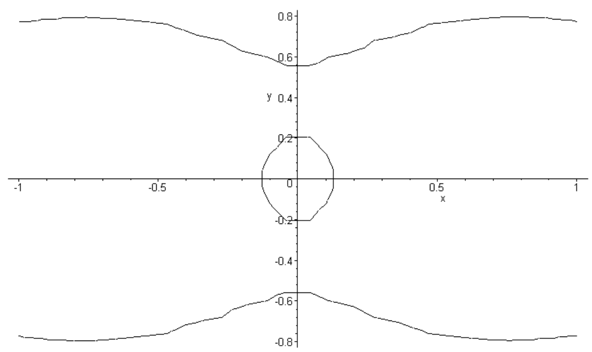

Fig. 13 Deformation of disk of radius 1

H, t=-10..10, {[-0.1,.1,-0.4,0.1,0]}:
poincare(%, stepsize=.05,iterations=5);
poincare(%%,stepsize=.05,iterations=5,scene=[p2,q2]);
F2 := poincare(H,t=-100..100, {[0,.1,-0.4,0.1,0] },stepsize=.1,iterations=4,scene=[p1=-1.5..1.5,q1=-1..1,q2=-1..1],3):
F2;
$ics_1 0 := generate_i c(H, \{t = 0, p2 = 2, q1 = 0, q2 = 0, energy = 10\}, 1) :$
poincare(H,t=-50..50,$ics_1$0,stepsize=.005,iterations=4,scene=[p2,q2]);
F3b := poincare(H,t=0..20,$ics_1$0,stepsize=.01,iterations=4,scene=[p2,q2,q1],3):
zoom(F3b,-3..3, -1..1, -1..1);

4 Appendix

For comparision, we present numeric evaluations made with "Mathematica". The definitions and the relations used for 10 terms in the sum are: $f[x, y] = x + 2 * b/Pi * [Sin[n * Pi * k/b] * Sinh[n * Pi * x/b] * Cos[n * Pi * y/b]/((n * Pi * k/b) * n * Cosh[n * Pi * a/b]), n, 1, 10] derfx[x, y] = D[f[x, y], x] derfy[x, y] = D[f[x, y], y] fenergy[x, y] = (derfx[x, y]^2 + derfy[x, y]^2)/2$

The constants have the following values: $a = 2, b = 1, k = 0.5$, or 0.1.

For k=0, the expression of the potential becomes:

$f[x, y] = x + 2 * b/Pi * Sum[Sinh[n * Pi * x/b] * Cos[n * Pi * y/b]/(n * Cosh[n * Pi * a/b]), n, 1, 1000]$

The vector fields are drawn using the standard package "Graphics":
PlotVectorField[derfx[x,y],derfy[x,y],x,-2,2,y,-1,1,PlotPoints15,
ScaleFactorAutomatic ,AspectRatio1/2,AxesTrue,FrameTrue];

The3D plots show the values of the two electric field components and of the electric energy:
Plot3D[derfx[x,y],x,-2.,2.,y,-1.,1.,PlotPoints30];
Plot3D[derfy[x,y],x,-2.,2.,y,-1.,1.,PlotPoints30];

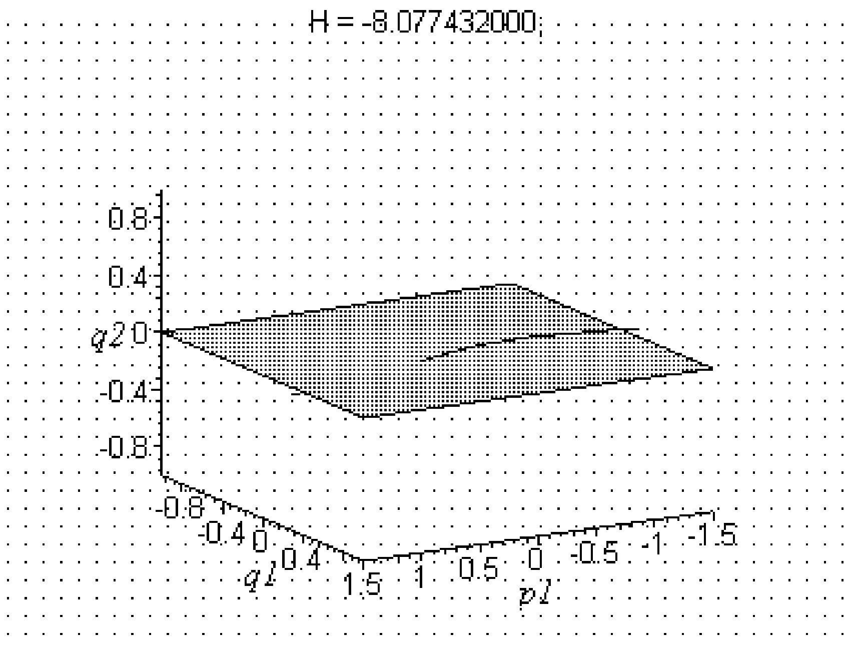

Fig. 14 Poincaré scene

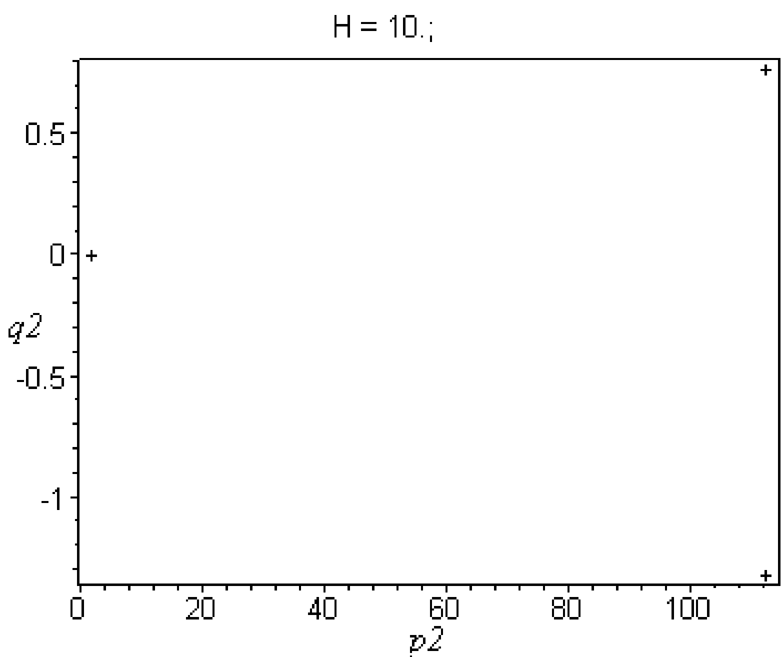

Fig. 15 Poincaré sections

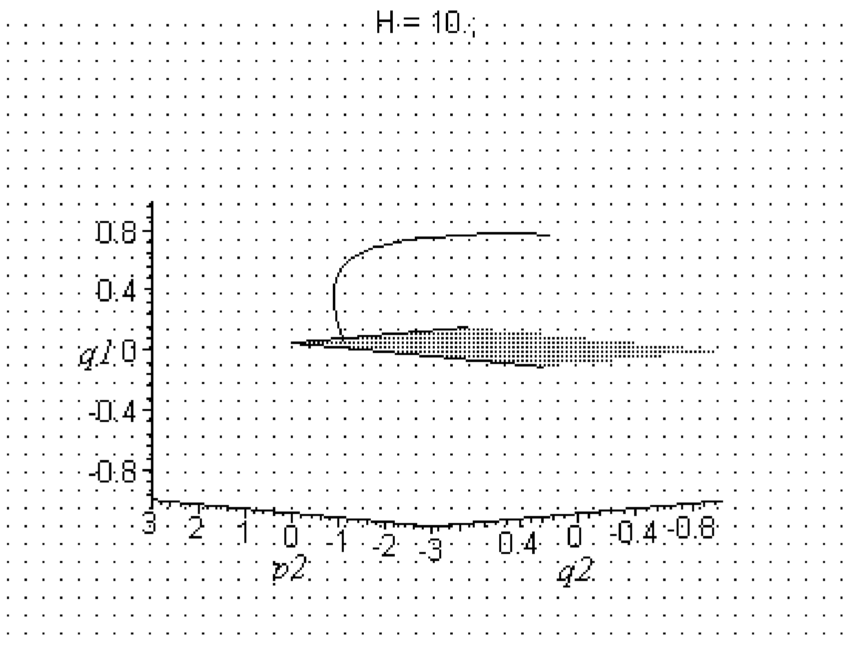

Fig. 16 Poincaré scene

Plot3D[fenergy[x,y],x,-2.,2.,y,-1.,1.,PlotPoints30];
The contour plots map lines of equal electric energy:
ContourPlot[fenergy[x,y],x,-2.,2.,y,-1.,1.,AspectRatio1/2,
Contours7,ContourShadingNone,PlotRange.45,.55,PlotPoints20];
ContourPlot[fenergy[x,y],x,-2.,2.,y,-1.,1.,AspectRatio1/2,Contours9,
ContourShadingNone,PlotRange.2,2.2,PlotPoints20];
Show[%,%%,PlotRangeAll];

Remarks:

1. - Results for $k = 0$ are very unstable and almost 3000 terms in the sum must be used. However, this case has no physical meaning.

2. - The E_2-component of the electric field has values very close to zero over large domains, because in the central region the field is almost horizontal.

3. - All the contour graphics of the electric energy exhibit straight lines passing through the points $y = -0.5$ and $y = 0.5$. On these lines the square of the field equals unity.

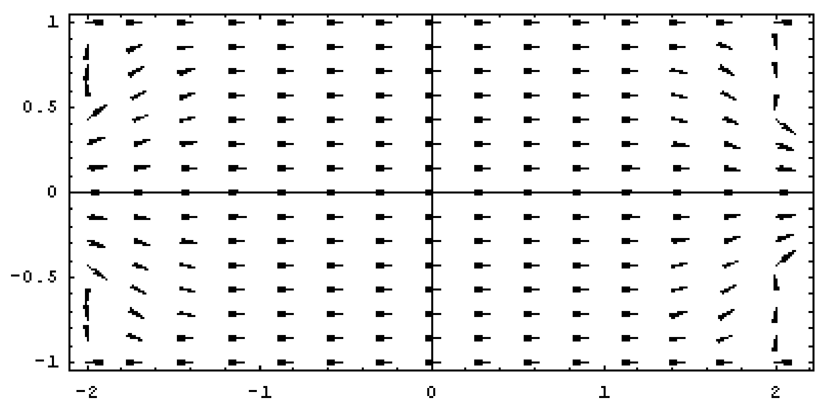

Fig. 17 Electric phase portrait of rectangular plate, $k = 0.5$ and 10 iterations

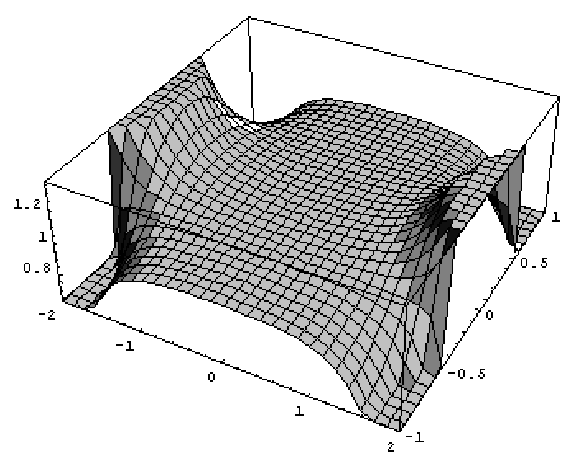

Fig. 18 Component E_1 of field for rectangular plate, $k = 0.5$ and 10 iterations

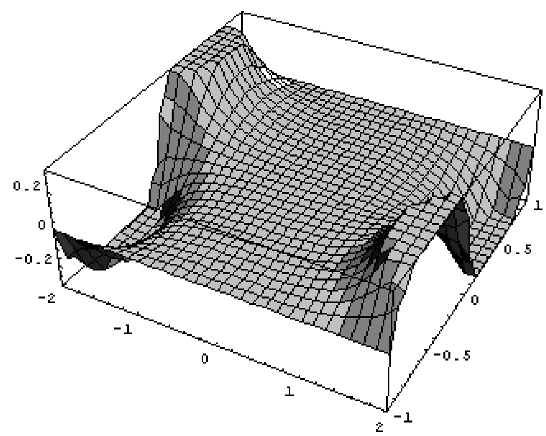

Fig. 19 Component E_2 of field for rectangular plate, $k = 0.5$ and 10 iterations

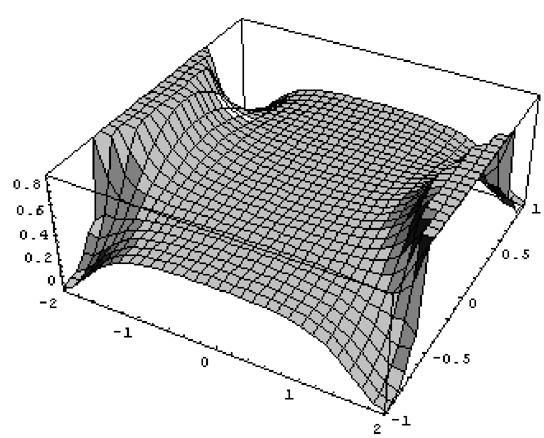

Fig. 20 Electric energy for rectangular plate, $k = 0.5$ and 10 iterations

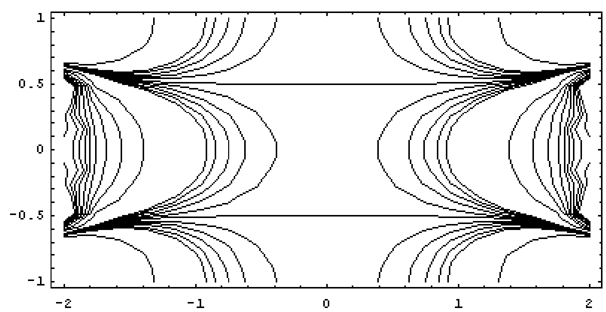

Fig. 21 Constant level sets of electric energy for rectangular plate, $k = 0.5$ and 10 iterations

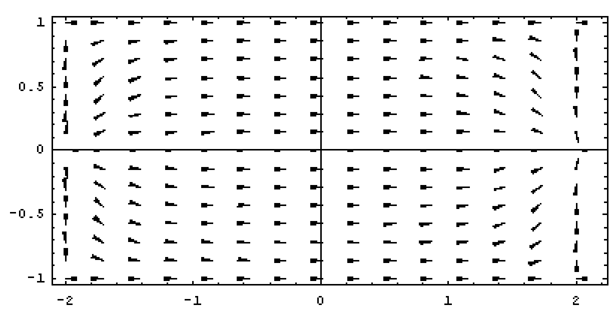

Fig. 22 Electric phase portrait of rectangular plate, $k = 0.1$ and 10 iterations

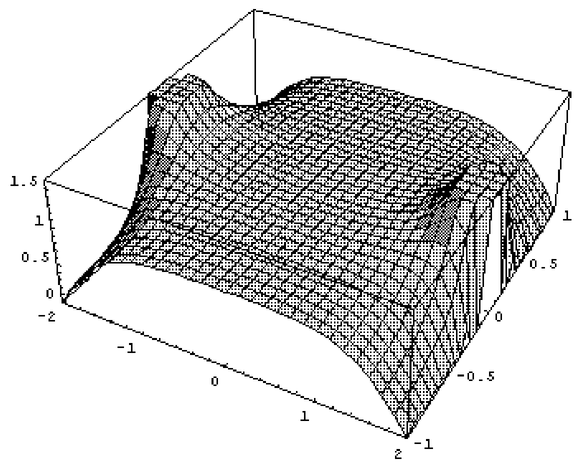

Fig. 23 Component E_1 of field for rectangular plate, $k = 0.1$ and 10 iterations

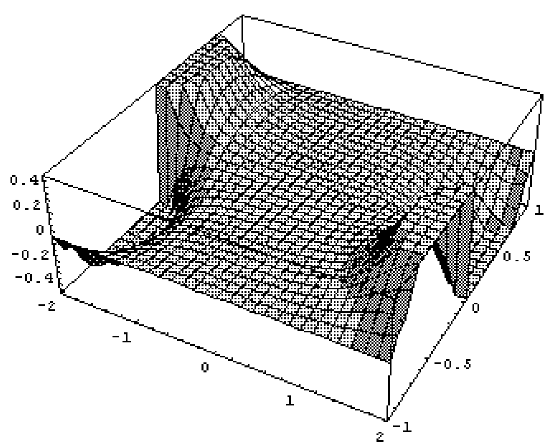

Fig. 24 Component E_2 of field for rectangular plate, $k = 0.1$ and 10 iterations

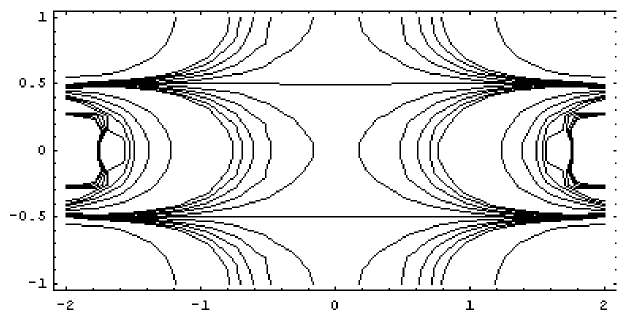

Fig. 25 Constant level sets of the electric energy for rectangular plate, $k = 0.1$ and 10 iterations

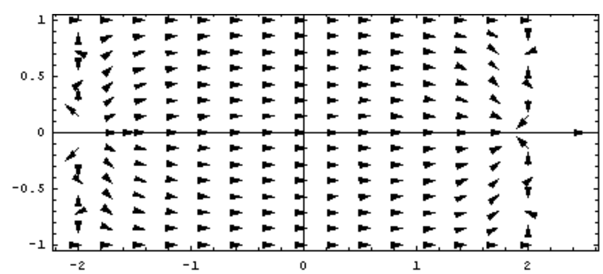

Fig. 26 Electric phase portrait of rectangular plate, $k \to 0$ and 10 iterations

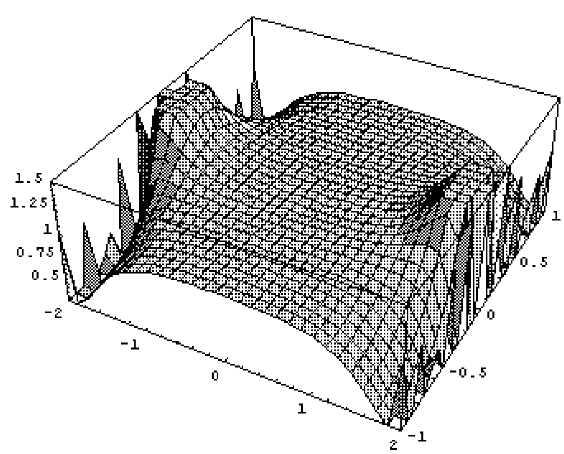

Fig. 27 Component E_1 of field for rectangular plate, $k \to 0$ and 10 iterations

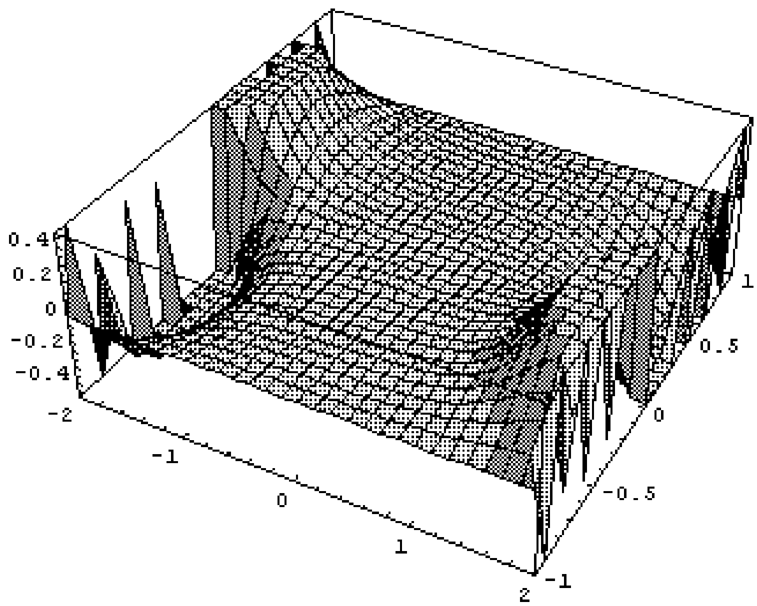

Fig. 28 Component E_2 of field for rectangular plate, $k \to 0$ and 10 iterations

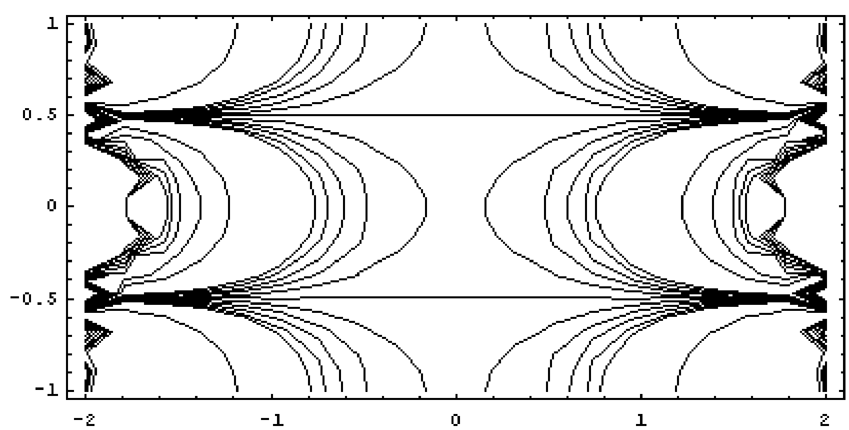

Fig. 29 Constant level sets of the electric energy for rectangular plate, $k \to 0$ and 10 iterations

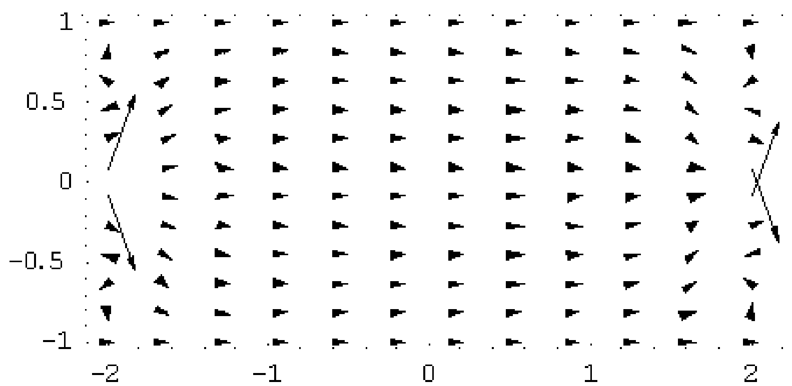

Fig. 30 Electric phase portrait of rectangular plate, $k \to 0$ and 3000 iterations

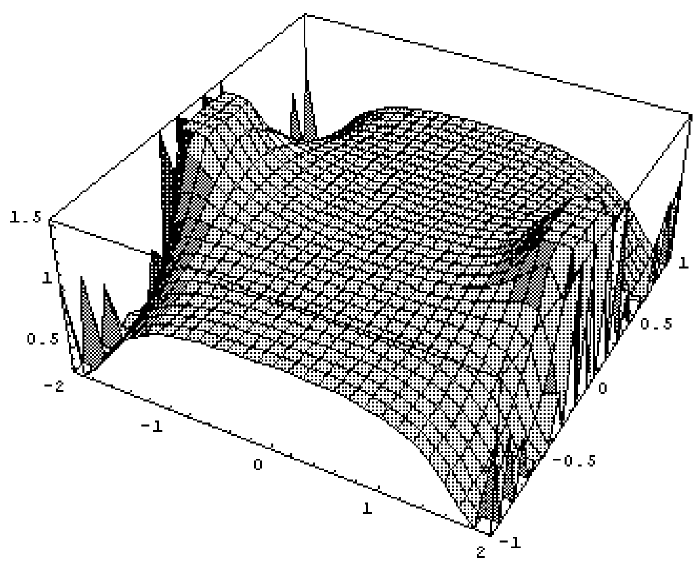

Fig. 31 Component E_1 of field for rectangular plate, $k \to 0$ and 3000 iterations

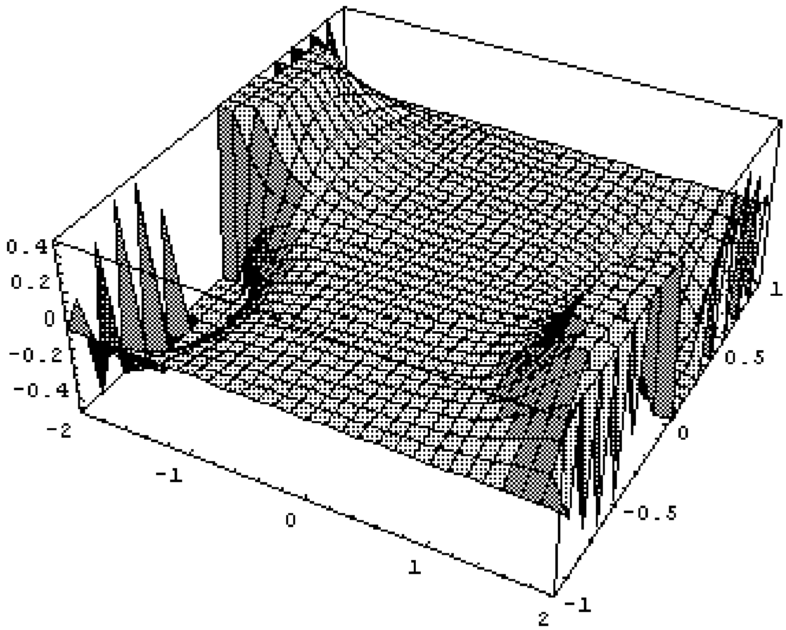

Fig. 32 Component E_2 of field for rectangular plate, $k \to 0$ and 3000 iterations

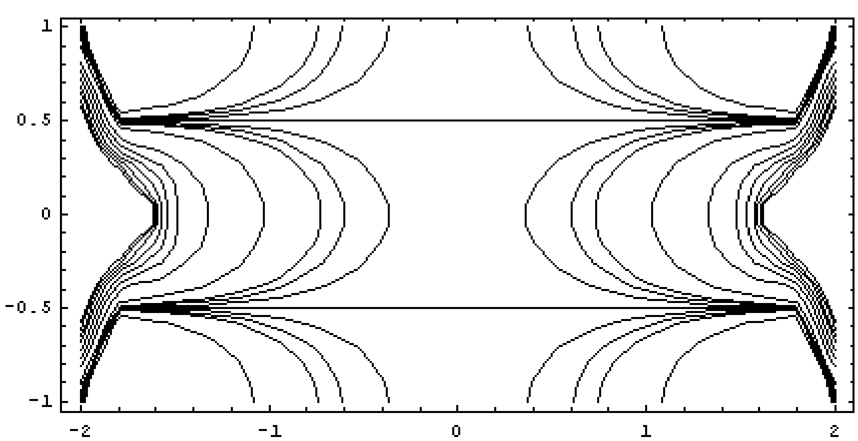

Fig. 33 Constant level sets of electric energy for rectangular plate, $k \to 0$ and 3000 iterations

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