A Geometrical Anisotropic Model of Space-time Based on Finslerian Metric

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Abstract

In this work we study an anisotropic model of general relativity based on the framework of Finsler geometry. The observed anisotropy of the microwave background radiation [6] is incorporated in the Finslerian structure of space-time.

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1 Introduction

A Finslerian geometrical structure of models which can correspond to aniso-tropic structures of regions of spacetime (radius$\leq 10^8$ light years) can be introduced. Our work was motivated by the observed anisotropy of the microwave cosmic radiation. This anisotropy is of dipole type, i.e. the intensity of the radiation is maximum at one direction and minimum at the opposite direction.

It is known that this anisotropy can be explained if we use Robertson-Walker metric and take into account the movement of our galaxy with respect to distant galaxies of the universe [7]. A small anisotropy is expected, however, due to the anisotropic distribution of galaxies in space [6].

From the above mentioned results, it is reasonable to seek for a Lagrangian which expresses this anisotropy. As such, we choose:

$$\mathcal{L} = \sqrt{a_{ij}y^i y^j} + \varphi(x)\hat{k}_a y^a. \tag{1}$$

The vector $\hat{k}_a$ expresses the observed anisotropy of the microwave background radiation.

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In §2 we give the necessary mathematical formalism, upon which we develop our theory.

In §3 we develop the geometric anisotropic structure of space-time based on the tangent bundle. Some physical interpretations are given.

2 Preliminaries

The framework in which we develop our present work is a Finsler tangent bundle. For this we consider a smooth 4-dimensional pseudoriemannian manifold M, (TM, π, M) its tangent bundle and $\tilde{TM} = TM \setminus \{0\}$, where 0 means the image of the null cross-section of the projection $\pi : TM \to M$. We also consider a local system of coordinates (x^i), $i = 0, 1, 2, 3$ and U a chart of M. Then the couple (x^i, y^a) is a local system of coordinates on $\pi^{-1}(U)$ in TM. A coordinate transformation on the total space TM is given by

$$\tilde{x}^i = \tilde{x}^i(x^0, \ldots, x^3), \quad \det \left\| \frac{\partial \tilde{x}^i}{\partial x^j} \right\| \neq 0, \quad \tilde{y}^a = \frac{\partial \tilde{x}^a}{\partial x^b} y^b, \quad x^a = \delta^a_i x^i. \tag{2}$$

By definition [4] a Finsler metric on M is a function $F : TM \to \mathbb{R}$ having the properties:

1. The restriction of F to $\tilde{TM}$ is of the class C^∞ and F is only continuous on the image of the null cross section in the tangent bundle to M.

2. The restriction of F to $\tilde{TM}$ is positively homogeneous of degree 1 with respect to (y^a).
$$F(x, ky) = kF(x, y), \quad k \in \mathbb{R}^*_+$$

3. The quadratic form on $\mathbb{R}^n$ with the coefficients
$$f_{ij} = \frac{1}{2} \frac{\partial^2 F^2}{\partial y^i \partial y^j}, \tag{3}$$
defined on $\tilde{TM}$, is non degenerate ($\det(f_{ij}) \neq 0$), with $\mathrm{rank}(f_{ij}) = 4$.

As it is known a non linear connection N on TM is a distribution on TM, supplementary to the vertical distribution V on TM :

$$T_{(x,y)}(TM) = N_{(x,y)} \oplus V_{(x,y)}.$$

In our case a non linear connection can be defined by

$$N^a_j = \frac{\partial G^a}{\partial y^j}, \tag{4}$$

where G^a are defined from

$$G^a = \frac{1}{4} f^{aj} \left(\frac{\partial^2 \mathcal{L}}{\partial y^j \partial x^k} y^k - \partial_j \mathcal{L} \right) \tag{5}$$

and the relation

$$\frac{dy^a}{ds} + 2G^a(x, y) = 0 \tag{6}$$

yields from the Euler-Lagrange equations:

$$\frac{d}{ds}\left(\frac{\partial \mathfrak{L}}{\partial y^a}\right) - \frac{\partial \mathfrak{L}}{\partial x^a} = 0. \tag{7}$$

The transformation rule of the non-linear connection coefficients is

$$\tilde{N}_i^a = \frac{\partial \tilde{x}^a}{\partial x^b}\frac{\partial x^j}{\partial \tilde{x}^i} N_j^b(x, y) + \frac{\partial \tilde{x}^a}{\partial x^h}\frac{\partial^2 x^h}{\partial \tilde{x}^i \partial \tilde{x}^b} y^b, \tag{8}$$

also

$$\frac{\delta}{\delta \tilde{x}^i} = \frac{\partial x^j}{\partial \tilde{x}^i}\frac{\delta}{\delta x^j}, \qquad \frac{\partial}{\partial \tilde{y}^a} = \frac{\partial x^b}{\partial \tilde{x}^a}\frac{\partial}{\partial y^b},$$

$$d\tilde{x}^i = \frac{\partial \tilde{x}^i}{\partial x^j} dx^j, \qquad \delta \tilde{y}^a = \frac{\partial \tilde{x}^a}{\partial x^b}\delta y^b.$$

A local basis of $T_{(x,y)}(TM)$, $(\delta_i, \dot{\partial}_a)$ adapted to the horizontal distribution N is

$$\delta_i = \partial_i - N_i^a(x, y)\dot{\partial}_a, \quad \text{where} \quad \partial_i = \frac{\partial}{\partial x^i}, \quad \dot{\partial}_a = \frac{\partial}{\partial y^a}, \tag{9}$$

where $N_i^a(x, y)$ are the coefficients of the non-linear Cartan connection N as we mentioned above.
The dual local basis is

$$\{d^i = dx^i,\, \delta^a = \delta y^a = dy^a + N_j^a dx^j\}_{i,a=\overline{0,3}} \equiv \{\delta^\beta\}_{\beta=\overline{0,7}}.$$

A d-connection on tangent bundle TM is a linear connection on TM which preserves by parallelism the horizontal distribution N and the vertical distribution V on TM.

Generally an h-v metric on the tangent bundle (TM, π, M) is given by

$$G = f_{ij}(x, y) dx^i \otimes dx^j + h_{ab}\delta y^a \otimes \delta y^b. \tag{10}$$

We consider a metrical d-connection $C\Gamma = (N_j^a, L_{jk}^i, C_{jk}^i)$ with the property

$$f_{ij|k} = \delta_k f_{ij} - L_{ik}^h f_{hj} - L_{jk}^h f_{ih} = 0, \tag{11}$$

$$f_{ij}|_k = \dot{\partial}_k f_{ij} - C_{ik}^h f_{hj} - C_{jk}^h f_{ih} = 0, \tag{12}$$

where

$$L_{jk}^i = \frac{1}{2} f^{ir}\left(\delta_j f_{rk} + \delta_k f_{jr} - \delta_r f_{jk}\right), \tag{13}$$

$$C_{jk}^i = \frac{1}{2} f^{ir}(\dot{\partial}_j f_{rk} + \dot{\partial}_k f_{jr} - \dot{\partial}_r f_{jk}). \tag{14}$$

The coordinate transformation of the objects L^i_{jk} and C^i_{jk} is:

$$\tilde{L}^i_{jk} = \frac{\partial \tilde{x}^i}{\partial x^h}\frac{\partial x^l}{\partial \tilde{x}^j}\frac{\partial x^r}{\partial \tilde{x}^k}L^h_{lr}(x,y) + \frac{\partial \tilde{x}^i}{\partial x^r}\frac{\partial^2 x^r}{\partial \tilde{x}^j \partial \tilde{x}^k}, \tag{15}$$

$$\tilde{C}^i_{jk} = \frac{\partial \tilde{x}^i}{\partial x^h}\frac{\partial x^l}{\partial \tilde{x}^j}\frac{\partial x^r}{\partial \tilde{x}^k}C^h_{lr}(x,y). \tag{16}$$

The Cartan torsion coefficients C_{ijk} are given by

$$C_{ijk} = \frac{1}{2}\dot{\partial}_k f_{ij}, \tag{17}$$

while the Christoffel symbols of the first and second kind for the metric f_{ij} are respectively:

$$\gamma_{ijk} = \frac{1}{2}\left(\frac{\partial f_{kj}}{\partial x^i} + \frac{\partial f_{ik}}{\partial x^j} - \frac{\partial f_{ij}}{\partial x^k}\right), \tag{18}$$

$$\gamma^l_{ij} = \frac{1}{2}f^{lk}\left(\frac{\partial f_{kj}}{\partial x^i} + \frac{\partial f_{ik}}{\partial x^j} - \frac{\partial f_{ij}}{\partial x^k}\right). \tag{19}$$

The torsions and curvatures which we use are given by [4, 3]:

$$T^i_{kj} = 0, \qquad S^i_{kj} = 0, \qquad R^i_{jk} = \delta_k N^i_j - \delta_j N^i_k, \tag{20}$$

$$P^i_{jk} = \dot{\partial}_k N^i_j - L^i_{kj}, \qquad P^i_{jk} = f^{im} P_{mjk}, \qquad P_{ijk} = C_{ijk|l}\, y^l, \tag{21}$$

$$R^i_{jkl} = \delta_l L^i_{jk} \delta_k L^i_{jl} + L^h_{jk} L^i_{hl} - L^h_{jl} L^i_{hk} + C^i_{jc} R^c_{kl}, \tag{22}$$

$$S_{jikh} = C_{iks} C^s_{jh} - C_{ihs} C^s_{jk}, \tag{23}$$

$$P_{ihkj} = C_{ijk|h} - C_{hjk|i} + C^r_{hj} C_{rik|l}\, y^l - C^r_{ij} C_{rkh|l}\, y^l, \tag{24}$$

$$S^l_{ikh} = f^{lj} S_{jikh}, \tag{25}$$

$$P^l_{ikh} = f^{lj} P_{jikh}. \tag{26}$$

3 The Geometrical Structure of the anisotropic model (based on TM)

In the following, the lowering and raising of the indices of the objects $\hat{k}_a, y^a$ and all related Riemannian tensors will be performed with the metric a_{ij}. For the related Finslerian tensors we shall use the Finsler metric f_{ij}.

The Lagrangian which gives the equation of geodesics in the case of (pseudo)-Riemannian space-time is given by:

$$L = \sqrt{a_{ij} y^i y^j}, \quad y^i = \frac{dx^i}{ds}, \tag{27}$$

or, equivalently, we may write for the line element:

$$ds_R = \sqrt{a_{ij} dx^i dx^j}, \tag{28}$$

where a_{ij} is the Riemannian metric with signature $(-,+,+,+)$. Because of the observed anisotropy, we must insert an additional term to the Riemannian line element (28). This term must fulfill the following requirements:

(a) It must give absolute maximum contribution for direction of movement parallel to the anisotropy axis.

(b) It must give zero contribution for movement in direction perpendicular to the anisotropy axis, i.e. the new line element must coincide with the Riemannian one for direction vertical to the anisotropy axis.

(c) It must not be symmetric with respect to replacement $y^a \to -y^a$. This requirement is necessary in order to express the anisotropy of dipole type of the Microwave Background Radiation (MBR). We need to have maximum (positive) contribution for direction that coincides with the direction of the anisotropy axis, and minimum (negative) contribution for the opposite direction.

We see that a term which satisfies the above conditions is $k_a(x)y^a$, where $k_a(x)$ expresses this anisotropy axis. For constant direction of $k_a(x)$ we may consider $k_a(x) = \varphi(x)\hat{k}_a$, where $\hat{k}_a$ is the unit vector in the direction $k_a(x)$. Then $\varphi(x)$ plays the role of "length" of the vector $k_a(x)$, $\varphi(x) \in \mathbb{R}$. Hence, we have the Lagrangian

$$\mathcal{L} = \sqrt{a_{ij} y^i y^j} + \varphi(x)\hat{k}_a y^a. \tag{29}$$

From (29) we define the Finsler metric function $F(x, y) = \mathcal{L}$. Setting $y^a = dx^a$ we have

$$ds_F = \sqrt{a_{ij} dx^i dx^j} + \varphi(x)\hat{k}_a dx^a; \tag{30}$$

ds_F is the Finslerian line element and ds_R is the Riemannian one. We see that the Finslerian line element is generated by an additional increment to the Riemannian one due to the anisotropy axis. Now

$$ds_F^2 = a_{ij} dx^i dx^j + 2\varphi(x)\hat{k}_a dx^a \sqrt{a_{ij} dx^i dx^j} + \varphi^2(x)\hat{k}_a dx^a \hat{k}_b dx^b. \tag{31}$$

In order for the Finslerian metric to be physically consistent with General Relativity theory, it must have the same signature with the Riemannian metric $(-,+,+,+)$. We have

$$ds_R = c\, d\tau = c\, \gamma dt = \gamma d(ct) = \gamma dx^0, \tag{32}$$

where $\gamma = \sqrt{1 - (v/c)^2}$ and v: 3-velocity in Riemannian space-time. From relations (32),(31) we obtain:

$$\begin{aligned} ds_F^2 &= a_{00} dx^0 dx^0 + 2a_{0\alpha} dx^0 dx^\alpha + a_{\alpha\beta} dx^\alpha dx^\beta + 2\varphi(x)\hat{k}_0 dx^0 ds_R \\ &\quad + 2\varphi(x)\hat{k}_\alpha dx^\alpha ds_R + \varphi^2(x)\hat{k}_0\hat{k}_0 dx^0 dx^0 + 2\varphi^2(x)\hat{k}_0\hat{k}_\alpha dx^0 dx^\alpha \\ &\quad + \varphi^2(x)\hat{k}_\alpha dx^\alpha \hat{k}_\beta dx^\beta, \end{aligned}$$

or

$$ds_F^2 = \left(a_{00} + 2\gamma\varphi(x)\hat{k}_0 + \varphi^2\hat{k}_0\hat{k}_0\right)dx^0dx^0 + \\ + \left(a_{\alpha\beta} + \varphi^2(x)\hat{k}_\alpha\hat{k}_\beta\right)dx^\alpha dx^\beta + 2\gamma\varphi(x)\hat{k}_\alpha dx^\alpha dx^0 \\ + 2a_{0\alpha}dx^0dx^\alpha + 2\varphi^2(x)\hat{k}_0\hat{k}_\alpha dx^0dx^\alpha, \tag{33}$$

where α, $\beta = 1, 2, 3$. From relation (33) it is evident that we must have

$$(k_0(x))^2 + 2\gamma k_0(x) + a_{00} < 0, \tag{34}$$

$$\delta^{\alpha\beta}\left(a_{\alpha\beta} + k_\alpha(x)k_\beta(x)\right) > 0, \tag{35}$$

for the signature to be preserved, where we have written $\varphi(x)\hat{k}_i = k_i(x)$. Relation (34) admits negative values for

$$-\gamma - \sqrt{\gamma^2 - a_{00}} < k_0(x) < -\gamma + \sqrt{\gamma^2 - a_{00}}, \tag{36}$$

while from (35) yields:

$$(k_\alpha(x))^2 > -a_{\alpha\alpha}, \tag{37}$$

which is true for any $k_a(x)$ since $a_{\alpha\alpha} > 0$.

Then for any physically acceptable vector, its 0 component $k_0(x)$ must lie in the interval (36). Relation (36) is a restriction upon the anisotropy of space-time, i.e. the anisotropy vector can not take arbitrary values.

The equation of geodesics is given by:

$$\frac{d^2x^l}{ds^2} + \overset{(a)}{\Gamma}{}^l_{ij}y^iy^j + \sigma a^{lm}(\partial_j\varphi\hat{k}_m - \partial_m\varphi\hat{k}_j)y^j = 0. \tag{38}$$

We observe that in the equation of geodesics we have an additional term, namely $\sigma a^{lm}(\partial_j(\varphi\hat{k}_m) - \partial_m(\varphi\hat{k}_j))y^j$ which expresses rotation of the aniso-tropy axis.

One possible explanation of the anisotropy axis could be that it expresses the resultant of the spin densities of the angular momenta of galaxies in a restricted region of space ($k_a(x)$ spacelike). It is known that the mass is anisotropically distributed for regions of space with radius$\leq 10^8$ light years [5]. Then an important kind of anisotropy might result from the ordering of the angular momenta of galaxies. As we move to greater distances (radius$\geq 10^8$ l.y.) the resultant of the spin densities is approximately zero, as it is expected for an isotropic universe.

$$k_a(x) = \sum_i \overset{(i)}{k}{}_a(x) \approx 0, \tag{39}$$

where $\overset{(i)}{k}{}_a(x)$ is the spin density tensor of each rotating mass distribution.

The spin is defined through the spin density tensor [2] from the relation

$$S_{ab} = \frac{\sqrt{-g}}{4\pi}\epsilon_{abc}k^c(x). \tag{40}$$

In the case that $\varphi(x)\hat{k}_a$ expresses spin density, the function $\varphi(x)$ is related to mass density (angular momenta depends upon angular velocity and mass distribution).

From equation (38) we see that for small variation of the resultant of the spin densities vector, the deviation from the Riemannian geodesics is very small, if not negligible.

From the equation of geodesics (38) we obtain for movement y^i perpendicular to k^i:

$$\frac{d^2x^a}{ds^2} + \overset{(a)}{\Gamma}{}^l_{ij} y^i y^j + \sigma a^{lm} \partial_j \varphi \hat{k}_m y^j = 0. \tag{41}$$

From (41) it is evident that although the contribution to the ds_R line element is zero for y^i vertical to k^i, the equation of geodesics is different from the Riemannian case. In the case, however, where $\partial_i \varphi(x)$is parallel to $\hat{k}_i$, i.e. the increment of anisotropy takes place only along the anisotropy axis, then the equation of geodesics is identical with the geodesics of the Riemannian space-time.

Using the notation $\beta = \hat{k}_a y^a$, $\sigma = \sqrt{a_{ij} y^i y^j}$, we calculate the metric tensor from (3):

$$f_{ij} = \frac{F}{\sigma} a_{ij} + \frac{\varphi(x)}{2\sigma} \underset{ij}{\mathbb{S}} \left(y_i \hat{k}_j \right) - \frac{\beta \varphi(x)}{\sigma^3} y_i y_j + \varphi^2(x) \hat{k}_i \hat{k}_j, \tag{42}$$

where $\underset{ij}{\mathbb{S}}$ is an operator and denotes symmetrization of the indices i, j, e.g.

$$\underset{ij}{\mathbb{S}}(A_{ikjl}) = \frac{1}{2}(A_{ikjl} + A_{jkil}).$$

Accordingly we define the antisymmetric operator

$$\underset{ij}{\mathbb{A}}(M_{ikjl}) = \frac{1}{2}(M_{ikjl} - M_{jkil}).$$

The inverse metric is

$$f^{ij} = \frac{\sigma}{F} a^{ij} - \frac{\sigma\varphi}{2F} \underset{ij}{\mathbb{S}} \left(y^i \hat{k}^j \right) + \frac{\varphi(\beta + m\sigma\varphi)}{F^3} y^i y^j, \tag{43}$$

as it may be verified by direct calculation, where $m = \hat{k}_a \hat{k}^a = 0, \pm 1$ according whether $\hat{k}_a$ is null, spacelike or timelike (in order to not loose generality, we do not identify $\hat{k}_a$ as spacelike). It must be noted, however, that if y^a represents the velocity of a particle (y^i timelike) then $\hat{k}^a$ is bound to be spacelike. This follows from the fact that one possible value of $y^a \hat{k}_a$ is zero.

The Cartan torsion coefficients which are given by (17), take the form:

$$C_{ijl} = \frac{3\beta\varphi}{2\sigma^5} y_i y_j y_l + \frac{3\varphi}{\sigma} \underset{ijl}{\mathbb{S}}(a_{ij}\hat{k}_l) - \frac{3\varphi}{\sigma^3} \underset{ijl}{\mathbb{S}}(y_i y_j y_l) - \frac{3\beta\varphi}{\sigma^3} \underset{ijl}{\mathbb{S}} a_{ij} y_l. \tag{44}$$

We observe from (44) that an increment of the anisotropy, i.e. increment of φ, results in a change in the values of the components of the Cartan coefficients. This is expected since the condition

$$C_{ijk} = 0 \tag{45}$$

is the condition for the Finsler metric to be Riemannian.

The finslerian Christoffel symbols of the first kind are given by (18)

$$\gamma_{ijl} = \frac{F}{\sigma}\overset{(a)}{\Gamma}_{ijl} + \Lambda_{ijl} + M_{ijl}, \tag{46}$$

where

$$\overset{(a)}{\Gamma}_{ijl} = \frac{1}{2}(\partial_i a_{lj} + \partial_j a_{il} - \partial_l a_{ij}) \tag{47}$$

are the Christoffel symbols corresponding to the metric a_{ij}.

$$\Lambda_{ijl} = \underset{ij\{l\}}{\mathcal{G}}\left[\left(\frac{3\beta\varphi}{2\sigma^5}y_i y_j - \frac{\varphi}{\sigma^3}\underset{ij}{\mathcal{S}} y_i \hat{k}_j - \frac{\varphi\beta}{4\sigma^3}a_{ij}\right)\partial_l a_{ab} y^a y^b\right] \tag{48}$$

and

$$M_{ijl} = \underset{ij\{l\}}{\mathcal{G}}\left[\left(\frac{\beta}{2\sigma}a_{ij} + \frac{1}{\sigma}\underset{ij}{\mathcal{S}} y_i \hat{k}_j - \frac{\beta}{\sigma^3}y_i y_j + 2\varphi \hat{k}_i \hat{k}_j\right)\partial_l \varphi\right]. \tag{49}$$

The operator $\underset{ij\{l\}}{\mathcal{G}}$ denotes an interchange of the indices in the form this interchange appears in the definition of the Christoffel symbols of a metric, e.g.

$$\underset{ij\{l\}}{\mathcal{G}} A_{ijl} = A_{lji} + A_{ilj} - A_{ijl},$$

$$\underset{ij\{l\}}{\mathcal{G}} \partial_l a_{ij} = 2\overset{(a)}{\Gamma}_{ijl}.$$

The Christoffel symbols of the second kind are found from (19):

$$\gamma^l_{ij} = \overset{(a)}{\Gamma}{}^l_{ij} + \left(\frac{\varphi(\beta + m\sigma\varphi)}{\sigma F^2}y^a y^l - \frac{2\varphi}{F}\underset{al}{\mathcal{S}}(y^a \hat{k}^l)\right)\overset{(a)}{\Gamma}_{ija} + \frac{\sigma}{F}\big(\Lambda^l_{ij} + M^l_{ij}\big) + (\Lambda_{ija} + M_{ija})\left(\frac{\varphi(\beta + m\sigma\varphi)}{F^3}y^a y^l - \frac{2\sigma\varphi}{F^2}\underset{al}{\mathcal{S}}(y^a \hat{k}^l)\right), \tag{50}$$

where $\Lambda^i_{jl} = \Lambda_{jlk}a^{ik}$ and $M^i_{jl} = M_{jlk}a^{ik}$. In relation (50) it is seen that, besides the $\overset{(a)}{\Gamma}{}^i_{jk} = 0$ terms, the rest express the anisotropic deviation from the Riemannian Christoffel symbols. When $\varphi = 0$, i.e. absence of anisotropy, the Finsler Christoffel symbols coincide with the Riemannian ones. From the above relation, for $\overset{(a)}{\Gamma}{}^i_{jk} = 0$ we have $\gamma^i_{jk} \neq 0$. This shows the dependence of γ^i_{jk} from the anisotropy terms.

From Euler-Lagrange equations we find for G^l (relation (5)):

$$G^l = \frac{1}{2}\overset{(a)}{\Gamma}{}^l_{ij} y^i y^j + \sigma a^{ml} y^j \underset{jm}{\mathcal{A}}(\partial_j \varphi(x)\,\hat{k}_m). \tag{51}$$

Using relation (4) we calculate the nonlinear connection coefficients:

$$N^l_k = \overset{(a)}{\Gamma}{}^l_{ik} y^i + \sigma a^{ml}\underset{km}{\mathcal{A}}(\partial_k \varphi(x)\,\hat{k}_m) + \frac{1}{\sigma}a^{ml} y^j \underset{jm}{\mathcal{A}}(\partial_j \varphi(x)\,\hat{k}_m) y_k, \tag{52}$$

or

$$N^l_k = \overset{(a)}{N}{}^l_j + \sigma a^{ml} \underset{km}{\mathcal{A}} (\partial_k \varphi(x)\, \hat{k}_m) + \frac{1}{\sigma} a^{ml} y^j \underset{jm}{\mathcal{A}} (\partial_j \varphi(x)\, \hat{k}_m) y_k. \tag{53}$$

Relation (53) clearly shows that the deviation from the Riemann non-linear connection is due to anisotropic terms. In the case of an irrotational anisotropic field, $\underset{km}{\mathcal{A}} (\partial_k \varphi(x)\, \hat{k}_m) = 0$, the non-linear connection is identical with the Riemannian one.

The connection coefficients C^l_{ij} are given by (14):

$$\begin{aligned} C^l_{ij} = {} & \frac{\varphi}{2F} a_{ij} \hat{k}^l + \frac{\varphi}{F} \underset{ij}{\mathcal{S}} (\hat{k}^i \delta^l_j) - \frac{\beta\varphi}{F\sigma^2} \underset{ij}{\mathcal{S}} (\delta^l_i y_j) - \frac{\varphi(\beta + m\sigma\varphi)}{2F^2\sigma} a_{ij} y^l - \\ & - \frac{\varphi}{2F\sigma^2} y_i y_j \hat{k}^l - \frac{\varphi(\sigma - \beta\varphi)}{F^2\sigma^2} y^l \underset{ij}{\mathcal{S}} (\hat{k}_i y_j) - \left(\frac{\varphi}{F}\right)^2 \hat{k}_i \hat{k}_j y^l + \\ & + \frac{\varphi(3\beta + m\sigma\varphi)}{2F^2\sigma^3} y_i y_j y^l. \end{aligned} \tag{54}$$

Correspondingly, using (13) we get:

$$\begin{aligned} L^i_{jk} = {} & \overset{(a)}{\Gamma}{}^l_{ij} + \left(\frac{\varphi(\beta + m\sigma\varphi)}{\sigma F^2} y^a y^l - \frac{2\varphi}{F} \underset{al}{\mathcal{S}} (y^a \hat{k}^l) \right) \overset{(a)}{\Gamma}{}_{ija} + \frac{\sigma}{F} \big(\Lambda^l_{ij} + \\ & + M^l_{ij} \big) + (\Lambda_{ija} + M_{ija}) \left(\frac{\varphi(\beta + m\sigma\varphi)}{F^3} y^a y^l - \frac{2\sigma\varphi}{F^2} \underset{al}{\mathcal{S}} (y^a \hat{k}^l) \right) - \\ & - \big(N^l_j C^i_{kl} + N^l_k C^i_{jl} - f^{ir} N^l_r C_{jkl} \big), \end{aligned} \tag{55}$$

where N^l_j and C^i_{kl} are given explicitly by relations (52), (54). The curvature of the non linear connection is (21):

$$
\begin{aligned}
R^i_{jk} = {} & \overset{(a)}{R}{}^i_{ajk} y^a + \frac{1}{2\sigma}(\partial_k a_{mn} \underset{jb}{\mathcal{A}}(\partial_j \varphi \hat{k}_b) - \partial_j a_{mn} \underset{kb}{\mathcal{A}}(\partial_k \varphi \hat{k}_b)) y^m y^n a^{bi} + \\
& + \sigma(\partial_k a^{bi} \underset{jb}{\mathcal{A}} \partial_j \varphi \hat{k}_b - \partial_j a^{bi} \underset{kb}{\mathcal{A}}(\partial_k \varphi \hat{k}_b)) - \sigma a^{bi} \underset{jk}{\mathcal{A}}(\partial_{bj} \varphi \hat{k}_k) + \\
& + \frac{1}{\sigma} a^{bi} y^c \left(\underset{cb}{\mathcal{A}}(\partial_{kc} \varphi \hat{k}_b) y_j - \underset{cb}{\mathcal{A}}(\partial_{jc} \varphi \hat{k}_b) y_k \right) + \\
& + \left(\frac{2}{\sigma} a^{bi} \underset{kj}{\mathcal{A}}(\partial_k y_j) - \frac{1}{\sigma^3} a^{bi} y^m y^n \underset{kj}{\mathcal{A}}(\partial_k a_{mn} y_j) \right) \underset{ab}{\mathcal{A}}(\partial_a \varphi \hat{k}_b) y^a \\
& - \sigma \left(\overset{(a)}{\Gamma}{}^i_{kb} a^{bc} \underset{cj}{\mathcal{A}}(\partial_c \varphi \hat{k}_j) + \overset{(a)}{\Gamma}{}^i_{jb} a^{bc} \underset{kc}{\mathcal{A}}(\partial_k \varphi \hat{k}_c) \right) - \beta \partial^i \varphi \underset{jk}{\mathcal{A}}(\partial_j \varphi \hat{k}_k) - \\
& - \frac{\beta^2 + m\sigma^2}{2\sigma^2} \partial^i \varphi \underset{jk}{\mathcal{A}}(\partial_j \varphi y_k) - \frac{\beta}{\sigma} \partial^a \varphi \underset{kj}{\mathcal{A}}(\overset{(a)}{\Gamma}{}^i_{ak} y_j) - \\
& - \frac{1}{\sigma} \underset{kj}{\mathcal{A}}(\overset{(a)}{\Gamma}_{kaj}) \left(\beta \partial^i \varphi y^a - (\partial_b \varphi y^b) y^a \hat{k}^i \right) - \\
& - \frac{1}{2}(\partial_b \varphi y^b) \left[\partial^i \varphi \underset{kj}{\mathcal{A}}(\hat{k}_k y_j) + \hat{k}^i \underset{kj}{\mathcal{A}}(\partial_k \varphi y_j) \right] - \frac{1}{2}(\partial_a \varphi \partial^a \varphi) \hat{k}^i \underset{jk}{\mathcal{A}}(\hat{k}_j y_k) - \\
& - (\partial_a \varphi y^a) \hat{k}^i \underset{kj}{\mathcal{A}}(\partial_k \varphi \hat{k}_j) - \frac{\beta}{2\sigma^2}(\partial_a \varphi y^a) \left[\partial^i \varphi \underset{kj}{\mathcal{A}}(\hat{k}_k y_j) + \hat{k}^i \underset{kj}{\mathcal{A}}(\partial_k \varphi y_j) \right] - \\
& - \frac{1}{\sigma}(\partial_a \varphi y^a) \hat{k}^b \underset{jk}{\mathcal{A}}(\overset{(a)}{\Gamma}{}^i_{bj} y_k) - \frac{1}{\sigma} \hat{k}_a y^b \partial^i \varphi \underset{jk}{\mathcal{A}}(\overset{(a)}{\Gamma}{}^a_{bj} y_k) - \\
& - \frac{1}{\sigma} \partial_a \varphi y^b \hat{k}^i \underset{kj}{\mathcal{A}}(\overset{(a)}{\Gamma}{}^a_{bk} y_j) - \frac{1}{2\sigma^2}(\partial_a \varphi y^a)^2 \hat{k}^i \underset{jk}{\mathcal{A}}(y_j \hat{k}_k),
\end{aligned}
\tag{56}
$$

where $\overset{(a)}{R}{}^i_{ajk}$ is the Riemannian curvature of the metric a_{ij}.

The torsion P^i_{jk} is given by (21):

$$
P^i_{jk} = \overset{(a)}{\Gamma}{}^i_{jk} + \frac{1}{\sigma} a^{mi} \left[\underset{lm}{\mathcal{A}}(\partial_l \varphi \, \hat{k}_m) y_j + a_{jl} \underset{rm}{\mathcal{A}}(\partial_r \varphi \, \hat{k}_m) y^r \right] - L^i_{kj},
\tag{57}
$$

then

$$
\begin{aligned}
P_{ijk} = {} & \frac{F}{\sigma} \overset{(a)}{\Gamma}_{jki} - \frac{\beta F}{2\sigma^2} a_{jk} \partial_i \varphi + \left[\frac{F + m\sigma\varphi^2}{2\sigma^2} \partial_i \varphi \, y^i - \frac{\beta \varphi^2}{2\sigma} \partial_i \varphi \hat{k}^i \right] a_{jk} \hat{k}_i \\
& + \varphi^2 \hat{k}_l \overset{(a)}{\Gamma}{}^l_{jk} \hat{k}_i + (\varphi^2 \hat{k}_l + \frac{\varphi}{\sigma} y_l) \overset{(a)}{\Gamma}{}^l_{jk} \hat{k}_i - \frac{F}{2\sigma^2} y_j \hat{k}_k \partial_i \varphi + \\
& + \left(\frac{\sigma + 2\beta\varphi + m\sigma\varphi^2}{2\sigma^2} \right) y_j \partial_k \varphi \, \hat{k}_i
\end{aligned}
$$

$$-\left(\frac{\varphi^2}{2\sigma}(\partial_i\varphi\,\hat{k}^i)+\frac{\varphi}{2\sigma^2}(\partial_i\varphi\,y^i)\right)y_j\hat{k}_k\hat{k}_i+\left(\frac{\varphi}{\sigma}\hat{k}_l-\frac{\beta\varphi}{\sigma^3}y_l\right)\overset{(a)}{\Gamma}{}^l_{jk}y_i+$$
$$+\left(\frac{m\varphi}{2\sigma^2}\partial_i\varphi\,y^i-\frac{\beta\varphi}{2\sigma^2}\partial_i\varphi\,\hat{k}^i\right)a_{jk}y_i+\frac{(m\sigma^2-\beta^2)\varphi}{2\sigma^4}y_j\partial_k\varphi\,y_i+$$
$$+\left(\frac{\beta\varphi}{2\sigma^4}\partial_i\varphi\,y^i-\frac{\varphi}{2\sigma^2}\partial_i\varphi\,\hat{k}^i\right)y_j\hat{k}_k y_i-\gamma_{kji}-\underset{jk\{i\}}{\mathcal{G}}(C_{jkl}N^l_i)-f_{ih}L^h_{kj}. \qquad (58)$$

The h-covariant derivative of the C_{ijk} coefficients is

$$C_{ijk|l}=\delta_l C_{ijk}-L^h_{il}C_{hjk}-L^h_{jl}C_{ihk}-L^h_{kl}C_{ijh}. \qquad (59)$$

From relations (21), (24), (44), (54), (55), (57), and (59) we can calculate the P_{ijkl} curvature.

Taking into account relations:

$$\delta_l L^i_{jk}=\delta_l f^{ir}\left(\gamma_{jkr}-\underset{jk\{r\}}{\mathcal{G}}(C_{jkh}N^h_r)\right)+f^{ir}\Big(\delta_l\gamma_{jkr}-\Big[(\delta_l N^h_j)C_{rkh}+$$
$$+N^h_j(\delta_l C_{rkh})+(\delta_l N^h_k)C_{jrh}+N^h_k(\delta_l C_{jrh})-(\delta_l N^h_r)C_{jkh}-$$
$$-N^h_r(\delta_l C_{jkh})\Big]\Big), \qquad (60)$$

$$\delta_l\gamma_{jkr}=\left(\frac{1}{\sigma}\delta_l F-\frac{F}{\sigma^2}\delta_l\sigma\right)\overset{(a)}{\Gamma}{}^r_{jk}+\frac{F}{\sigma}\delta_l\overset{(a)}{\Gamma}{}^r_{jk}+\delta_l\Lambda_{jkr}+\delta_l M_{jkr}, \qquad (61)$$

$$\delta_k N^i_j=\frac{\partial\overset{(a)}{\Gamma}{}^i_{jr}}{\partial x^k}y^r+\left(\frac{1}{2\sigma}\frac{\partial a_{mn}}{\partial x^k}y^m y^n a^{hi}+\sigma\frac{\partial a^{hi}}{\partial x^k}\right)\underset{jh}{\mathcal{A}}(\partial_j\varphi\,\hat{k}_h)+$$
$$+\sigma a^{mi}\left[\underset{jm}{\mathcal{A}}\left((\partial^2_{kj}\varphi)\,\hat{k}_m\right)\right]+\frac{1}{\sigma}a^{mi}\left[\underset{rm}{\mathcal{A}}\left((\partial^2_{kr}\varphi)\,\hat{k}_m\right)\right]y^r y_j+$$
$$+\left(\frac{1}{\sigma}\frac{\partial a^{mi}}{\partial x^k}-\frac{1}{2\sigma^3}\frac{\partial a_{pn}}{\partial x^k}y^p y^n a^{mi}\right)\underset{rm}{\mathcal{A}}\left(\partial_r\varphi\,\hat{k}_m\right)y^r y_j-$$
$$-\frac{\beta}{2}\partial^i\varphi\underset{jk}{\mathcal{A}}\left(\partial_j\varphi\,\hat{k}_k\right)-\sigma a^{hl}\overset{(a)}{\Gamma}{}^i_{jh}\underset{kl}{\mathcal{A}}\left(\partial_k\varphi\,\hat{k}_l\right)+\frac{m}{4}\partial^i\varphi\,\partial_k\varphi\,y_j-$$
$$-\frac{1}{4}(\partial_a\varphi\,\hat{k}^a)\partial^i\varphi\,\hat{k}_k y_j-\frac{1}{2}\partial_h\varphi\hat{k}^i\left[a^{hl}\underset{kl}{\mathcal{A}}(\partial_k\varphi\,\hat{k}_l)y_j+y^h\underset{kj}{\mathcal{A}}(\partial_k\varphi\,\hat{k}_j)\right]-$$
$$-\left(\frac{\beta}{2\sigma}\right)^2\partial^i\varphi\partial_j\varphi y_k-\frac{\beta}{2\sigma}\left[a_{mj}\overset{(a)}{\Gamma}{}^m_{ka}y^a\partial^i\varphi-\partial^a\varphi\overset{(a)}{\Gamma}{}^i_{ja}y_k\right]-$$
$$-\frac{\beta}{2\sigma^2}\partial_a\varphi\underset{ia}{\mathcal{A}}(\partial^i\varphi\,\hat{k}^a)y_j y_k+\frac{\beta}{4\sigma^2}(\partial_a\varphi y^a)\partial^i\varphi\,\hat{k}_j y_k-$$
$$-\frac{1}{\sigma}y^b\overset{(a)}{\Gamma}{}_{kba}y_j\underset{ai}{\mathcal{A}}(\partial^a\varphi\,\hat{k}^i)-\frac{1}{2\sigma}y^a\Big(\partial_a\varphi\,\hat{k}^b\overset{(a)}{\Gamma}{}^i_{jb}y_k+$$
$$+(\partial_b\varphi\,y^b)a_{jl}\overset{(a)}{\Gamma}{}^l_{ka}y^a\hat{k}^i\Big)+\frac{\beta}{4\sigma^2}(\partial_b\varphi\,y^b)\hat{k}^i\partial_j\varphi\,y_k+$$

$$+\frac{m}{4\sigma^2}(\partial_b\varphi\, y^b)\partial^i\varphi\, y_j y_k - \overset{(a)}{\Gamma}{}^{l}_{kb}\overset{(a)}{\Gamma}{}^{i}_{jl} y^b -$$
$$-\frac{1}{2\sigma^2}(\partial_b\varphi\, y^b)\partial_a\varphi\, a^{al}\hat{k}^i y_k \underset{lj}{\mathcal{S}}(\hat{k}_l y_j), \tag{62}$$
$$\delta_l\beta = -\hat{k}_h N^h_l, \tag{63}$$
$$\delta_l\sigma = \frac{1}{2\sigma}\partial_l a_{ij} y^i y^j - N^h_l(\frac{1}{\sigma}a_{ih}y^i), \tag{64}$$
$$\delta_l F = \frac{1}{2\sigma}\partial_l a_{ij} y^i y^j + \partial_l\varphi\,\beta - N^h_l(\frac{1}{\sigma}a_{ih}y^i + \varphi(x)\hat{k}_h), \tag{65}$$

$$\delta_l y_i = -a_{ih}N^h_l, \tag{66}$$
$$\delta_l C_{ijk} = \frac{3}{2}\left(\frac{\varphi}{\sigma^5}\delta_l\beta + \frac{\beta}{\sigma^5}\partial_l\varphi - 5\frac{\beta\varphi}{\sigma^6}\delta_l\sigma\right) y_i y_j y_k -$$
$$-\frac{3\beta\varphi}{2\sigma^5}\left(a_{ih}N^h_l y_j y_k + a_{jh}N^h_l y_i y_k + a_{kh}N^h_l y_i y_j\right) +$$
$$+3\left(\frac{1}{\sigma}\partial_l\varphi - \frac{\varphi}{\sigma^2}\delta_l\sigma\right)\underset{ijk}{\mathcal{S}}(a_{ij}\hat{k}_k) + 3\frac{\varphi}{\sigma}\partial_l[\underset{ijk}{\mathcal{S}}(a_{ij}\hat{k}_k)] -$$
$$-3\left(\frac{1}{\sigma^3}\partial_l\varphi - 3\frac{\varphi}{\sigma^4}\delta_l\sigma\right)\underset{ijk}{\mathcal{S}}(y_i y_j\hat{k}_k) - \frac{3\varphi}{\sigma^3}\delta_l(\underset{ijk}{\mathcal{S}}(y_i y_j \hat{k}_k)) -$$
$$-3\left(\frac{\varphi}{\sigma^3}\delta_l\beta + \frac{\beta}{\sigma^3}\partial_l\varphi - 3\frac{\beta\varphi}{\sigma^4}\delta_l\sigma\right)\underset{ijk}{\mathcal{S}}(a_{ij}y_k) - \frac{3\beta\varphi}{\sigma^3}\delta_l\left(\underset{ijk}{\mathcal{S}}(a_{ij}y_k)\right), \tag{67}$$
$$\delta_l\Lambda_{ijk} = \frac{3}{2}\left(\frac{\varphi}{\sigma^5}\delta_l\beta + \frac{\beta}{\sigma^5}\partial_l\varphi - 5\frac{\beta\varphi}{\sigma^6}\delta_l\sigma\right)\underset{ij\{k\}}{\mathcal{G}}(y_i y_j \partial_k a_{ab})y^a y^b +$$
$$+\frac{3\beta\varphi}{2\sigma^5}\delta_l\left[\underset{ij\{k\}}{\mathcal{G}}(y_i y_j\partial_k a_{ab})y^a y^b\right] -$$
$$-\left(\frac{1}{\sigma^3}\partial_l\varphi - 3\frac{\varphi}{\sigma^4}\delta_l\sigma\right)\underset{ij\{k\}}{\mathcal{G}}\left((y_i\hat{k}_j + y_j\hat{k}_i)\partial_k a_{ab}\right)y^a y^b -$$
$$-\frac{\varphi}{\sigma^3}\delta_l\left[\underset{ij\{k\}}{\mathcal{G}}\left((y_i\hat{k}_j + y_j\hat{k}_i)\partial_k a_{ab}\right)y^a y^b\right] -$$
$$-\left(\frac{\varphi}{4\sigma^3}\delta_l\beta + \frac{\beta}{4\sigma^3}\partial_l\varphi - 3\frac{\beta\varphi}{4\sigma^4}\delta_l\sigma\right)\underset{ij\{k\}}{\mathcal{G}}(a_{ij}\partial_k a_{ab})\, y^a y^b -$$
$$-\frac{\varphi\beta}{4\sigma^3}\delta_l\left[\underset{ij\{k\}}{\mathcal{G}}(a_{ij}\partial_k a_{ab})\, y^a y^b\right], \tag{68}$$
$$\delta_l M_{ijk} = \frac{1}{2}\left(\frac{1}{\sigma}\delta_l\beta - \frac{\beta}{\sigma^2}\delta_l\sigma\right)\underset{ij\{k\}}{\mathcal{G}}(a_{ij}\partial_k\varphi) + \frac{\beta}{2\sigma}\delta_l\left[\underset{ij\{k\}}{\mathcal{G}}(a_{ij}\partial_k\varphi)\right] -$$
$$-\frac{1}{\sigma^2}\delta_l\sigma\underset{ij\{k\}}{\mathcal{G}}\left((y_i\hat{k}_j + y_j\hat{k}_i)\partial_k\varphi\right) + \frac{1}{\sigma}\delta_l\left[\underset{ij\{k\}}{\mathcal{G}}\left((y_i\hat{k}_j + y_j\hat{k}_i)\partial_k\varphi\right)\right] -$$

$$- \left(\frac{1}{\sigma^3} \delta_l \beta - 3 \frac{\beta}{\sigma^4} \delta_l \sigma \right) \underset{ij\{k\}}{\mathcal{G}} (y_i y_j \partial_k \varphi) - \frac{\beta}{\sigma^3} \delta_l \left[\underset{ij\{k\}}{\mathcal{G}} (y_i y_j \partial_k \varphi) \right] +$$
$$+ 2\partial_l \varphi \underset{ij\{k\}}{\mathcal{G}} \left(\hat{k}_i \hat{k}_j \partial_k \varphi \right) + 2\varphi \partial_l \left[\underset{ij\{k\}}{\mathcal{G}} \left(\hat{k}_i \hat{k}_j \partial_k \varphi \right) \right] \tag{69}$$

and (22), (55), (54) we may calculate the R^i_{jkl} curvature explicitly from (56).

The S-curvature (23) is:

$$S_{jikh} = \frac{\varphi^2(m\sigma^2 - \beta^2)}{2F\sigma^3} \underset{ji}{\mathcal{A}}(a_{hj} a_{ik}) + \frac{\varphi^2}{2F\sigma} \left((\underset{ij}{\mathcal{A}}(a_{ki} \hat{k}_j) \hat{k}_h + \underset{ji}{\mathcal{A}}(a_{hj} \hat{k}_i) \hat{k}_k) \right) +$$
$$+ \frac{\beta\varphi^2}{2F\sigma^3} \left((\underset{ji}{\mathcal{A}}(a_{kj} \hat{k}_i) y_h + \underset{kh}{\mathcal{A}}(a_{jk} \hat{k}_h) y_i) \right) + \frac{\varphi^2}{2F\sigma^3} \left(\hat{k}_h y_k \underset{ij}{\mathcal{A}}(\hat{k}_i y_j) + \right.$$
$$\left. + \hat{k}_k y_h \underset{ji}{\mathcal{A}}(\hat{k}_j y_i) \right) + \frac{\beta\varphi^2}{2F\sigma^3} \left(\underset{hk}{\mathcal{A}}(a_{ih} \hat{k}_k) y_j + \underset{ij}{\mathcal{A}}(a_{hi} \hat{k}_j) y_k \right) +$$
$$+ \frac{\varphi^2(m\sigma^2 - 2\beta^2)}{4F\sigma^5} \left(\underset{hk}{\mathcal{A}}(a_{ih} y_k) y_j + \underset{kh}{\mathcal{A}}(a_{jk} y_h) y_i \right), \tag{70}$$

$$S^r_{ikh} = \frac{(m\sigma^2 - \beta^2)}{2F^2\sigma^2} \underset{hk}{\mathcal{A}}(\delta^r_h a_{ki}) + \frac{\varphi^2}{2F^2} \left(\hat{k}_i \underset{hk}{\mathcal{A}}(\delta^r_h \hat{k}_k) + \hat{k}^r \underset{kh}{\mathcal{A}}(a_{ki} \hat{k}_h) \right) +$$
$$+ \frac{\beta\varphi^2}{2F^2\sigma^2} \left(\delta^r_k \underset{ih}{\mathcal{S}}(\hat{k}_i y_h) - \delta^r_h \underset{ik}{\mathcal{S}}(\hat{k}_i y_k) - \right) + \frac{\beta\varphi^2}{2F^2\sigma^2} \hat{k}^r \underset{hk}{\mathcal{A}}(a_{ih} y_k) +$$
$$+ \frac{(m\sigma^2 - 2\beta^2)\varphi^2}{2F^2\sigma^4} y_i \underset{kh}{\mathcal{A}}(\delta^r_k y_h) + \frac{\varphi^2}{2F^2\sigma^2} \left(\hat{k}^r y_i \underset{kh}{\mathcal{A}}(\hat{k}_k y_h) + y^r \hat{k}_i \underset{hk}{\mathcal{A}}(\hat{k}_h y_k) \right) +$$
$$+ \frac{\varphi^2(\beta\sigma - \beta^2\varphi + 2m\sigma^2\varphi)}{2F^3\sigma^2} y^r \underset{hk}{\mathcal{A}}(a_{ih} \hat{k}_k) +$$
$$+ \frac{2\beta^2\varphi^2 - m\sigma^2\varphi^2 + \beta m\sigma\varphi^3}{2F^3\sigma^3} y^r \underset{kh}{\mathcal{A}}(a_{ik} y_h) +$$
$$+ \frac{(m\sigma^2 - \beta^2)\varphi^3}{F^3\sigma^4} y^r y_i \underset{kh}{\mathcal{A}}(\hat{k}_k y_h), \tag{71}$$

$$S_{ih} = -\frac{3(m\sigma^2\varphi^2 - \beta^2\varphi^2)}{4F^2\sigma^2} a_{ih} - \frac{\varphi^2}{4F^2} \hat{k}_i \hat{k}_h +$$
$$+ \frac{\beta\varphi^2}{2F^2\sigma^2} \underset{ih}{\mathcal{S}}(\hat{k}_i y_h) + \frac{3m\sigma^2\varphi^2 - 4\beta^2\varphi^2}{4F^2\sigma^4} y_i y_h, \tag{72}$$
$$S = \frac{5(\beta^2 - m\sigma^2)\varphi^2}{2\sigma F^3}. \tag{73}$$

From a physical point of view the S-curvature can be considered as a curvature parameter of anisotropy as it is evident from relation (73). In the absence of anisotropy $\varphi = 0$, we have $S = 0$. In other words, S represents the measure of anisotropy of matter [6].

4 Conclusion

The observed anisotropy of the microwave cosmic rediation, represented by a vector $k_a(x)$, can be incorporated in the framework of Finsler geometry. The equations of geodesics are generalized in a Finsler anisotropic space-time. The calculation of a curvature parameter of anisotropy is performed explicitly by the contraction of the S^i_{jkl} curvature.

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