An algebraic computing program for studying cosmological models without singularities

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Abstract

In order to study cosmological models, which involve tensorial operations, we have conceived an analytical program using GRTensorII package for Maple 8. This program use a de-Sitter gauge teory of gravitational field over a spherical symmetric Minkowski space-time. We define the gauge fields with GRTensorII, we choose a particular ansatz and using special commands we compute the components of the strength tensor and of other quantities defined with gauge fields and strength tensor. Using some invariants of strength tensor like higher derivative terms in the integral of action we obtain field equations with nonsingular solutions.

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1 Introduction

The gauge theory of gravitation allows to describe the gravity in a similar way with other interactions (electromagnetic, weak, strong). As gauge group of gravitation we use the de-Sitter group in order to obtain models with cosmological constant for the gravitational field. The Poincaré gauge theory is obtained as a limit of de-Sitter model when the cosmological constant vanishes.

The Section 2 is devoted to the formulation of the de-Sitter gauge model on a spherical symmetric Minkowski space-time. The general expressions for the components $F^A_{\mu\nu}$ of the strength tensor of the gauge fields are obtained. A particular ansatz for the gauge fields is chosen and the corresponding components $F^A_{\mu\nu}$ are presented in Section 3. The tensorial operations involve a great number of calculations, and that imposes computer implementation. From this point of view, the symbolic programs, as Maple, are appropriate. The calculations are performed with analitical programs using the GRTensorII computer algebra package, running on the Maple 8 platform. An algebraic computing program for the developed gauge theory of gravitation is presented in Section 3.

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In order to obtain solutions without singularities in the gauge theory of gravitation in Section 4 we choose to develope models with higher derivative corrections in the case of null torsion. One of these begin with an action with two invariants I_1 and I_2 coupled by two Lagrange multiplier $\psi_1(t)$, $\psi_2(t)$ and a potential $V(\psi_1, \psi_2)$. The other model, for heterotic string cosmology, is a gravidilaton model with string α' corrections. The both models implement restrictions on the invariants like the so-called "limiting curvature hypothesis" in the cosmology with metrics.

For the particular ansatz of spherically symmetric gauge fields of Section 3 we write the equations of motion in Section 4. Solutions for the field equations are presented in Section 5. These solutions are nonsingular.

2 Gauge theory of gravitation

We consider a gauge theory of gravitation having deSitter (DS) group as local symmetry. Let X_A, $A = 1, 2, ..., 10$, be a basis of DS Lie algebra with the corresponding equations of structure given by [3]:

$$[X_A, X_B] = i f_{AB}^{C} X_C, \tag{2.1}$$

where $f_{AB}^{C} = -\ f_{BA}^{C}$ are the constants of structure whose concrete expressions will be given below [see eq.(2.4)]. We envision space-time as a four-dimensional manifold M_4; at each point we have a copy of DS group. Introduce, as usually, 10 gauge fields $h_\mu^A(x)$, $A = 1, 2, ..., 10$, $\mu = 0, 1, 2, 3$, where (x) denotes the local coordinates on M_4. Then, we construct the tensor of the gauge fields (strength tensor) $F_{\mu\nu} = F_{\mu\nu}^A X_A$ which takes its values in the Lie algebra of the DS group (Lie algebra-valued tensor). The components of this tensor are given by:

$$F_{\mu\nu}^A = \partial_\mu h_\nu^A - \partial_\nu h_\mu^A + f_{BC}^A h_\mu^B h_\nu^C. \tag{2.2}$$

In order to write the constants of structure f_{AB}^C, we use the following notation for the index A:

$$A = \begin{cases} a = 0, 1, 2, 3, \\ [ab] = [01], [02], [03], [12], [13], [23]. \end{cases} \tag{2.3}$$

This means that A can stand for a single index like 2 as well as for a pair of indices like $[01], [12]$ etc. The infinitesimal generators X_A are interpreted as: $X_a \equiv P_a$ (energy-momentum operators) and $X_{[ab]} \equiv M_{ab}$ (angular momentum operators) with property $M_{ab} = -M_{ba}$, $a, b = 0, 1, 2, 3$ [1]. For the constants of structure f_{AB}^C we find the following expressions:

$$\begin{aligned} f_{bc}^a &= f_{c[de]}^{[ab]} = f_{[bc][de]}^a = 0, \\ f_{cd}^{[ab]} &= 4\lambda^2 \left(\delta_c^b \delta_d^a - \delta_c^a \delta_d^b\right) = -f_{dc}^{[ab]}, \\ f_{b[cd]}^a &= -f_{[cd]b}^a = \frac{1}{2}\left(\eta_{bc}\delta_d^a - \eta_{bd}\delta_c^a\right), \\ f_{[ab][cd]}^{[ef]} &= \frac{1}{4}\left(\eta_{bc}\delta_a^e\delta_d^f - \eta_{ac}\delta_b^e\delta_d^f + \eta_{ad}\delta_b^e\delta_c^f - \eta_{bd}\delta_a^e\delta_c^f\right) - e \longleftrightarrow f, \end{aligned} \tag{2.4}$$

where λ is a real parameter, and $\eta_{ab} = diag(1, -1, -1, -1)$ is the Minkowski metric. In fact here we have a deformation of DS Lie algebra having λ as parameter. When $\lambda \to 0$, we obtain the Poincaré Lie algebra, i.e. the DS group contracts to the Poincaré group.

We will denote the gauge fields (or potentials) $h_\mu^A(x)$ by $e_\mu^a(x)$ (tetrad fields) if $A = a$ and by $\omega_\mu^{ab}(x) = -\omega_\mu^{ba}(x)$ (spin connection) if $A = [ab]$. Then, introducing the relations (2.4) into the definition (2.2), we find the following expressions of the strength tensor components:

$$F^a_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu + \left(\omega^{ab}_\mu e^c_\nu - \omega^{ab}_\nu e^c_\mu\right)\eta_{bc}, \tag{2.5}$$

$$F^{ab}_{\mu\nu} = \partial_\mu \omega^{ab}_\nu - \partial_\nu \omega^{ab}_\mu + \left(\omega^{ac}_\mu \omega^{db}_\nu - \omega^{ac}_\nu \omega^{db}_\mu\right)\eta_{cd} - 4\lambda^2\left(e^a_\mu e^b_\nu - e^a_\nu e^b_\mu\right). \tag{2.6}$$

The action associated to the gravitational gauge fields, quadratic in the components $F^A_{\mu\nu}$, is writen in the form [4]:

$$S_g = \int d^4x \epsilon^{\mu\nu\rho\sigma} F^A_{\mu\nu} F^B_{\rho\sigma} Q_{AB}, \tag{2.7}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita symbol of rang four. This action is independent of any specific metric on M_4. The quantities Q_{AB} are constants, symmetric with respect to the indices A, B: $Q_{AB} = Q_{BA}$. If we chose [1]:

$$Q_{AB} = \begin{cases} \epsilon_{abcd}, \quad for \quad A = [ab], \quad B = [cd] \\ 0 \qquad\qquad\qquad\qquad\quad otherwise \end{cases} \tag{2.8}$$

then we obtain the action of the General Relativity (GR).

3 An analytical program for de-Sitter gauge theory of gravitation

We develop a gauge theory of the DS group in a 4-dimensional Minkowski space-time M_4, endowed with spherical symmetry:

$$ds^2 = dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{3.1}$$

In addition we chose a particular form of gauge fields of the DS group $e^a_\mu(x)$ and $\omega^{ab}_\mu(x)$ given by the following ansatz:

$$\begin{aligned} e^0_\mu &= (N(t), 0, 0, 0), \qquad e^1_\mu = \left(0, \frac{a(t)}{\sqrt{1-kr^2}}, 0, 0\right), \\ e^2_\mu &= (0, 0, ra(t), 0), \qquad e^3_\mu = (0, 0, 0, ra(t)\sin\theta) \end{aligned} \tag{3.2}$$

respectively

$$\begin{aligned} \omega^{01}_\mu &= (0, U(t,r), 0, 0), \qquad \omega^{02}_\mu = (0, 0, V(t,r), 0), \\ \omega^{03}_\mu &= (0, 0, 0, W(t,r)\sin\theta), \qquad \omega^{12}_\mu = (0, 0, Y(t,r), 0), \\ \omega^{13}_\mu &= (0, 0, 0, Z(t,r)\sin\theta), \qquad \omega^{23}_\mu = (0, 0, 0, \cos\theta), \end{aligned} \tag{3.3}$$

where $N(t)$ and $a(t)$ are functions only of the time variable t; k is a constant; $U(t,r), V(t,r), W(t,r), Y(t,r)$ and $Z(t,r)$ are functions of time t and three-dimensional radius r. We use the above expressions to compute the components of the tensors, $F^a_{\mu\nu}$ and $F^{ab}_{\mu\nu}$. The calculations are performed using an analytical program conceived by us using GRTensorII package for Maple 8.

The program call the GRTensorII package with *grtw( )* and load the Minkowski metric in spherical coordinates (with *grload( )*), which already exists in *Metrics* director of *Grtii(6)*. The gauge fields e^a_μ (denoted by ev) and ω^{ab}_μ(denoted by omega) are defined in the program by *grdef( )*. After the definition, these potentials are introduced during of the running program by the command *grcalc( )*. For our purpose it is necessary to define and introduce $\eta_{ab} = diag(1,-1,-1,-1)$ and we will use Kronecker delta *kdelta(up,dn)*. GRTensorII allows to define new tensors outside of the standard GRTensorII library and so we can calculate the components of the strength tensor field $F^a_{\mu\nu}$, respectively $F^{ab}_{\mu\nu}$. In program we denoted $F^a_{\mu\nu}$ by Famn and $F^{ab}_{\mu\nu}$ by Fabmn and we define these components with (2.5) and (2.6).

Program "gauge-theory.mws"

```
restart: grtw():
grload(minknou, `c:/grtii(6)/metrics/minknou.mpl`);
grdef(`ev{^a miu}`);   grcalc(ev(up,dn));
grdef(`omega{[^a ^b] miu}`);  grcalc(omega(up,up,dn));
grdef(`eta1{(a b)}`);  grcalc(eta1(dn,dn));
grdef(`Famn{^a miu niu} := ev{^a niu,miu} - ev{^a miu,niu}
                    + omega{^a ^b miu}*ev{^c niu}*eta1{b c}
                    - omega{^a^b niu}*ev{^c miu}*eta1{b c}`);
grcalc(Famn(up,dn,dn)); grdisplay(_);
grdef(`Fabmn{^a ^b miu niu} := omega{^a ^b niu, miu}
      -omega{^a ^b miu, niu} + (omega{^a ^c miu}*omega{^d ^b niu}
      -omega{^a ^c niu}*omega{^d ^b miu})*eta1{c d}
      -4*lambda^2*(ev{^a miu}*ev{^b niu}-ev{^b miu}*ev{^a niu})`);
grcalc(Fabmn(up,up,dn,dn)); grdisplay(_);
```

The non-null components of $F^a_{\mu\nu}$ and $F^{ab}_{\mu\nu}$ are:

$$\begin{aligned} F^1_{01} &= \frac{\dot a}{\sqrt{1-kr^2}} + UN, \quad F^2_{02} = r\dot a + VN, \quad F^3_{03} = (r\dot a + WN)\sin\theta, \\ F^2_{12} &= a - \frac{V}{\sqrt{1-kr^2}}, \quad F^3_{13} = a\sin\theta\left(1 - \frac{Z}{\sqrt{1-kr^2}}\right), \end{aligned} \tag{3.4}$$

respectively

$$
\begin{aligned}
F^{12}_{12} &= -\left(\frac{\partial Y}{\partial r} + UV + \frac{4\lambda^2 r a^2}{\sqrt{1-kr^2}}\right), \qquad F^{02}_{12} = \frac{\partial V}{\partial r} - UY,\\
F^{13}_{13} &= \left(\frac{\partial Z}{\partial r} - UW - 4\lambda^2 r a^2\right)\sin\theta, \qquad F^{03}_{13} = \left(\frac{\partial W}{\partial r} - UZ\right)\sin\theta,\\
F^{01}_{01} &= \frac{\partial U}{\partial t} - \frac{4\lambda^2 N a}{\sqrt{1-kr^2}}, \qquad F^{03}_{23} = (W-V)\cos\theta,\\
F^{23}_{23} &= \left(-1 + ZY + 4\lambda^2 r^2 a^2 + WV\right)\sin\theta, \qquad F^{13}_{23} = \frac{\partial Z}{\partial t}\sin\theta,\\
F^{12}_{02} &= \frac{\partial Y}{\partial t}, \qquad F^{02}_{02} = \frac{\partial V}{\partial t} - 4\lambda^2 N r a,\\
F^{03}_{03} &= \left(\frac{\partial W}{\partial t} - 4\lambda^2 N r a\right)\sin\theta, \qquad F^{13}_{03} = \frac{\partial Z}{\partial t}\sin\theta.
\end{aligned}
\tag{3.5}
$$

where $\dot{a}$ is the derivative of $a(t)$ with respect to the variable t.

If we assume that all the components $F^a_{\mu\nu}$ of the strength tensor vanish and if we remember the Riemann-Cartan theory of gravitation, then the torsion tensor $T^\rho_{\mu\nu} = \bar{e}^\rho_a F^a_{\mu\nu}$ vanish, in acccord with GR theory. Here, $\bar{e}^\rho_a$ denotes the invers of e^a_μ with the properties:

$$e^a_\mu \bar{e}^\mu_b = \delta^a_b, \qquad e^a_\mu \bar{e}^\nu_a = \delta^\nu_\mu.$$

From this condition and Eq. (3.4) we obtain the following constraints:

$$
\begin{aligned}
U(t,r) = -\frac{\dot{a}(t)}{N(t)\sqrt{1-kr^2}} \quad &, \qquad V(t,r) = W(t,r) = -\frac{r\dot{a}(t)}{N(t)}\\
Y(t,r) \quad &= \quad Z(t,r) = \sqrt{1-kr^2}.
\end{aligned}
\tag{3.6}
$$

Using these constraints the program compute the resulting components $F^{ab}_{\mu\nu}$ of the strength tensor and the quantities $F^a_\mu = F^{ab}_{\mu\nu}\bar{e}^\nu_b, \quad F = F^{ab}_{\mu\nu}\bar{e}^\mu_a\bar{e}^\nu_b, \quad e = det(e^a_\mu)$:

Program "gauge-theory.mws" (continued)

```
U(t,r):=-diff(a(t),t) /(N(t)*sqrt(1-k*r^2));
V(t,r):=-r*diff(a(t),t)/N(t);
W(t,r):=-r*diff(a(t),t)/N(t);
Y(t,r):=sqrt(1-k*r^2);
Z(t,r):=sqrt(1-k*r^2);
grdef(`Fabmn{^a ^b miu niu} := omega{^a ^b niu, miu}
      -omega{^a ^b miu, niu} + (omega{^a ^c miu}*omega{^d ^b niu}
      -omega{^a ^c niu}*omega{^d ^b miu})*eta1{c d}
      -4*lambda^2*(ev{^a miu}*ev{^b niu}-ev{^b miu}*ev{^a niu})`);
grcalc(Fabmn(up,up,dn,dn)); grdisplay(_);
grdef(`evi{^miu a}`);  grcalc(evi(up,dn));
grdef(`F:=Fabmn{^a ^b miu niu}*einv{a ^miu}*einv{b ^niu }`);
grcalc(F);   grdisplay(_);
```

```
grdef(‘Fam{^a miu}:=Fabmn{^a ^b miu niu}*einv{b^niu }‘);
grcalc(Fam(up,dn));    grdisplay(_);
grdef(‘de‘);  grcalc(de);
```

With this results the components $F^{ab}_{\mu\nu}$ of the strength tensor becomes:

$$F^{12}_{12} = -\frac{r}{N^2\sqrt{1-kr^2}}\left(kN^2+4\lambda^2a^2N^2+\dot{a}^2\right),$$

$$F^{13}_{13} = \frac{r\sin\theta}{N^2\sqrt{1-kr^2}}\left(kN^2+4\lambda^2a^2N^2+\dot{a}^2\right), \tag{3.7}$$

$$F^{01}_{01} = -\frac{\ddot{a}N-\dot{a}\dot{N}+4\lambda^2aN^3}{N^2\sqrt{1-kr^2}}, \qquad F^{23}_{23} = -\frac{r^2\sin\theta}{N^2}\left(kN^2+4\lambda^2a^2N^2+\dot{a}^2\right),$$

$$F^{02}_{02} = -\frac{r}{N^2}\left(\ddot{a}N-\dot{a}\dot{N}+4\lambda^2aN^3\right), \quad F^{03}_{03} = -\frac{r\sin\theta}{N^2}\left(\ddot{a}N-\dot{a}\dot{N}+4\lambda^2aN^3\right),$$

and the scalar F is:

$$F = -6\frac{a\ddot{a}N-a\dot{a}\dot{N}+kN^3+\dot{a}^2N+8\lambda^2a^2N^3}{a^2N^3}. \tag{3.8}$$

4 Actions and field equations for cosmological models without singularities

Using proper invariants of the strength tensor in an action with higher derivatives terms [5, 6, 7] we obtain field equations with nonsingular solutions.

First we work with an action with two invariants I_1, I_2 and correspondently two Lagrange multiplier ψ_1, ψ_2:

$$S = -\frac{1}{16\pi G}\int d^4xe\left[F+\psi_1f_1(I_1)+\psi_2f_2(I_2)+V(\psi_1,\psi_2)\right], \tag{4.1}$$

where the functions f_1 and f_2 are:

$$f_1(I_1)=I_1, \qquad f_2(I_2)=-\sqrt{I_2}. \tag{4.2}$$

We chose the invariants I_1, I_2 in the form:

$$I_1 = F-\sqrt{3}\left(4F^a_\mu F^\mu_a-F^2\right)^{1/2}, \tag{4.3}$$

respectively

$$I_2 = 4F^a_\mu F^\mu_a - F^2, \tag{4.4}$$

and we set the potential $V(\psi_1,\psi_2)$ to be separated:

$$V(\psi_1,\psi_2)=V_1(\psi_1)+V_2(\psi_2). \tag{4.5}$$

Then, the action (4.1) becomes:

$$S = -\frac{1}{16\pi G}\int d^4xe\left[F + \psi_1 I_1 - \psi_2\sqrt{I_2} + V_1(\psi_1) + V_2(\psi_2)\right]. \tag{4.6}$$

In order to use the variational principle $\delta S = 0$ we must express the Lagrangian in terms of $N(t)$, $\psi_1(t)$, $\psi_2(t)$ and those derivatives. The analytical program aids us to do these using the aboves calculated quantities and the commands *grdef( )*, *grcalc( )*:

Program "gauge-theory.mws" (continued)

```
grdef(`I2:=4*Fam{^a miu}*Faminv{a ^ miu-F^ 2)`);
grcalc(I2);  grdisplay(_);
grdef(`I1:=F-sqrt(3)*sqrt(I2)`);
grcalc(I1);  grdisplay(_);
grdef(`Lg:=(F+psi1(t)*I1-psi2(t)*sqrt(I2)+ V1(psi1)+V2(psi2))*de`);
grcalc(Lg));    grdisplay(_);
```

We obtain the following expressions of the invariants I_1, I_2:

$$I_1 = -12\frac{kN^2 + \dot{a}^2 + 4\lambda^2 a^2 N^2}{a^2N^2}, \tag{4.7}$$

$$I_2 = 12\frac{\left(kN^3 + \dot{a}^2N - a\ddot{a}N + a\dot{a}\dot{N}\right)^2}{a^4N^6}, \tag{4.8}$$

where $\ddot{a}$ is the second derivative of $a(t)$ with repect to t.

From the variational principle, for the particular case $N(t) = 1$, we obtain the following field equations:

$$\begin{aligned} -\frac{1}{2}(V_1+V_2) &+ 3H^2(1-2\psi_1) + 3\frac{k}{a^2}(1+2\psi_1) - 2\Lambda = \\ &= \sqrt{3}(\dot{\psi}_2 + 3H\psi_2 - \frac{k}{Ha^2}\psi_2), && (4.9)\\ \frac{k}{a^2} &+ H^2 - \frac{\Lambda}{3} = \frac{1}{12}\frac{dV_1}{d\psi_1}, && (4.10)\\ \dot{H} - \frac{k}{a^2} &= -\frac{1}{2\sqrt{3}}\frac{dV_2}{d\psi_2}, && (4.11) \end{aligned}$$

with

$$H = \frac{\dot{a}}{a}, \qquad \dot{H} = \frac{dH}{dt} = \frac{\ddot{a}a - \dot{a}^2}{a^2}, \tag{4.12}$$

where $\dot{\psi}_2$ is the derivative of $\psi_2(t)$ with respect to t, and $\Lambda = -12\lambda^2$ is interpreted as cosmological constant [4].

For heterotic string theory, in the four-dimensional Einstein frame, we work with an action with time-dependent homogenous dilaton $\phi(t)$, a potential for dilaton $V(\phi)$ and α' expansions (truncated to first order) [8, 9] given by:

$$S_s = -\frac{1}{16\pi G}\int d^4xe\left[F - 2(\nabla\phi)^2 + \frac{\alpha'}{8}\exp(-2\phi)\mathcal{L}_2 - V(\phi)\right], \tag{4.13}$$

A proper choice of invariant $\mathcal{L}_2$ is

$$\mathcal{L}_2 = F^{ab}_{\mu\nu}F^{\mu\nu}_{ab} - \frac{1}{6}F^2 \tag{4.14}$$

With the following commands:

Program "gauge-theory.mws" (continued)

```
grdef(‘L2:=Fabmn{^a ^b miu niu }*Fabmninv{a b ^ miu ^niu}-
                               (1/6)F^ 2)‘);
grcalc(L2);  grdisplay(_);
grdef(‘de‘);  grcalc(de);
grdef(‘Lgs:=(F-2*diff(phi(t),t)^2  +
                  c*exp(-2*phi(t))*L2- V(phi(t))*de‘);
grcalc(Lgs));    grdisplay(_);
```

we obtain $\mathcal{L}_2$:

$$\mathcal{L}_2 = 6\frac{\left(kN^3 + \dot{a}^2N - a\ddot{a}N + a\dot{a}\dot{N}\right)^2}{a^4N^6}, \tag{4.15}$$

and the Lagrangian of the string model. Imposing the variational principle $\delta S_s = 0$ with respect to $\phi(t)$ and $N(t)$ we obtain the corresponding field equations. These field equations for the particular case $N(t) = 1$ are:

$$12\frac{\alpha'}{8}\left(\frac{k}{a^2} - \dot{H}\right)^2 = -\exp(2\phi)\left(\frac{\partial V(\phi)}{\partial\phi} + 4\ddot{\phi} + 12H\dot{\phi}\right), \tag{4.16}$$

$$6\frac{\alpha'}{8}\left[\dot{H}^2 - 2H\ddot{H} - 6H^2\dot{H} + 4H\dot{H}\dot{\phi} + \frac{k}{a^2}\left(2H\dot{\phi} + \dot{H} - H^2 - \frac{k}{a^2}\right)\right] =$$
$$= -\exp(2\phi)\left(V(\phi) + 2\dot{\phi}^2 + 6H^2 - 4\Lambda + \frac{k}{a^2}\right) \tag{4.17}$$

where $\Lambda = -12\lambda^2$ is also interpreted as cosmological constant [3].

If we consider the limit $\lambda \to 0$ or equivalently $\Lambda = 0$, in the case of model with two Lagrange multipliers we obtain the results in Ref.[2] and for the model of heterotic string theory we obtain the results in Ref.[8]; but, for $\Lambda \neq 0$ we can study in addition the dependence on the cosmological constant of the solutions (without singularities) obtained by solving (4.9)-(4.11) and (4.16)-(4.17).

5 Example of solutions without singularities

The solution of Eqs. (4.9)-(4.11) includes a dependence on the cosmological constant Λ. We suppose that the Lagrange-multiplier function $\psi_1(t)$ is absent, and consider the cas when $k = 0$. Then, denoting $\psi_2(t) = \psi(t)$ and $V_2(\psi_2) = V(\psi)$, the eqs. (4.9)-(4.11) becomes:

$$\begin{aligned} \dot{H} &= -\frac{1}{2\sqrt{3}}\frac{dV}{d\psi}, \\ \dot{\psi} &= -3H\psi + \sqrt{3}H - \frac{1}{2\sqrt{3}H}V - \frac{2\Lambda}{\sqrt{3}H}. \end{aligned} \tag{5.1}$$

We consider the potential $V(\psi)$ of the form:

$$V(\psi) = 2\sqrt{3}\lambda^2\left(\frac{\psi^2}{1+\psi^2} + \frac{24}{\sqrt{3}}\right), \tag{5.2}$$

where λ is the real parameter that determines the cosmological constant Λ. This parameter coincides with the constant H_0 in Ref. [2] that has been interpreted as a Planck scale of the model. Therefore, in our first example the Planck scale is related to the cosmological constant Λ [4]. For small values of H and ψ, the eqs. (5.1) can be written as:

$$\begin{aligned} \dot{H} &\simeq -2\lambda^2\psi, \\ \dot{\psi}(t) &\simeq \frac{\sqrt{3}H^2 - \lambda^2\psi^2}{H}. \end{aligned} \tag{5.3}$$

These equations have the periodic solution:

$$\psi(t) = \psi_0\sin(\omega t), \qquad H(t) = \frac{\omega\psi_0}{2\sqrt{3}}[\cos(\omega t) - 1], \tag{5.4}$$

where ψ_0 is an integration constant and $\omega = 2 \times 3^{1/4}\lambda$ is the frequency of oscillation of the corresponding gravitational field described by the gauge potentials $e^a_\mu(x)$ şi $\omega^{ab}_\mu(x)$. This solution has no singularities and it is valid if the cosmological constant is negative ($\Lambda < 0$).

For the model of the heterotic string theory developed above, the solution of equations of motion (4.16)-(4.17) also includes a dependence on the cosmological constant Λ. For the case with $k = 0$ these equations of motion becomes:

$$12\frac{\alpha'}{8}\dot{H}^2 = -\exp(2\phi)\left(\frac{\partial V(\phi)}{\partial\phi} + 4\ddot{\phi} + 12H\dot{\phi}\right), \tag{5.5}$$

$$\begin{aligned} &6\frac{\alpha'}{8}\left(\dot{H}^2 - 2H\ddot{H} - 6H^2\dot{H} + 4H\dot{H}\dot{\phi}\right) = \\ &\qquad -\exp(2\phi)\left(V(\phi) + 2\dot{\phi}^2 + 6H^2 - 4\Lambda\right). \end{aligned} \tag{5.6}$$

If we set $\phi(t)$ and $V(\phi)$ to have a proper expression with some properties we obtain nonsingular solutions of equations of motion . At late time we demand $\phi \to \phi_0$

and $V(\phi) \to 4\Lambda$. In this case a de-Sitter type solution $a(t) = a_0 \exp(Ht)$ which correspond to constant $H = H_0$ satisfy the equations of motion (5.5)-(5.6) and the constraint equation (imposing the variational principle $\delta S = 0$ with respect to $a(t)$) if $H = 0$ and, hence $a(t) = const$. At $t = 0$ we will force the solutions to be with $a(t) = a_0 \exp(Ht)$. In accord with all the above properties we choose to set the dilaton $\phi(t)$ [8] to be:

$$\phi(t) = \phi_0 \tanh\left(\frac{t - t_1}{t_0}\right), \tag{5.7}$$

and the potential $V(\phi)$ in the form:

$$V(\phi) = V_0\left[(\phi + 1)^2 - 4\right] - 48\lambda^2. \tag{5.8}$$

Setting $\phi_0 = 1$, $V_0 = \frac{3}{2}H_0^2$, $t_0 = 1$ and t_1 large enough so that $\phi(t = 0) \approx -1$ and inserting (5.7) and (5.8) into (5.5), at $t \to 0$ we obtain: $\dot{H}^2 = 0$, which imply $H = const$ and using (5.6) we conclude that the parameter λ have the order of H_0 (interpreted in [4] as a Planck scale of the model). Therefore, this second model has no singularities and it is also valid if the cosmological constant is negative.

6 Concluding remarks

The gauge theory of gravitation allows a complementary description of the gravitational effects in which the mathematical structure of the underlying space-time is not affected by physical events. Only the gauge potentials $e_\mu^a(x)$ and $\omega_\mu^{ab}(x)$ of the gravitational field change as functions of coordinates. This is important when we consider a quantum gauge theory of gravitation.

The cosmological models in this paper represent solutions of equations of motion in two different higher derivative corrections of the action, but both with some invariants which depend on the strength tensor field. In ours examples the Planck scale is related to the cosmological constant Λ and the solutions has no singularities and its are valid if the cosmological constant is negative.

A gauge theory imply tensorial operations with a great number of calculations, and that imposes computer implementation, especially when we apply to some models. The invariants of the models presented in this paper request an algebraic computing program and the commads and facilities of GRTrnsorII for Maple was appropriate.

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