

# Considerations regarding the reliability of a manufacturing system

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**Abstract.** The states of a manufacturing cell are analyzed in this paper from the functional point of view. A new model has been conceived based on the graph Markov properties, and residue classes, which further offers the possibility to determine the system availability and productivity.

**M.S.C. 2000:** 60K20, 90B25, 13C20.

**Key words:** reliability, manufacturing system, Markovian property, transition probabilities, residue class, availability, productivity.

One of the main challenges of the Extended Enterprise is to maintain and to optimize the quality of the services delivered by industrial objects in a dynamic way along their life cycle. The purpose is to conceive decision aiding systems to maintain the system in operation [12]. Nevertheless, most of the automated systems do not provide the means of intelligent interpretation of the information when great process disturbances have to be considered. Moreover, decisions can be taken without a perfect perception of state of the system. This partial perception argues in favour of using a probabilistic estimation of the system state [12].

In most manufacturing systems, decision making is involved at various stages of design, planning, and operation. The role of performance modeling is to aid the decision making in an effective way. However, disruptive events such as machine breakdowns, changes in part type and volume, tool replacement, raw material and various short interruptions will cause the performance less desirable than the predicted value before disruptions [1]. This inherent problem is vital since it affects the true production capacity of the system [8]. Data concerned with automated manufacturing systems show that machine failure problem is the major cause of system under utilization, compared with other factors such as raw material, control equipment, software and workers, etc. [7]. The presence of unreliable machines clearly establishes the need for performance models that capture the reliability of the system [1].

Let it be a manufacturing system composed of two numeric controlled machine-tools  $M_1$  and  $M_2$ , which are performing the same work, at the same work piece, an industrial robot  $R$ , which pick up pieces from the feeder  $A$ , carries them to the two

machine-tools for working and in the end it places them into the evacuation device (fig.1).

$M_1$	$M_2$	$A$	$e$	$R$
1	2	3	4	5

**Fig. 1.**

Let be the set  $U = \{(f, d), f = \text{working}, d = \text{failure}\}$ ; it results that the set system states, notated  $S$ , is a subset of the Cartesian product:

$$U^5 = \{(u_1, u_2, u_3, u_4, u_5) / u_i = f \text{ or } u_i = d, \quad \forall i \in \overline{1, 5}\},$$

where  $u_i = f, i = \overline{1, 5}$ , means that the  $i$  subsystem works, while  $u_i = d, i = \overline{1, 5}$ , means that the  $i$  subsystem breakdowns.

The states set is usually strictly included in  $U^5$ , because from practical considerations it eliminates some of they elements since it exists different elements in  $U^5$ , identical functionally, and others can't be practically possible [10].

The set of failure states is noted as  $S_I, S_I \subset S$  and the set of functional states is noted as  $S_{II}, S_{II} \subset S$ ; it follows:

$$S = S_I \cup S_{II}, \quad S_I \cap S_{II} = \Phi.$$

In many performance models, calculation of probabilities of a system at different states has always been a major objective. While much of the work on performance modeling is based on the operational research techniques of queuing theory, Markovian property plays an important part in many of these models [1, 3, 5, 11].

This article presents a model about reliability based on Markovian property.

If at the moment  $t$ , the Markov chain is in the state  $k$ , the probability to remain in the same state  $k$  at the moment  $t + \Delta t$ , denoted as  $p_k(t + \Delta t)$ , is given by:

$$p_k(t + \Delta t) = \sum_{i \in \Gamma_k^{-1}} p_i(t)[p_{ik}\Delta t + o_1(\Delta t)] + p_k t \left[ 1 - \sum_{i \in \Gamma_k} p_{ik}\Delta t + o_2(\Delta t) \right] + o_3(\Delta t)$$

where:  $\Gamma_k^{-1}$  = the set of states from which can be reached the  $k$  state

$\Gamma_k$  = the set of states which can be reached from the  $k$  state

$p_i(t)$  = the probability that the process to be at the moment  $t$  in the state  $i$

$p_{ik}$  = the transition probability from the state  $i$  in the state  $k$  (break down probabilities)

$p_{ki}$  = the transition probability from the state  $k$  in the state  $i$  (repair probabilities).

Taking into account the real aspects of the problem and continuing the mathematical reasoning, it obtains the system:

$$(0.1) \quad \sum_{i \in \Gamma_k^{-1}} p_i p_{ik} = p_k \sum p_{ki}, \quad \forall k \in S \quad \sum p_k = 1, \quad k \in S,$$

where the probabilities  $p_k$ ,  $k \in S$ , are the unknowns of the system.

In the paper it proposed the following work modality: to repair the system only when the whole assembly is down; obviously there are many other possibilities.

In fig.2 it is presented the transition diagram for the proposed model; every vertex represents a state of the system. Considering the attached graph and the theoretical equations (0.1), it results the system:

$$\left\{ \begin{array}{l} \text{for } k = 1 \text{ it results : } p_{6;1} * p_6 + p_{13;1} * p_{13} + p_{17;1} * p_{17} + p_{19;1} * p_{19} + \\ p_{20;1} * p_{20} + p_{22;1} * p_{22} + p_{23;1} * p_{23} + p_{24;1} * p_{24} = p_1(p_{1;2} + p_{1;3} + p_{1;4} + p_{1;5}) \\ \text{for } k = 2 \text{ it results : } p_{1;2} * p_1 = p_2(p_{2;6} + p_{2;7} + p_{2;8} + p_{2;9}) \\ \dots\dots\dots \\ \text{for } k = 23 \text{ it results : } p_{16;23} * p_{16} = p_{23} * p_{23;1} \\ \text{for } k = 24 \text{ it results : } p_{21;24} * p_{21} = p_{24} * p_{24;1} \\ \sum_{k=1}^{24} p_k = 1. \end{array} \right.$$

The transition probabilities of a Markov chain "measure" the changes of state at successive moments, in only one step; the transition probabilities in exactly  $n$  steps, at successive moments, notated  $p(n, i, j)$ , are the elements of the  $n$  power of the transition matrix [2]. The probabilities of appearance of the unsuccessful states, at given moments, should be calculated with the help of the above probabilities [4;6].

For a complete structure of the system time evolution and to make possible the calculus of different probabilities, which represents interest from the practical view of point, the exposed model will be framed into general theory of the Markov chains pulling out some specifically properties of the model [5].

The states of a Markov chain can be grouped in classes, considering the communication criterion. It says that the state  $j$  is accessible from the state  $i$  and is noted  $i \rightarrow j$ , if exists  $n \geq 1$ , so that  $p(n, i, j) > 0$ . It says that the states  $i$  and  $j$  communicate and it is noted  $i \leftrightarrow j$ , if  $i \rightarrow j$  and  $j \rightarrow i$ .

It is defined a class of states as a maximal subset of the states space, so that any two elements of this subset communicate. A class property is the class period.

In the case of the given example the states  $S$  form a periodical class, with the period  $c = 6$ .

For the fixed  $i \in S$ , let it be defined:

$S_b = \{j \in S / p(n, i, j) > 0 \text{ implies } n \in b \pmod{6}\}$ , for  $0 \leq b \leq 5$ . The sets  $S_b$  are mutually exclusive sets and their reunion is  $S$ .

In this case it will be obtained the subclasses:

$$\begin{aligned} S_0 &= \{1\}, \\ S_1 &= \{2, 3, 4, 5\}, \\ S_2 &= \{6, 7, 8, 9, 10, 11, 12\}, \\ S_3 &= \{13, 14, 15, 16, 17, 18, 19\}, \\ S_4 &= \{20, 21, 22, 23\}, \\ S_5 &= \{24\}. \end{aligned}$$

Because  $S$  is a closed periodical class, the evolution of the Markov chain, though random, follows an certain cycle, more precisely: if  $i \in S_b$  and  $p(i, j) > 0$ , then  $j \in S_{b+1}$  (obviously with the index reduced modulo 6); thus the Markov chain starting from state  $\{1\}$  it moves from  $S_0$  to  $S_1, \dots$ , from  $S_4$  to  $S_5$ , from  $S_5$  to  $S_0$  and so on. It results that the transition matrix of the chain will take the form:

$$P = \begin{array}{c} S0 \ S1 \ S2 \ S3 \ S4 \ S5 \\ \begin{pmatrix} 0 & \underline{x} & 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{x} & 0 & 0 \\ 0 & 0 & 0 & 0 & \underline{x} & 0 \\ 0 & 0 & 0 & 0 & 0 & \underline{x} \\ \underline{x} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

where the symbol  $\underline{x}$  indicates a stochastic matrix and the symbol 0 indicates the null matrix. With the obtained data it can be calculate also the availability of the whole system, denoted  $A$ :

$$A = \sum_{k \in S_{II}} p_k;$$

other says it can be calculated the system availability, on the basis of his subsystems availabilities [6;9]. Performance measure is a critical factor used to judge the effectiveness of a system. In many cases, the ideal performance measure tends to be distorted by failure of machines. Ignoring reliability of equipment when modeling a system will result in overestimation of performance measures. Methods available for modeling unreliable systems involving determination of system-state probabilities are generally difficult and inefficient, especially when limits are set on the buffer size [1]. A possible solution is to use the availability to determine the productivity.

The productivity of the whole system, denoted  $m$ , has the expression:

$$m = \sum_{k \in S} p_k * m_k,$$

where  $m_k$  is the state  $k$  productivity;  $m_k \neq m_j, \forall k, j \in S, k \neq j$  [10].

As a conclusion the proposed model has rigorously analyzed, from the mathematical and technical point of view, one of the repairing alternatives, the real possibility of the monitoring the whole evolution of the system.

In a fixed technical situation it will be analyzed all possible options, it will calculate the each one productivity, on a given time, for example the repayment time of the system, and is possible to chose the optimal variant also considering the productivity [10].

On the other side, for the chosen variant, it exists the possibility to optimum modify the reliability and maintainability of the different subsystems so that the productivity of the whole system will increase.

What is here to emphasize in the end, is that the proposed analysis proves its efficiency if is executed early in the design stage, when it is possible to conceive and model varied constructive alternatives and to chose the best one.

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