# Soft computing for viscoanelastic media

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**Abstract.** In this work, using Neural Networks, a model identification for the measure of phenomenological coefficients which occur in a mathematical description for viscoanelastic media is proposed. This approach of identification is developed to substitute the process of measure, made in laboratory (C.N.R. - Messina), applied to polymeric materials for the sake of testing the applicability of viscoanelastic description. The aim of this system identification is to simulate the behaviour of the viscoanelastic model, and allows us to determine univocally phenomenological coefficients as functions of experimentally determinable coefficients only. The Neural Netowork model is realized as a "soft sensor" which supply the system. The best results, in terms of quality indices and reliability, are obtained through the potential of neural networks, Linear and nonlinear Moving Average models based on MLP neural networks have been designed and the models obtained should be implemented on line in the laboratory to be tested.

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## 1 Introduction

In the present work, the model identification of laboratory process is considered, in particular to determine the prediction of the measure of phenomenological coefficients which occur in the mathematical description for viscoanelastic media proposed by Ciancio, in [1]. With this aim, models for realising a "soft sensor" are implemented, which can supply the behaviour of the polymeric materials. In particular we have take in consideration the behaviour of poly-isobutilene. Using the potential of Neural Networks we show that can be obtained a alternative representation of experimental measurements.

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## 2 System identification

When physical modelling of the system is not possible the attention must be focused on strategies which directly use the measurements of the signals produced by the system. These are defined by the term "*system identification*". When the structure of the system to be modelled is not known previously, one attempts to characterise the output of the system to a given instant ($k$) as an expansion of the input/output data through a non linear static function $S$ (**NARMAX model**) [2]:

$$y(k) = S[y(k), u(k)] \, . \tag{2.1}$$

A useful approach in approximating the S function of 1.1 is represented by neural networks ([3], [4], [5]). These models are composed of many non linear computational elements operating in parallel and arranged in patterns reminiscent of biological neural nets. Computational elements or nodes are connected via weights that are typically adapted during use to improve performance. The weight of networks constitutes the parameters to be identified, and the learning algorithm, which minimises the quadratic error between the output of the model and the real output of the process, assumes the role of an identification algorithm. The inputs of the network correspond to the *input/output* samples of the system, while the unknown $S$ function is determined repeatedly through the learning algorithm as a combination of sigmoidal function. To be able to evaluate if the error signal, that represents the difference between the simulated output of the implemented model and the real output of the process, is of a random type, it was necessary to determine some parameters. These parameters are the autocorrelation function of the error and the probability density function. The "*best conditions*" present themselves in the case in which: the average of the error tends towards zero, the autocorrelation function assumes a trend as similar as possible to a white noise, the correlation coefficient tends towards one, the simulated output of the model as similar as possible towards the real output of the process.

## 3 Viscoanelastic media and phenomenological coefficients

The fundamental problem of continuum dynamics [6, 7, 8] has no general mathematical solution since the number of differential equations that follow from general physical principles is insufficient for their integration when compared to the number of unknown functions that appear in these equations. We recall that the aforementioned problem consists in the integration of the following four equations:

$$\varrho \frac{dv_i}{dt} = \varrho F_i - \frac{\partial \phi_{ik}}{\partial x_k} \, , \qquad (i, k = 1, 2, 3) \, , \tag{3.1}$$

$$\frac{\partial \varrho}{\partial t} + div(\varrho \mathbf{v}) = 0 \tag{3.2}$$

with ten unknown functions ($\varrho$ = mass density, $\mathbf{v}$ = velocity vector, $\phi_{ik}$ = Cauchy stress tensor) and a known function $\varrho\mathbf{F}$ (density of force) provided that other six equations containing the unknown functions are added to the equations (3.1) and (3.2).

Introducing one thermodynamical tensorial internal variable in entropy function the behaviour of an isotropic viscoanelastic medium with memory can be described, shown in [9, 10, 11] with the following rheological equation if cross-effect between viscous and inelastic flows are neglected:

$$\frac{d\tilde{\tau}_{\alpha\beta}}{dt} + R_0^{(\tau)}\tilde{\tau}_{\alpha\beta} = R_0^{(\varepsilon)}\tilde{\varepsilon}_{\alpha\beta} + R_1^{(\varepsilon)}\frac{d\tilde{\varepsilon}_{\alpha\beta}}{dt} + R_2^{(\varepsilon)}\frac{d^2\tilde{\varepsilon}_{\alpha\beta}}{dt^2}\,, \tag{3.3}$$

where $\tilde{\tau}_{\alpha\beta}$ and $\tilde{\varepsilon}_{\alpha\beta}$ are the deviators of the stress and strain tensors, respectively. The following hold

$$\begin{cases} R_0^{(\tau)} = a^{(1,1)}\eta_s^{(1,1)} \\ R_0^{(\varepsilon)} = a^{(0,0)}\left(a^{(1,1)} - a^{(0,0)}\right)\eta_s^{(1,1)} \\ R_1^{(\varepsilon)} = a^{(0,0)} + a^{(1,1)}\eta_s^{(1,1)}\eta_s^{(0,0)} \\ R_2^{(\varepsilon)} = \eta_s^{(0,0)} \end{cases} \tag{3.4}$$

in which $a^{(0,0)}$ and $a^{(1,1)}$ are state coefficients while $\eta_s^{(0,0)}$ and $\eta_s^{(1,1)}$ are phenomenological coefficients related to the following physical phenomena

$$\begin{array}{lcl} a^{(0,0)} & \Rightarrow & elasticity \\ a^{(1,1)} & \Rightarrow & inelasticity \\ \eta_s^{(0,0)} & \Rightarrow & viscosity \\ \eta_s^{(1,1)} & \Rightarrow & fluidity \end{array} \tag{3.5}$$

It is also possible to verify these inequalities that follows from the principle of entropy production:

$$\begin{array}{r} a^{(0,0)} > 0 \\ a^{(1,1)} > 0 \\ \eta_s^{(1,1)} > 0 \\ \eta_s^{(0,0)} > 0 \end{array} \tag{3.6}$$

Applying [1] experimental approach of linear response theory the solution of system allow us to obtain the following phenomenological coefficients

$$a^{(0,0)}(\omega) = \frac{G_1\left(1+\omega^2\sigma^2\right) - G_{1R/H}}{\omega^2\sigma^2}\,, \tag{3.7}$$

$$a^{(1,1)}(\omega) = \frac{1}{\omega^2\sigma^2}\left\{\frac{\left[G_1\left(1+\omega^2\sigma^2\right) - G_{1R/H}\right]^2}{\left(G_1 - G_{1R/H}\right)\left(1+\omega^2\sigma^2\right)}\right\}\,, \tag{3.8}$$

$$\eta_s^{(1,1)}(\omega) = \omega^2\sigma\left\{\frac{\left(G_1 - G_{1R/H}\right)\left(1+\omega^2\sigma^2\right)}{\left[G_1\left(1+\omega^2\sigma^2\right) - G_{1R/H}\right]^2}\right\}\,, \tag{3.9}$$

$$\eta_s^{(0,0)}(\omega) = \frac{G_{1R/H} + G_2\omega\sigma - G_1}{\omega^2\sigma}\,. \tag{3.10}$$

In this way we have obtained the variables $a^{(0,0)}$, $a^{(1,1)}$, $\eta_s^{(0,0)}$, $\eta_s^{(1,1)}$, for low and high frequency, as functions of the frequency dependent quantities $G_1$ and $G_2$ which are experiemtally determinable; while $\sigma$ is calculated by specific considerations on $G_1$ and $G_2$.

## 4 Soft sensors with neural models

Two series of models, using neural networks, were realised and more precisely: the first considering one layer of neurons; the second, using two layer of neurons. The best results, in terms of reliability of the model and quality indices, were obtained by considering a neural network with two layers and in particular with five neurons in the first hidden layer and seven neurons in the second hidden layer.
In the following figure we remark the importance of the process to envelope a soft sensor. After the phase of harvest of measures that is historical data of process in

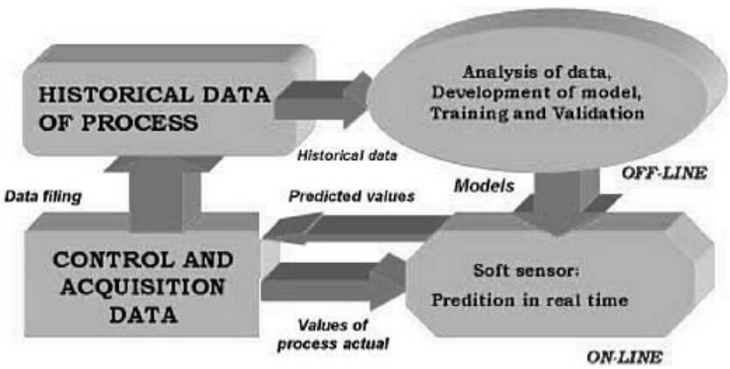


*Fig.* 1: Development of a soft sensor.

which the aim is to have data more representative of operational conditions, there is the phase of inspection data, in which the information more significative are obtained through the removal dates related to malfunctions. Then it is necessary to normalize data, so important variable with small amplitudes they maintain the weight necessary.

## 5 Results and conclusion

To obtain the neural network, we first must process data of CNR-Laboratory in order to have the possibility to use one part for training process and the other one to test the data, so we can evaluate all of the candidate Neural Networks.
This is described in the scheme of fig.2

To obtain a good model is necessary to have a set of data that is qualitative and also is important that information of Input/Output have a pre - processing treatment. So the data of poly - isobutylene imported in MATLAB, are processed as we see in the plot of Matlab window (fig.3).

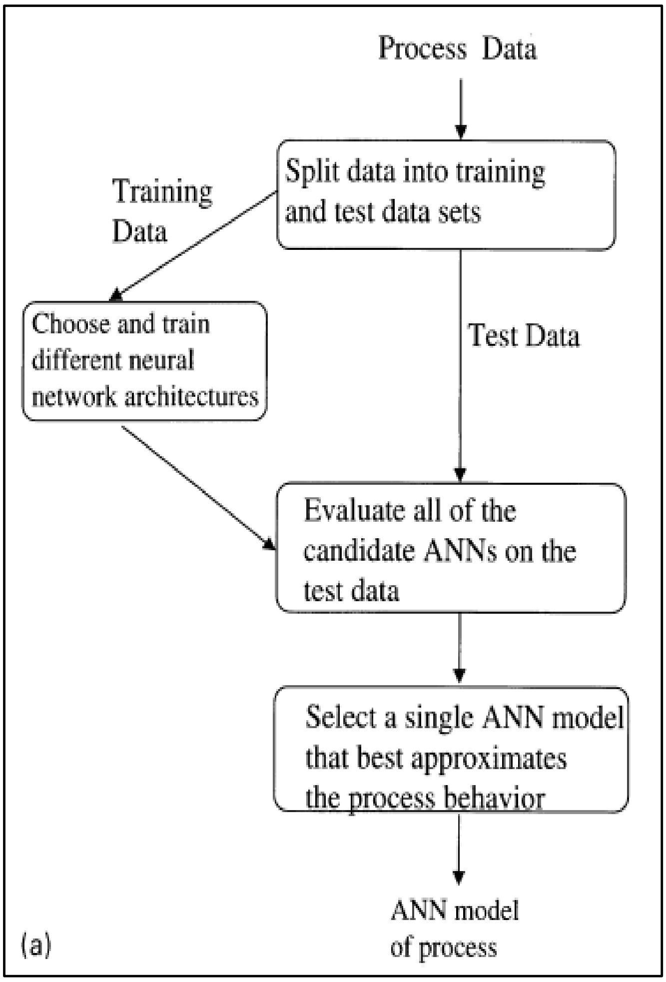


*Fig.* 2:

In the successive figures there are represented the results of the best neural networks obtained.
In fig.4 is showed, a Matlab plot of the comparison between the real output (red), the output stimated through a neural network with 2 layers and 5 neurons in both layers (black) and the output measured through a neural network with 1 hidden layer with 5 neurons(green).

From a grafic point of view it is possibile to see that there are some plays of the plot in wich the model that have best results, in terms of reliability of the model and quality indices, is the one with 2 layers and parts of the plot in wich the model that have best results is the one with 1 layer.

Otherwise both graphs and indexes (table 1) said that best results are obtained with neural network with 2 layers, in fact the average of signal error is inferior respect

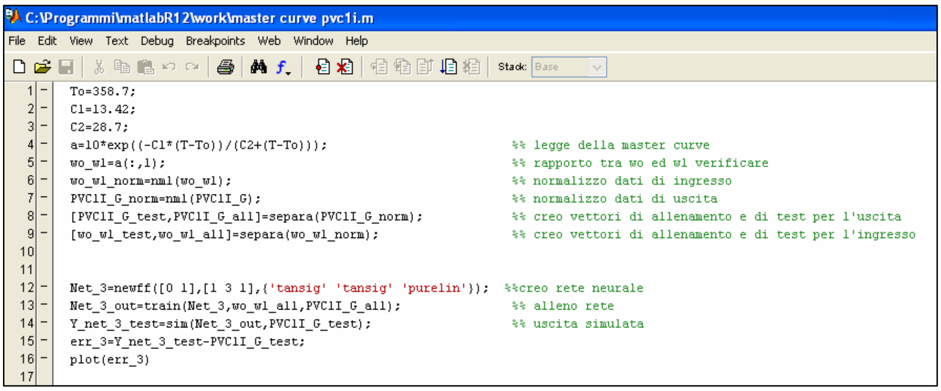


*Fig.* 3: Matlab window with processement of data (Poly-isobutilene)

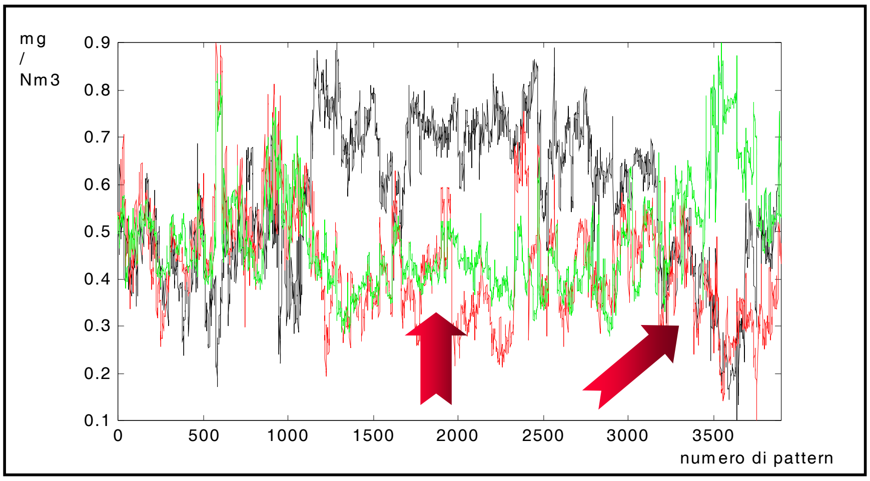


*Fig.* 4: Comparison between outputs: real,2 layers Neural network, 1 layer neural network

| | Average of signal error | Autocorrelation coeff. |
|---|---|---|
| NN 1 layer | **0.1397** | **0.0327** |
| NN 2 layer | **0.0506** | **0.2323** |

*Fig.* 5: Table 1 - Index of quality.

the average signal error of the model with one layer. In conclusion we can say that the definitive best results, as we can see in figg.6 - 8, and table 2, are obtained with

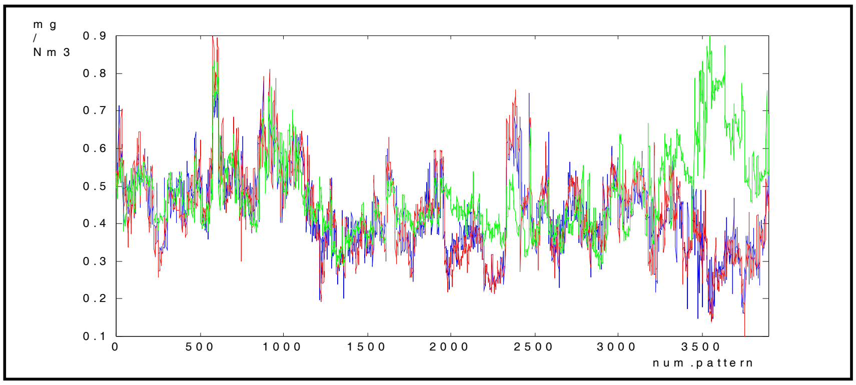


*Fig.* 6: Comparison between outputs: real,2 layers (5.7 neurons) neural network, 1 layer neural network.

| | Average of signal error | Autocorrelation coeff. |
|---|---|---|
| 2 layer (5,7) | **5.3360e-004** | **0.9130** |
| 1 layer (5) | **0.0506** | **0.2323** |

*Fig.* 7: Table 2 - Index of quality.

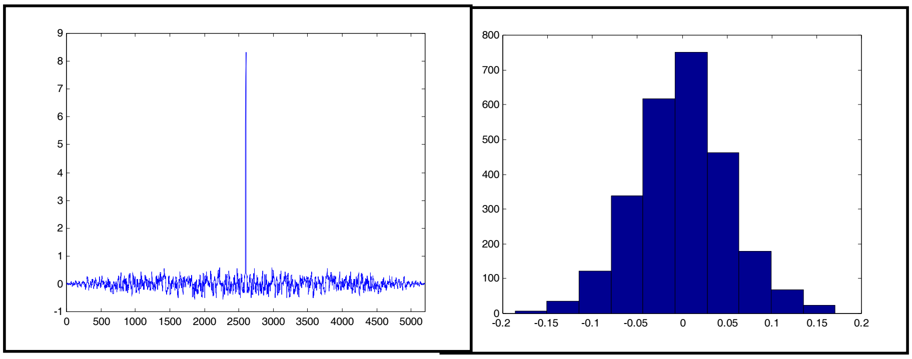


*Fig.* 8:

the model with 2 layers, 5 neurons in the first hidden layer and 7 neurons in the second hidden layer. To remark these results in fig. 8 are plotted the function of autocorrelation of the error and the histogram of the error

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