Phenomenological approach to a model of viscoanelastic media

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Abstract. The aim of this work is to report some general properties of phenomenological and state coefficients in Kluitenberg-Ciancio's theory for viscoanelastic media of order one obtained as function of frequency in a previous paper. In particular, after the specification of the low and high frequencies region, under physical considerations characterized by limiting the values of some functions connected to mechanical phenomena, the trend of the aforementioned coefficients is investigated as function of the frequency.

It has been possible, admitting the principle of entropy production, to determine some characteristics of the coefficients as function of frequency in order that this principle is satisfied. The experimental confirmation on a polymeric material, as PolyIsobutilene, of the so obtained results in the regions in which, in a first approximation, the medium shows a linear behaviour (low and high frequency region), has been obtained.

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1 Introduction

In a previous paper [1] we have determined phenomenological and state coefficients in Kluitenberg-Ciancio's theory for viscoanelastic media of order one [2],[3] as function of two coefficients G_1 and G_2 experimentally determined. Now we report the results obtained by analysis of aforementioned coefficients comparing these results with experimental determination for a polymeric material as PolyIsobutilene. There we have introduced particular values of frequency ω_R and ω_U which are well described in fig.1 and we have limited our considerations, for a fixed relaxation times, to the following set of frequencies

- *low frequencies*:

$$A_L = (\omega_R < \omega < \omega_L) \cap \left(10^{-4} Hz < \omega < 1 Hz\right) \cap \left(\omega\sigma < 10^{-2}\right)$$

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- *high frequencies*:

$$A_H = (\omega_{_H} < \omega < \omega_{_U}) \cap \left(10^9 Hz < \omega < 10^{14} Hz\right) \cap \left(\omega\sigma > 10^2\right)$$

We will remember [4],[5] that $G_1(\omega)$ is an increasing function for $\omega \in A_L$ and $\omega \in A_H$ while $G_2(\omega)$ is an increasing function for $\omega \in A_L$ but it is a decreasing one for $\omega \in A_H$ as it is shown in Fig.1.

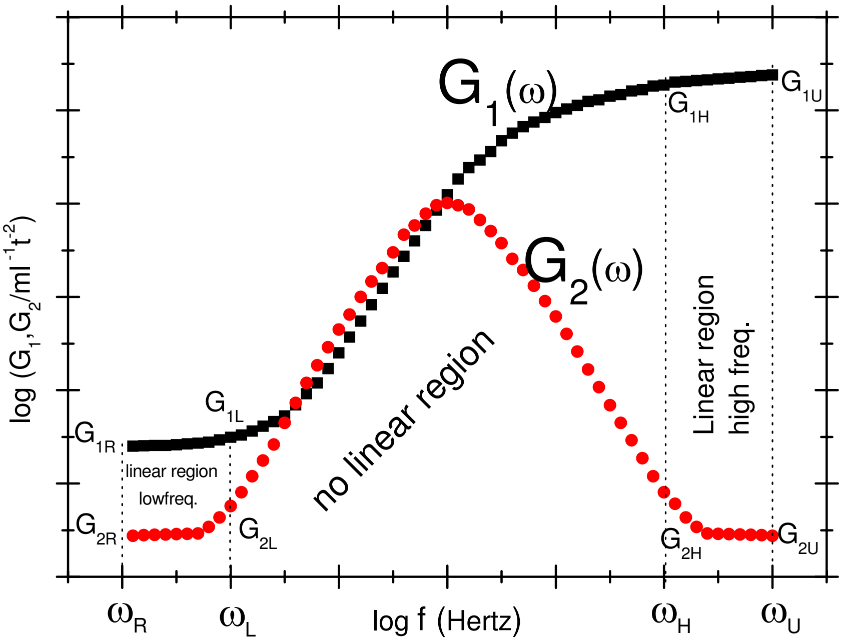

Figure 1: Generic $G_1(\omega)$ and $G_2(\omega)$.

2 Phenomelogical coefficient $a^{(0,0)}(\omega)$.

2.1 Low frequency ($\omega \in A_L$).

We remember that:

$$a^{(0,0)}(\omega) = \frac{G_1\left(1+\omega^2\sigma^2\right) - G_{1R}}{\omega^2\sigma^2} \tag{2.1}$$

It is easy to verify that

$$a^{(0,0)}(\omega) > 0 \quad a^{(0,0)}(\omega) > G_{1R}\,, \qquad (\forall\omega \in A_L)\,. \tag{2.2}$$

Introducing a particular approximation fr G_1 and G_2 it easy to write

$$a^{(0,0)}(\omega) = G_{1R}\left(\frac{1.001^{\frac{\omega-\omega_R}{\chi}}\left(1+\omega^2\sigma^2\right)-1}{\omega^2\sigma^2}\right) \tag{2.3}$$

where

$$\chi = \frac{\omega_L - \omega_R}{263}$$

and it can be shown that

$$\frac{d}{d\omega}\left[a^{(0,0)}(\omega)\right] > 0 \tag{2.4}$$

if

$$\left(1+\omega^2\sigma^2\right)\omega > 2001\chi\left(1-1.001^{-\frac{\omega-\omega_R}{\chi}}\right) \tag{2.5}$$

is never verifed.
The important result which we have obtained is that coefficient $a^{(0,0)}$ is a decreasing function which don't depends by materials. All these establishments are confirmed by experiments carried out on PolyIsobutilene as it is shown in ref.[1].

2.2 High frequency ($\omega \in A_H$).

We remember that

$$a^{(0,0)}(\omega) = \frac{G_1\left(1+\omega^2\sigma^2\right)-G_{1H}}{\omega^2\sigma^2} \tag{2.6}$$

It easy to see that

$$a^{(0,0)}(\omega) > G_{1H} \qquad \forall\, \omega \in A_H \tag{2.7}$$

By aforementioned approximation for G_1 and G_2 we have:

$$a^{(0,0)}(\omega) < G_{1U} \tag{2.8}$$

and

$$\frac{d}{d\omega}\, a^{(0,0)}(\omega) > 0 \tag{2.9}$$

if

$$(1+\omega^2\sigma^2)\omega > 2001\cdot\Gamma(1-1.001^{\frac{\omega-\omega_H}{\Gamma}}) \tag{2.10}$$

where

$$\Gamma = \frac{\omega_U - \omega_H}{357}$$

It can be shown that the equation (2.10) is always verified obtaining the important result that the coefficient $a^{(0,0)}$ is an increasing function of the frequency for every material. All these establishments are confirmed by experiments carried out on Poly-Isobutilene as it is shown in ref. [1].

3 Phenomelogical coefficient $a^{(1,1)}(\omega)$, $\eta_s^{(1,1)}(\omega)$.

3.1 Low frequency ($\omega \in A_L$).

We remember that

$$a^{(1,1)}(\omega) = \frac{1}{\omega^2\sigma^2}\left\{\frac{\left[G_1\left(1+\omega^2\sigma^2\right)-G_{1R}\right]^2}{\left(G_1-G_{1R}\right)\left(1+\omega^2\sigma^2\right)}\right\} \qquad \forall\omega \in A_L\,. \tag{3.1}$$

Taking in account equatione (2.1) the expression (3.1) can be rewritten as:

$$a^{(1,1)}(\omega) = \left\{\frac{G_1\left(1+\omega^2\sigma^2\right)-G_{1R}}{\left(G_1-G_{1R}\right)\left(1+\omega^2\sigma^2\right)}\right\}a^{(0,0)} = \frac{\left[a^{(0,0)}(\omega)\right]^2}{a^{(0,0)}(\omega)-G_{1R}} > G_{1R}\,. \tag{3.2}$$

It can be shown that

$$\frac{d}{d\omega}[a^{(1,1)}(\omega)] > 0 \quad \text{if} \quad \frac{d}{d\omega}[a^{(0,0)}(\omega)] > 0 \tag{3.3}$$

This mean that $a^{(1,1)}(\omega)$ and $a^{(0,0)}(\omega)$ have the same trend for every material. Moreover it can be shown that

$$a^{(1,1)}(\omega) \cong a^{(0,0)}(\omega)$$

All these establishments are confirmed by experiments carried out on PolyIsobutilene as it is shown in ref. [1].

3.2 High frequency ($\omega \in A_H$).

We remember that

$$a^{(1,1)}(\omega) = \frac{1}{\omega^2\sigma^2}\left\{\frac{\left[G_1\left(1+\omega^2\sigma^2\right)-G_{1H}\right]^2}{\left(G_1-G_{1H}\right)\left(1+\omega^2\sigma^2\right)}\right\} \qquad \forall\omega \in A_H\,. \tag{3.4}$$

Taking in account equation (2.6), the exspression (3.4) may be rewritten as

$$a^{(1,1)}(\omega) = \left\{\frac{G_1\left(1+\omega^2\sigma^2\right)-G_{1H}}{\left(G_1-G_{1H}\right)\left(1+\omega^2\sigma^2\right)}\right\}a^{(0,0)} = \frac{\left[a^{(0,0)}(\omega)\right]^2}{a^{(0,0)}(\omega)-G_{1H}} > G_{1H}\,. \tag{3.5}$$

It follows that

$$a^{(1,1)}(\omega) > G_{1U} \tag{3.6}$$

if

$$\left[a^{(0,0)}(\omega)\right]^2 - a^{(0,0)}(\omega)\,G_{1U} + G_{1H}G_{1U} > 0$$

which is always verified since it results

$$\Delta = G_{1U}^2 - 4G_{1H}G_{1U} = G_{1U}^2\left(1-4\cdot 0.7\right) < 0$$

It follows that

$$\frac{d}{d\,\omega}\,a^{(1,1)}(\omega) > 0 \tag{3.7}$$

is not verified since it can be shown that $a^{(0,0)} < 2G_{1H}$.
So we obtain the important result that $a^{(1,1)}$ is a decreasing function for every material. All these establishments are confirmed by experiments carried out on PolyIsobutilene as it is shown in ref. [1].
Observing the coefficient $\eta_s^{(1,1)}$ which has the form

$$\eta_s^{(1,1)} = \frac{1}{\sigma\, a^{(1,1)}} \tag{3.8}$$

it is easy to deduce his properties as the reciprocal of the coefficient $a^{(1,1)}$, since σ is a constant.

4 Phenomelogical coefficient $\eta_s^{(0,0)}(\omega)$.

4.1 Low frequency ($\omega \in A_L$).

We remember that

$$\eta_s^{(0,0)}(\omega) = \frac{G_{1R} + G_2\omega\sigma - G_1}{\omega^2\sigma} \tag{4.1}$$

It can be shown that it will be

$$\eta_S^{(0,0)}(\omega) < 0 \tag{4.2}$$

This inequality is in contrast with the physical principle of production of entropy. This is reasonable if we remember that for low frequencies viscous effect occur in the medium and so no linear effects. All this is in agreement with experimental data on PolyIsobutilene as it is shown in ref. [1].

4.2 High frequency ($\omega \in A_H$).

We remember that

$$\eta_S^{(0,0)} = \frac{G_{1H} + G_2\omega\sigma - G_1}{\omega^2\sigma} \tag{4.3}$$

and can be shown that

$$\eta_S^{(0,0)}(\omega) > 0 \qquad \text{and} \qquad \frac{d}{d\omega}\,\eta_s^{(0,0)} > 0 \tag{4.4}$$

will not be verified obtaining the important result that the function $\eta_s^{(0,0)}$ will be decreasing for every material. All this is in agreement with experimental data on PolyIsobutilene as it is shown in ref. [1].

References

[1] V. Ciancio, A. Bartolotta, F. Farsaci, *Experimental confirmation on a thermodynamical theory for viscoanelastic media with memory*, Physica B 394 (2007), 8-13.

[2] G.A. Kuitenberg and V. Ciancio, *On linear dynamical equation of state for isotropic media I*, Physica 93 A (1978), 273–286.

[3] V. Ciancio and G.A. Kluitenberg, *On linear dynamical equation of state for isotropic media II*, Physica 99A (1979), 592–600.

[4] I.M. Ward, D.W. Hadley, *An Introduction to the Mechanical Properties of Solid Polymers*, John Wiley-Sons 1993.

[5] N.G. McCrum, B.E. Read, G.Williams, *Anelastic and Dielectric Effects in Polymeric Solids*, John Wiley-Sons, London-New York, Sydney 1967.

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