Kerr-Nut seeds for cosmic strings

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Abstract. We analytically solve the Einstein-Maxwell equations with a particular choice of the Ernst potential, to derive a new exact solution to the Einstein-Maxwell equations depending on five parameters: the mass (m), the angular-momentum (α), the electromagnetic field strength (k), the parameter p and the Kerr-NUT parameter (l). This (Petrov Type D) solution is cylindrically-symmetric and represents the curved background around an electrified, rotating cosmic string interacting with gravitational and electromagnetic waves.

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1 Introduction

Cosmic strings are one-dimensional objects that can be formed as linear defects at a symmetry breaking phase transition (for a detailed analysis see Hindmarsh and Kibble [7] as well as Vilenkin and Shellard [20] . If they exist, they may help us to explain some of the large-scale structures seen in the Universe today, such as gravitational lenses [18, 19]. They may also serve as *seeds* for density perturbations [25], as well as potential sources of relic gravitational radiation [2].

The curved space-time around a straight, isolated cosmic string is constructed by a flat background from which a wedge has been (locally) cut off. The resulting metric tensor acquires a *conical singularity* located on the axis of symmetry and the corresponding *angle-deficit* is given by $\delta\phi = 8\pi\mu$, where μ is the *mass-density per unit length* [7] . The cosmic-string radius is extremely small, of the order 10^{-27} m [8, 9]. Hence, from the macroscopic point of view, the cosmic string is described as a *line-source* and the gravitational field produced by it has *cylindrical symmetry* [18] .

Cylindrically-symmetric solutions to the Einstein-Maxwell equations, pertinent to the linear defects produced by phase transitions in the early Universe [10, 11], have been modelled by several authors [3], ([6], [13]-[15], [17], [21]-[23]. A particular method, developed by Xanthopoulos [21]-[23], is based on the concept of the *Ernst potential* and the existing analogy between plane-waves and cylindrically symmetric solutions

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to the Einstein field equations [12]. Xanthopoulos [22] derived a solution representing a rotating cosmic string surrounded by cylindrical gravitational waves, which (later) he extended to include and electromagnetic waves [23].

The most interesting solutions to the Einstein equations can be generalized to include infinite cosmic strings. In the present article, a linear topological defect is embedded in the so-called *Kerr-NUT space-time* [12]. This is a class of axially symmetric space-times with an additional (the Newman-Unti-Tamburino) parameter (l). In this context, the space-time surrounding the cosmic string is formed by removing a wedge of given deficit from the Kerr-NUT metric and then gluing together the resulting edges.

2 A new exact solution to the Einstein-Maxwell equations

Adopting the notation of Xanthopoulos [22, 23], we consider a curved space-time with line-element in the form

$$ds^2 = e^{\nu+\mu_3}\sqrt{\Delta}[\frac{d\eta^2}{\Delta} - \frac{d\mu^2}{\delta}] - \frac{\Delta\delta}{\Psi}d\phi^2 - \Psi(dz - q_2 d\phi)^2 \tag{2.1}$$

where, Ψ is the real-part of the Ernst potential and the gauge choice

$$\sqrt{\Delta\delta} = e^{\Psi+\mu_2}\ ,\ \Delta = \eta^2 + 1\ ,\ \delta = \mu^2 - 1 \tag{2.2}$$

has been taken into account. The metric (2.1) admits two space-like, commuting Killing fields and the Einstein-Maxwell equations reduce to the Ernst equations [12]

$$\begin{aligned} & (ReZ - |H|^2)[(\Delta Z_{,\eta})_{,\eta} - (\delta Z_{,\mu})_{,\mu}] \\ = \ & \Delta(Z_{,\eta})^2 - \delta(Z_{,\mu})^2 - 2H^*(\Delta Z_{,\eta}H_{,\eta} - \delta Z_{,\mu}H_{,\mu}), \end{aligned} \tag{2.3}$$

$$\begin{aligned} & (ReZ - |H|^2)[(\Delta H_{,\eta})_{,\eta} - (\delta H_{,\mu})_{,\mu}] \\ = \ & \Delta H_{,\eta}Z_{,\eta} - \delta H_{,\mu}Z_{,\mu} - 2H^*[\Delta(H_{,\eta})^2 - \delta(H_{,\mu})^2], \end{aligned} \tag{2.4}$$

where

$$Z - HH^* = \Psi + i\Phi \tag{2.5}$$

with Φ being the imaginary-part of the Ernst potential and H is measuring the strength of the electromagnetic field. In this gauge, for every solution to equations (2.3) and (2.4), the metric coefficients q_2 and $e^{\nu+\mu_3}$ are obtained by the equations

$$q_{2,\eta} = \frac{\delta}{\Psi^2}[\Phi_{,\mu} + 2Im(HH^*_{,\mu})], \tag{2.6}$$

$$q_{2,\mu} = \frac{\Delta}{\Psi^2}[\Phi_{,\eta} + 2Im(HH^*_{,\eta})], \tag{2.7}$$

and

$$\begin{aligned} &\frac{\mu}{\delta}M_{,\eta} + \frac{n}{\Delta}M_{,\mu} = \frac{1}{\Psi^2}[\Psi_{,\eta}\Psi_{,\mu} \\ + \quad &(\Phi_{,\eta} + I_{(\eta)})(\Phi_{,\mu} + I_{(\mu)})] + \frac{2}{\Psi}(H_{,\eta}H^*_{,\mu} + H^*_{,\eta}H_{,\mu}) \end{aligned} \tag{2.8}$$

$$\begin{aligned} &2\eta M_{,\eta} + 2\mu M_{,\mu} = (4 - \frac{3\eta^2}{\Delta} - \frac{\mu^2}{\delta}) + \frac{4}{\Psi}(\Delta H_{,\eta}H^*_{,n} + \delta H_{,\mu}H^*_{,\mu}) \\ + \quad &\frac{1}{\Psi^2}\{\Delta[\Psi^2_{,\eta} + (\Phi_{,\eta} + I_{(\eta)})^2] + \delta[\Psi^2_{,\mu} + (\Phi_{,\mu} + I_{(\mu)})^2]\} \end{aligned} \tag{2.9}$$

where, we have set

$$M = \nu + \mu_3 + \ln\frac{\Psi}{\sqrt[4]{\Delta\delta}}\ , \quad I_{(\alpha)} = 2Im(HH^*_{,\alpha})\ , \ \alpha = n,\ \mu \tag{2.10}$$

A first family of solutions to the Einstein-Maxwell equations describing an electrified cosmic string, can be derived by imposing the ansatz $Z = 1$. In this case, the corresponding metric is identical to that of Economou and Tsoubelis [4] and it will not be considered any further. On the other hand, imposing the ansatz

$$H = Q(1 + Z), \quad k = (1 - 4QQ^*)^{1/2} \tag{2.11}$$

where Q is a complex constant, a second family of electromagnetic string solutions can be obtained, as

$$ds^2 = e^{\nu+\mu_3}_{(e)}\sqrt{\Delta}[\frac{d\eta^2}{\Delta} - \frac{d\mu^2}{\delta}] - \frac{\Delta\delta}{\Psi_{(e)}}d\phi^2 - \Psi_{(e)}(dz - q_{2(e)}d\phi)^2 \tag{2.12}$$

We denote by E the Ernst potential corresponding to the vacuum solution

$$\Psi + i\Phi = \frac{1 + E}{1 - E} \tag{2.13}$$

of the Ernst equation

$$(1 - EE^*)[(\Delta E_{,\eta})_\eta - \delta E_{,\mu})_\mu] = -2E^*(\Delta E^2_{,\eta}) - \delta E^2_{,\mu}) \tag{2.14}$$

Accordingly, we find

$$\Psi_{(e)} = \frac{k^2(1 - EE^*)}{|1 - kE|^2}, \quad \Phi_{(e)} = \frac{ik(E^* - E)}{|1 - kE|^2} \tag{2.15}$$

$$Z_{(e)} = \frac{(1 + kE)}{(1 - kE)}, \quad k = (1 - 4QQ^*)^{1/2}, \quad H = \frac{2Q}{1 - kE} \tag{2.16}$$

and therefore

$$e^{(\nu+\mu_3)(e)} = \frac{1}{4k^2}[(1 - k)^2(\Psi^2_{(\upsilon)} + \Phi^2_{(\upsilon)}) + 2(1 - k^2)\Psi_{(\upsilon)} + (1 + k^2)]e^{(\nu+\mu_3)(\upsilon)} \tag{2.17}$$

where the indices (e) and (υ) stand for the electric and the vacuum solution, respectively. As regards $q_{2(e)}$, it is obtained by the equation

$$q_{2(e)} = \frac{(1+k)^2}{4k^2} q_{2(\upsilon)} + \frac{(1-k)^2}{4k^2} q^{2(e)} \tag{2.18}$$

where, $q^{2(e)}$ is a solution to the system of differential equations

$$q^{(e)}_{2,\eta} = \frac{\delta}{\Psi^2_{(\upsilon)}} [(\Phi^2_{(\upsilon)} - \Psi^2_{(\upsilon)})\Phi_{(\upsilon),\mu} + 2\Phi_{(\upsilon)}\Psi_{(\upsilon)}\Psi_{(\upsilon),\mu}] \tag{2.19}$$

$$q^{(e)}_{2,\mu} = \frac{\Delta}{\Psi^2_{(\upsilon)}} [(\Phi^2_{(\upsilon)} - \Psi^2_{(\upsilon)})\Phi_{(\upsilon),n} + 2\Phi_{(\upsilon)}\Psi_{(\upsilon)}\Psi_{(\upsilon),n}] \tag{2.20}$$

Demanding that equations (2.19) and (2.20) satisfy the integrability condition $q^{(e)}_{2,\eta\mu} = q^{(e)}_{2,\mu\eta}$, we obtain

$$q^{(e)}_2 = \frac{2}{pY}[q\delta(1+p\eta) + lq\delta(l+q) - p^2\Delta(\mu-1)] \tag{2.21}$$

where the resulting integration constant is suitably chosen so that $q^{(e)}_2$ vanishes for $\mu = 1$ (on the azimuthal axis). Since the metric coefficient $q^{(e)}_2$ is determined up to an additive constant, we use this freedom to simplify the expression of $q^{(e)}_2$. Eventually, equations (2.15)-(2.21) give us all the metric coefficients of (2.12), in the form

$$\begin{aligned} q_{2(e)} &= \frac{(1+k)^2}{2k^2pY}[q\delta(1-\eta p) + q\delta l(l-q) + lp^2(\mu-1)\Delta] \\ &+ \frac{(1-k)^2}{2k^2pY}[q\delta(1+p\eta) + q\delta l(l+q) - lp^2(\mu-1)\Delta] \end{aligned} \tag{2.22}$$

$$\Psi_{(e)} = k^2\frac{Y}{\Pi}, \quad \Phi_{(e)} = 2k\frac{(q\mu - lp\eta)}{\Pi} \tag{2.23}$$

$$e^{(\nu+\mu_3)(e)}\sqrt{\Delta} = \frac{\alpha^2\Pi}{k^2} \tag{2.24}$$

with $q = \sqrt{(1+p^2+l^2)}$, $\Pi = (k-p\eta)^2 + (q\mu - kl)^2$ and $Y = p^2\Delta + q^2\delta$. In accordance, the line-element (2.12) reads

$$ds^2 = \frac{\alpha^2\Pi}{k^2}[\frac{d\eta^2}{\Delta} - \frac{d\mu^2}{\delta}] - \frac{\Delta\delta\Pi}{k^2Y}d\phi^2 - \frac{k^2Y}{\Pi}[dz - q_{2(e)}d\phi]^2 \tag{2.25}$$

The metric (2.25) satisfies the Rainich-Wheeler-Misner conditions (see p. 529 in [1]) and therefore is a new exact solution to the Einstein-Maxwell equations, depending on five parameters: the mass m, the angular-momentum α, the strength of the electromagnetic field k, the parameter p and the Kerr-NUT parameter l. This solution describes the space-time around an electrified, rotating linear defect in the presence of electromagnetic and gravitational waves. In fact, the ansatz (2.11) imposed to construct the Kerr-Newman solution from the corresponding Kerr one, has been often used to generate solutions to the Einstein-Maxwell equations describing the space-time resulting from the collision of gravitational and electromagnetic waves [3].

3 The metric at $\omega \to 0^+$, $\omega \to \infty$

To investigate the behavior of the metric coefficients $q_{2(e)}$, $e^{(\nu+\mu_3)(e)}$ and $\Psi_{(e)}$ near the axis and at infinity, we express the metric (2.25) in cylindrical coordinates. The result consists of lengthy expressions which are simplified considerably in the limits $\omega \to 0^+$ and $\omega \to \infty$. Thus, near the axis ($\omega \ll t$) we have

$$\eta = t - \frac{\omega^2 t}{2(1+t^2)} + O(\omega^4), \quad \mu = 1 + \frac{\omega^2}{2(1+t^2)} + O(\omega^4) \tag{3.1}$$

and the line-element reads

$$ds^2 = \frac{\alpha^2 N}{k^2}[dt^2 - d\omega^2 - \frac{\omega^2}{\alpha^2 p^2} d\phi^2] - \frac{k^2 p^2}{N}[dz - \frac{\omega^2 B}{k^2(1+t^2)^2 p^3} d\phi]^2 \tag{3.2}$$

It is evident that, in this region, the curvature is smooth and the two Killing vectors are $\mid \frac{\partial}{\partial z} \mid^2 = O(1)$ and $\mid \frac{\partial}{\partial \phi} \mid^2 = O(\omega^2)$, which are first-order orthogonal, that is $\frac{\partial}{\partial z}\frac{\partial}{\partial \phi} = O(\omega^2)$. Equation (3.2) implies that, given a small circle lying on the hypersurface $dt = 0 = dz$ and having its center at $\omega = 0$, the ratio circumference/radius differs from 2π, unless $\mid ap \mid = 1$. When $\mid ap \mid \neq 1$ the region near the symmetry axis is characterized by an angle-deficit and the metric (3.2) exhibits a conical singularity. In particular, the angle-deficit around this linear defect is given by [24]

$$\delta\phi_{axis} = 2\pi - \lim_{\rho \to 0} \frac{\int_0^{2\pi} \sqrt{g_{\phi\phi}} d\phi}{\int_0^{\rho} \sqrt{g_{\rho\rho}} d\rho} = 2\pi[1 - \frac{1}{\mid \alpha p \mid}] \tag{3.3}$$

and the corresponding mass-density is $\mu_0 = \frac{1}{4}[1 - \frac{1}{|\alpha p|}]$. In this case, the Kerr-NUT parameter does not make any contribution to the problem and the result is the same as in the $l = 0$ case.

Far away from the axis ($\omega \gg t$) we have

$$\eta = \frac{t}{\omega} + \frac{t(t^2-1)}{2\omega^3} + O(\omega^{-4}), \quad \mu = \omega + \frac{(1-t^2)}{2\omega} + O(\omega^{-2}) \tag{3.4}$$

and the line-element (2.25) reads

$$\begin{aligned} ds^2 &= \frac{\alpha^2 q^2}{k^2}[1 - \frac{2kl}{q\omega} + O(\omega^{-2})][dt^2 - d\omega^2 - \frac{\omega^2}{\alpha^2 q^2} d\phi^2] \\ &- k^2[1 + \frac{2kl}{q\omega} + O(\omega^{-2})][dz - \frac{1}{qpk^2}\Lambda d\phi]^2 \end{aligned} \tag{3.5}$$

where, $\Lambda = [(1+k^2)(1+\frac{l^2}{2}) + kl(l-2q)]$. The angle-deficit of the metric (3.5) is given by the formula

$$\delta\phi_\infty = 2\pi - \lim_{\rho \to \infty} \frac{\int_0^{2\pi} \sqrt{g_{\phi\phi}} d\phi}{\int_0^{\rho} \sqrt{g_{\rho\rho}} d\rho} = 2\pi \left\{ 1 - \frac{1}{\mid \alpha q \mid} \right\} \tag{3.6}$$

Equation (3.6) implies that the electromagnetic field does not contribute to the angle-deficit. In this case, the mass-density of the linear defect is given by the equation

$$\mu_0 = \frac{1}{4}\left\{1 - \frac{1}{\mid \alpha q \mid}\right\}$$

As regards the metric (3.5), the two Killing vectors are $\mid \frac{\partial}{\partial z} \mid^2 = O(1)$ and $\mid \frac{\partial}{\partial \phi} \mid^2 = O(\omega^2)$. These are not hyper-surface orthogonal [not even in the first order since $\frac{\partial}{\partial z}\frac{\partial}{\partial \phi} = O(\omega)$] unless $l = 0$ or $k = 1$. Combining equations (3.3) and (3.6) we find that

$$\delta\phi_{asym} > \delta\phi_{axis}$$

since $q^2 = 1 + p^2 + l^2$. In other words, the angle-deficit as measured asymptotically is always greater than the corresponding deficit as measured near the axis. This excess in the deficit is attributed to the contribution of the energy of the intervening gravitational and electromagnetic waves (colliding waves). Thus the choice $\mid \alpha \mid = q^{-1}$, which would erase the asymptotic deficit, requires a string with negative mass-density. Therefore, although near the axis the string can be erased for a suitable choice of the parameter α (e.g. $\mid \alpha \mid = p^{-1}$), the asymptotic angle-deficit can not be eliminated in a physically acceptable situation.

4 The C - energy

As regards to the cylindrically-symmetric solutions of the Einstein equations, the quantity

$$C = \nu_e + \frac{1}{2}\ln[\Psi_{(e)}] \tag{4.1}$$

is often referred to as their *C - energy* [16]. It is proportional to the energy-density per unit length contained in a cylinder of radius ω. With the aid of equations (2.23) and (2.24), equation (4.1) is written in the form

$$C = \frac{1}{2}\ln[\frac{\alpha^2 Y}{(\eta^2 + \mu^2)}] \tag{4.2}$$

It is easy for someone to see that, near the axis (as $\omega \to 0$) $C_{axis} \sim \ln \mid ap \mid$, while asymptotically (as $\omega \to \infty$) $C_{asym} \sim \ln \mid aq \mid$. Since $q^2 = 1 + p^2 + l^2$, we have $C_{asym} > C_{axis}$, which is a manifestation that the intervening gravitational waves contribute to the curved space-time a positive energy amount.

Furthermore, the behavior of the C - energy flux along the null directions reveals that the presently considered solution, although it is quite tedious, exhibits the same radiative behavior and the same fall-off away from the null direction of propagation, as the solution of Economou and Tsoubelis [4, 5].

In particular, to examine the radiative behavior of the metric (2.25), we introduce the so-called *retarded* and *advanced* null-coordinates $u = t - \omega$ and $v = t + \omega$, respectively and we express the C - energy and its first derivatives $\partial C/\partial u = C_{,u}$ and $\partial C/\partial v = C_{,v}$ in terms of u and v. We obtain:

Behavior at past null-infinity:

$$\begin{aligned} \lim_{u\to-\infty} C_{,u} &= O(1/u^2) \\ \lim_{u\to-\infty} C_{,v} &= \frac{1 + l^2}{[(p^2 + q^2)\sqrt{(1 + v^2)} + v(1 + l^2)](1 + v^2)} + O(1/u) \end{aligned} \tag{4.3}$$

Behavior at future null-infinity:

$$\begin{aligned} \lim_{v\to\infty} C_{,u} &= -\frac{1+l^2}{[(p^2+q^2)\sqrt{(1+u^2)}-u(1+l^2)](1+u^2)} + O(1/v) \\ \lim_{v\to\infty} C_{,v} &= +O(1/v^2) \end{aligned} \tag{4.4}$$

We observe that, at the past null-infinity there is a flux of incoming radiation toward the axis, with its profile given by equation (4.3). For large values of v, equation (4.3) behaves as $\lim_{u\to-\infty} C_{,v} \sim \frac{1+l^2}{(1+l^2+q^2)v^3}$. Thus, the original pulse of incoming radiation is concentrated around $v = 0$ and for large values of v it falls of quite rapidly, as v^{-3}. Therefore, it may be interpreted that, near the past null-infinity there is a beam of incoming radiation. Similarly, for large values of u, equation (4.4) behaves as $\lim_{u\to\infty} C_{,u} \sim -\frac{1+l^2}{p^2u^3}$, suggesting that near the future null-infinity there is only outgoing null radiation, which is beamed around $u = 0$.

These results indicate that the entire space-time (2.25) is filled with a mixture of incoming and outgoing radiation originated at the past null-infinity, which is reflected by the cosmic string and propagate toward the future null-infinity. Both waves, incoming and outgoing, are beamed around the null-direction in the radiation zone.

5 Discussion

In the present article, we have solved analytically the Einstein-Maxwell equations with the particular choice of the Ernst potential $E = \frac{1-il}{E_k}$, where $E_k = p\eta - iq\mu$. Accordingly, we have derived a new exact solution to the Einstein-Maxwell equations depending on five parameters: the mass (m), the angular-momentum (α), the electromagnetic field strength (k), the parameter p and the Kerr-NUT parameter (l). The Petrov Type D solution (2.25) has cylindrical symmetry and represents the curved background around an electrified, rotating cosmic string which interacts with gravitational and electromagnetic waves under the influence of the Kerr-NUT parameter. In what follows, we summarize the most important mathematical and/or physical properties of this space-time.

In the absence of electromagnetic fields, i.e. for $k = 1$, the line-element (2.25) is reduced to the corresponding solution obtained by Economou and Tsoubelis [4, 5], while, for $k = 1$ and $l = 0$ it results in the solution obtained by Xanthopoulos [22], describing the interaction of a cosmic string with gravitational waves. On the other hand, for $l = 0$ and $k \neq 1$ one finds the solution obtained also by Xanthopoulos [23], regarding a linear defect in the space-time of colliding gravitational and electromagnetic waves.

In our case, i.e. for $l \neq 0$ and $k \neq 1$, the space-time is smooth everywhere and it exhibits a conical singularity near the axis and at infinity. The angle-deficit near the axis is always smaller than the corresponding deficit as determined asymptotically. The deficit near the axis signals the existence of a cosmic string with mass per unit length given by equation (3.3). The deficit at infinity, on the other hand, is attributed to the combined effect of the string and of the energy carried by the gravitational and the electromagnetic radiation. The particular choice $\mid \alpha \mid = p^{-1}$ erases the string at small distances. On the other hand, a similar choice could erase the asymptotic deficit, but this would require a linear defect with negative mass per unit length. Therefore, the angle deficit at infinity cannot be erased.

A study of the C-energy in the radiation zone, suggests that both the incoming and the outgoing radiation is gravitational, strongly focused around the null direction and preserving its profile. The absence of the parameter k from the C - energy and its derivatives implies that there is no electromagnetic flux either near or far from the linear defect.

Other remarkable features of the space-time (2.25), both close and far away from the axis-region are:

1. At time-like infinity, i.e. for $\mid \alpha p \mid > 1$, the axis region of the metric (3.2) is flat, acquiring a conical singularity, while the axis is occupied by a static string.

2. The original Killing vectors $\frac{\partial}{\partial z}, \frac{\partial}{\partial \phi}$ of the metric (2.25), although being hyper-surface orthogonal near the axis, at large distances are not (due to non-zero values of the Kerr-NUT parameter l and the strength of the electromagnetic field k). However, upon a transformation of the form $z \to \tilde{z} = z - \Lambda\phi$, where

$$\Lambda = \frac{1}{qpk^2}[(1+k^2) + \frac{l^2}{2}(1+k^2) + kl(l-2q)]$$

the last term in equation (3.5) can be (locally) gauged away and the corresponding Killing vectors $\frac{\partial}{\partial \tilde{z}}, \frac{\partial}{\partial \phi}$ become hyper-surface orthogonal. In any other case, the last term in the metric (3.5) will give rise to global effects analogous to those arising in stationary space-times, where the time-like Killing vector is not hyper-surface orthogonal [4].

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