Anisotropic fluid inside the black hole

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Abstract. A spacetime endowed with an anisotropic fluid is proposed for the interior of the event horizon of a black hole. The geometry is a solution of Einstein's equations with a stress tensor on the r.h.s. which obeys all the energy conditions. The interior fluid is compressible and the coefficients of viscosity are time dependent. The energy density ρ and the "radial" pressure p are time dependent (with $p + \rho = 0$, as for dark energy), with no pressures on $\theta-$ and $\phi-$ directions.

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1 Introduction

Many authors expressed the idea that the interior of a black hole can be thought of as an anisotropic collapsing cosmology. The interior of a Schwarzschild black hole may be considered as a homogeneous anisotropic cosmology of the Kantowski - Sachs family. When $r < 2m$, a remarkable change occurs in the nature of spacetime : the temporal coordinate for the outside observer becomes a spacial coordinate for the inside observer, the interior geometry being time dependent. In addition, the area of the two-surface of constant "radial" and time coordinates increases at a rate proportional with time.

We conjecture in the present article the interior of a black hole is filled with an anisotropic fluid, its equation of state being given by $p(t) = -\rho(t)$, where the negative "radial" pressure p and the energy density ρ are time dependent.

Following an idea of Doran et al. [4] we chose the line element inside the horizon to be Minkowskian for constant angular variables (i.e., the mass of the black hole is proportional with time). We show that the fluid is characterized by time dependent coefficients of dynamic and bulk viscosities, a follow up of the pressure and energy density time dependance. We found that the Raychaudhuri equation is fulfilled for a congruence of particles having timelike velocities u^α as they fall under their own gravity.

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2 The time dependent interior spacetime

We know that the geometry inside the horizon of a Schwarzschild black hole is dynamic since the radial coordinate becomes timelike and the metric is time dependent. It is given by

$$ds^2 = -\left(\frac{2m}{t} - 1\right)^{-1} dt^2 + \left(\frac{2m}{t} - 1\right) dz^2 + t^2 d\Omega^2 \tag{2.1}$$

where m is the mass of the black hole, z plays the role of the radial coordinate, with $-\infty < z < +\infty$ and $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$. Throughout the paper we take the velocity of light $c = 1$ and Newton's constant G = 1.

Doran et al. [4] have taken into consideration the case of a time-dependent $m(t)$, when the metric (2.1) is no longer a solution of the vacuum Einstein equations. A time dependent mass inside the horizon is justified by the fact that it is equivalent to a r - dependent mass $m(r)$ outside the horizon. Moreover, Lundgren, Schmekel and York, Jr. [7] showed that the quasilocal energy of a black hole depends on the radial position of the observer outside the horizon and on time inside it [9].

Doran et al. observed that the spacetime acquires an instantaneous Minkowski form for $m(t) = t$

$$ds^2 = -dt^2 + dz^2 + t^2 d\Omega^2 \tag{2.2}$$

The line element (2.2) has a curvature singularity at $t = 0$ since the scalar curvature $R^\alpha_\alpha = 4/t^2$ is infinite there. However, a geodesic particle with constant angular coordinates moves exactly as in flat space.

In order to be a solution of Einstein's equations, we must have an energy - momentum tensor (a source) on the r.h.s. We consider the spacetime (2.2) is endowed with an anisotropic fluid with the stress tensor

$$T^\nu_\mu = \rho u_\mu u^\nu + p s_\mu s^\nu \tag{2.3}$$

where $u_\mu = (1, 0, 0, 0)$ is the fluid 4 - velocity (the components are in order t, z, θ, ϕ), $s_\mu = (0, 1, 0, 0)$ is the unit spacelike vector in the direction of anisotropy (with $u_\alpha u^\alpha = -1$, $s_\alpha s^\alpha = 1$ *and* $s_\alpha u^\alpha = 0$). Hence, we have only one principal pressure p and the transverse pressures measured in the orthogonal direction to s^μ are vanishing.

With (2.3) on the r.h.s. of the Einstein equations, one obtains, for the energy density and pressure on the z - direction

$$\rho(t) = -T^t_t = \frac{1}{4\pi t^2}, \quad p(t) = T^z_z = -\frac{1}{4\pi t^2}, \quad T^\theta_\theta = T^\phi_\phi = 0. \tag{2.4}$$

(The convention $R_{\alpha\beta} = \partial_\nu \Gamma^\nu_{\alpha\beta} - ...$ has been used). We note that $p(t) = -\rho(t)$, as for dark energy stars.
It is an easy task to check that the weak, strong and dominant energy conditions for $T_{\mu\nu}$ are obeyed.

3 The anisotropic stress tensor

Let us consider the general expression of the stress-energy tensor for a compressible, viscous fluid with heat conduction [8]

$$T_{\alpha\beta} = \rho u_\alpha u_\beta + (p - \varsigma\Theta)h_{\alpha\beta} - 2\eta\sigma_{\alpha\beta} + q_\alpha u_\beta + q_\beta u_\alpha \tag{3.1}$$

where Θ is the expansion of the fluid worldlines (the rate of increasing of a fluid volume element) given by the divergence of u_α

$$\Theta = \nabla_\alpha u^\alpha, \tag{3.2}$$

$h_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$ is the projection tensor and $\sigma_{\alpha\beta}$ expresses the distorsion of the fluid in shape without change in volume [1]. It is orthogonal to $u^\alpha (i.e., u^\alpha\sigma_{\alpha\beta} = 0)$, tracefree and may be computed from

$$\sigma_{\alpha\beta} = \frac{1}{2}(h^\mu_\beta \nabla_\mu u_\alpha + h^\mu_\alpha \nabla_\mu u_\beta) - \frac{1}{3}\Theta h_{\alpha\beta} + \frac{1}{2}(a_\alpha u_\beta + a_\beta u_\alpha). \tag{3.3}$$

η *and* ς are the coefficients of dynamical and, respectively, bulk viscosity, with $\eta > 0$, $\varsigma > 0$. q_α is the heat flux 4-vector and a_α is the acceleration due to nongravitational forces

$$a_\alpha = u^\beta \nabla_\beta u_\alpha. \tag{3.4}$$

Before to compare the tensors (2.3) and (3.1), we note that (2.3) is a particular case of the locally anisotropic fluid with nonzero transversal pressure p_t and a stress tensor given by [3]

$$\tau_{\alpha\beta} = (\rho + p_t)u_\alpha u_\beta + p_t g_{\alpha\beta} + (p - p_t)s_\alpha s_\beta \tag{3.5}$$

We see that a vanishing tangential pressure in (3.5) leads to (2.3), with p as the pressure along the anisotropy.

The geometry (2.2) has the following nonzero Christoffel symbols :

$$\Gamma^t_{\theta\theta} = t, \quad \Gamma^\theta_{t\theta} = \Gamma^\phi_{t\phi} = \frac{1}{t}, \quad \Gamma^\phi_{\theta\phi} = cot\theta, \quad \Gamma^t_{\phi\phi} = tsin^2\theta. \tag{3.6}$$

Therefore, the components of the Ricci tensor are given by [2]

$$R^z_z = R^t_t = 0, \quad R^\theta_\theta = R^\phi_\phi = \frac{2}{t^2} \tag{3.7}$$

By means of (3.7) one obtains, from (3.2)

$$\Theta = -\frac{2}{t}, \quad \dot{\Theta} \equiv \frac{d\Theta}{d\tau} = -\frac{2}{t^2} \tag{3.8}$$

The nonzero components of the shear tensor are

$$\sigma_{zz} = \frac{2}{3t}, \quad \sigma_{\theta\theta} = \frac{\sigma_{\phi\phi}}{sin^2\theta} = -\frac{t}{3} \tag{3.9}$$

with $g^{\alpha\beta}\sigma_{\alpha\beta} = 0$. From (3.4) and (3.6) we find that $q_\alpha = 0$ *and* $a_\alpha = 0$ (we have a congruence of timelike geodesics). The same conclusion is valid for the vorticity (rotation without change in shape).

Using (3.8), it is an easy task to check that the Raychaudhuri equation

(3.10) $$\dot{\Theta} - \nabla_\alpha a^\alpha + 2(\sigma^2 - \omega^2) + \frac{1}{3}\Theta^2 = R_{\alpha\beta} u^\alpha u^\beta$$

is obeyed. We have here $2\sigma^2 \equiv \sigma_{\alpha\beta}\sigma^{\alpha\beta} = 2/3t^2$ *and* $2\omega^2 \equiv \omega_{\alpha\beta}\omega^{\alpha\beta} = 0$.

We are now in a position to see whether the stress tensor (2.3) could be put in the general form (3.1), with the identification of the parameters η *and* ς. By a direct inspection, we conclude that we should have :

(3.11) $$2\eta(t) = 3\varsigma(t) = \frac{1}{4\pi t}$$

The fact that $2\eta = 3\varsigma$ (G.G. Stokes relation) is well-known from the kinetic theory of gases [5].

The time dependence of the coefficients of viscosity might be justified on the basis of Landau's and Lifshitz's remark [6] that in general the coefficients of viscosity may depend on the pressure of the fluid which, in our situation is a function of time.

Keeping in mind that $T_{\alpha\beta}$ may be decomposed in the following manner

(3.12) $$T_{\alpha\beta} = T_{\alpha\beta}^{pf} + T_{\alpha\beta}^{visc},$$

where $T_{\alpha\beta}^{pf}$ corresponds to the perfect fluid part of the stress tensor and the contribution of the viscosity is given by (see also [8])

(3.13) $$T_{\alpha\beta}^{visc.} = -2\eta\sigma_{\alpha\beta} - \varsigma\Theta h_{\alpha\beta}.$$

4 Conclusions

A black hole interior filled with a compressible, anisotropic fluid is conjectured in this paper. The coefficients of dynamical and bulk viscosities are time dependent, a property which is justified by a remark of Landau and Lifshitz that in general η and ς may be functions of the pressure of the fluid.

The fact that the equation of state is $p + \rho = 0$ remind us of many dark energy models with a similar behaviour.

Our geometry (2.2) has infinite scalar curvature on the hypersurface $t = 0$ and, therefore, the parameters $\rho,\ p,\ \eta,$ *and* ς are also singular there.

We have also checked that the Raychaudhuri equation for a congruence of timelike geodesics is satisfied.

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