Uni-and multivariate loss functions and the Taguchi theory

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Dedicated to the 70-th anniversary
of Professor Constantin Udriste

Abstract. The main goal of the paper is to propose approach symmetric/asymmetric quality loss models using Pareto and beta stochastic laws. In addition a few numerical examples illustrate that the adaptive functions are ubiquitous for describing the quality loss. The calculation of the parameters was made using CurveExpert and Lab Fit software. The proposed models can be easy adapted for the similar cases.

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Key words: Taguchi methods; quality loss function; Pareto and beta distributions.

1 Introduction

The quality quantification paradigm proposed by Taguchi is based on two fundamentals concepts: the quality loss function and the signal/noise ratio [10]. Although there are several loss functions that have proposed and studied in the literature, it is room for improvement [6].

Depending of type of tolerance for product specifications there are three categories of quality loss models [11]:

- the-smaller-the-better (S-type)
- the-larger-the-better (L-type)
- the-nominal-the-best (N-type.)

In the case of S-type loss functions, quality losses appear due to highest-as intended outcomes. In the L-type quality losses appear due to lower -as intended outcomes [7]. In the N-type case, the target specification is fixed. Taguchi postulated the quality loss to be zero if and only if the product is on target, increasing function with growing the distance from the target, and can be different to the left or to the right of target point. For N-type loss functions Taguchi used quadratic approximation for the loss function, as model for measuring losses of society including the loss of the producer and that of the customer [1].

For a sample of product the average loss can be decomposed as a sum of the variance and bias. According to this approach, a manufacturing process must to

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have two complementary goals: zero bias and the smallest possible variance. Taguchi philosophy of quality control focuses on the design stage. The deviations from control target should be evaluated in terms of the loss of quality they cause. A loss occurs even if the outcome is still within pre-specified tolerance bounds.

The analysis of experimental data were performed on CurveExpert [5] and LAB Fit [9] software developed for curve fitting (nonlinear regression - least squares method, Levenberg-Marquardt algorithm).

2 Pareto and beta laws used as quality loss functions

The quality loss functions try to estimate the loss of the quality in financial terms. Any continuous function which has a minimum point at the target value, decreasing/increasing for the smaller/larger values than target is the loss function ([2], [3]). In the following it is proposed some types of models using Pareto, generalized Pareto and beta pattern ([1], [4]). The canonical Pareto has unbounded right range, while the quality loss function can be non-zero only on bounded interval. The distribution analysis of experimental data points indicated that curve profiles can be described more adequately different of the degrees of skewness and peakedness of the curves.

In this paper were used the following functions:

A. Symmetrical models

a. Polynomial model $y_1 = a + bx + cx^2 + dx^3 + ex^4$. For the experimental data it results: $a = 2574576$, $b = -1021483$, $c = 151956$, $d = -10045$, $e = 249$.

b. Shifted, adapted beta function $y_2 = a[(x-9.5)(10.5-x)]^b - \frac{a}{4^b}$. For the experimental data it results: $a = -2503$; $b = 21.45$

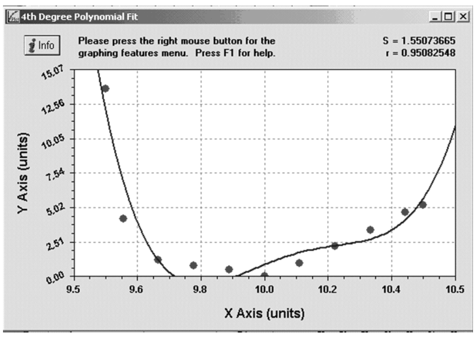

Fig. 1. Polynomial graph.

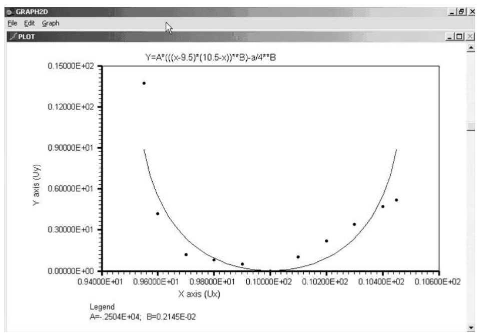

Fig. 2. Shifted, adapted beta graph.

B. Left defined models

a. Left shifted Pareto model $y_3 = \frac{a}{x^b} - \frac{a}{10^b}$. For the experimental data it results: $a = 47651635$; $b = 6.3883625$.

b. Shifted, adapted beta function. For the experimental data it results: $a = -91.15$; $b = 1.826$

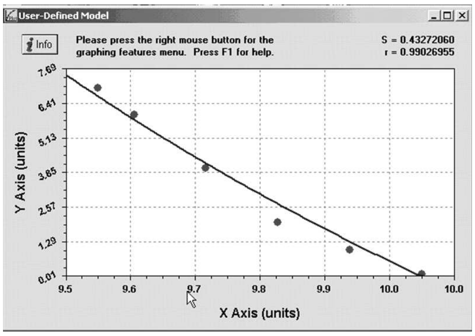

Fig. 3. Left shifted Pareto graph

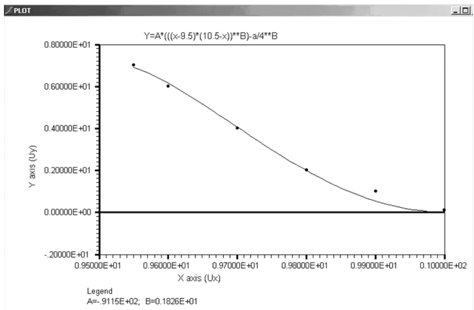

Fig. 4. Shifted, adapted beta graph

C. Right defined models

a. Right shifted Pareto model $y_4 = \frac{a}{(20-x)^4} - \frac{a}{10^4}$. For the experimental data it results: $a = 383894$,

b. Shifted, adapted beta function. For the experimental data it results: $a = 0.12464637$; $b = -1.5761434$.

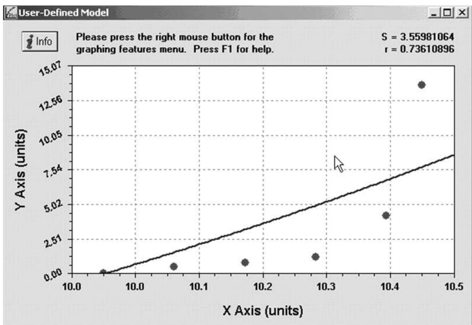

Fig. 5. Right shifted Pareto graph

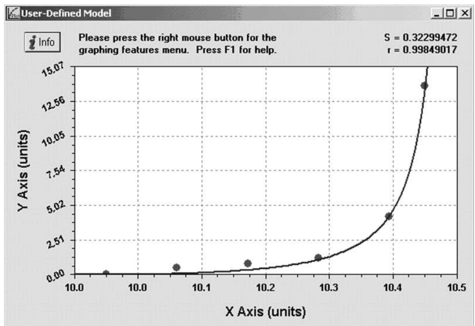

Fig. 6. Shifted, adapted beta graph

D. Mixed asymmetrical defined models

The solution of the problem is based on a piecewise-defined function. In the first step it is solved the left side of data (smaller than the target value) and in the second step follows the right side.

3 Conclusions

In the paper following issues will be pursued:

- the comparative analysis of the proposed models for experimental data;
- the identification of most adequate stochastic law.

The best results were obtained with the adaptive shifted beta model, since, for example, the values of the correlation coefficient are in most of the cases closed to one.

Hence the modeling by truncated shifted beta model on a given interval is an adequate quality loss function for symmetrical/asymmetrical cases. The analyzed laws prove their efficiency if are applied in the design stage. The results can be extended in the n-dimensional space, taking into consideration multidimensional variables with/without the potential interactions, which can cause a fraction of quality loss too ([8], [12]). It exists the possibility to adapted the obtain results in the reliability area ([13], [14]).

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