Anisotropic metric for the gravitation theory: new ways to interpret the classical GRT tests

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Dedicated to the 70-th anniversary of Professor Constantin Udriste

Abstract. The use of the non-Riemannian geometry can help to solve the fundamental problems that are known nowadays in the general relativity theory (GRT) which is unable to explain a row of observed astrophysical phenomena on large scales. These observations include flat rotation curves of spiral galaxies, Tully-Fisher law, globular clusters behavior and some others. It turns out that the introduction of the anisotropic metric corresponding to the generalized Lagrange space solves those problems and leads to the modification of the gravitation theory. It is important to underline that the introduction of the mentioned metric is not arbitrary or aimed just at fitting the observations. It is performed on the base of the equivalence principle understanding and with account to the general geometrical identity. Therefore, the classical GRT and its results remain valid in its region of applicability. It turns out that on the galactic and larger scales, the observed phenomena can be interpreted as the revelation of the classical GRT tests and this leads to the promising cosmological consequences. It also appears possible to reveal the fundamental (geometrical) origin of the cH acceleration value appearing in equations which is usually mentioned as an empirical coincidence.

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1 Introduction

The systematic mathematical approach to the so called geometric dynamics was developed in [13]. Here we will start with the specific astrophysical problems in order to show not the possibility but rather the inevitability of the anisotropic metric introduction stemming from the main paradigm of the GRT. As it is well known, any theory that combines Newtonian gravity (inverse square law) together with Lorentz invariance in a consistent way must include a "gravitomagnetic" field, which is generated by mass current. And there exists the so called GEM theory (gravitoelectromagnetism) which follows this receipt introducing scalar and vector gravitational

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potentials describing gravitation in the Riemannian space and producing the set of dynamics equations similar to that of electromagnetism

$$(1.1)\quad \begin{cases} \vec{F} = m\vec{E}_G + m\left[\dfrac{\vec{v}}{c}, \vec{B}_G\right], & \vec{E} = -\nabla\Phi - \dfrac{1}{2c}\dfrac{\partial\vec{A}}{\partial t}, & \vec{B} = rot\vec{A}, \\ \Phi = -\dfrac{GM}{r}, & A^l = \dfrac{G}{c}\dfrac{S^n x^k}{r^3}\varepsilon^l_{nk}, & M = \displaystyle\int \rho d^3x, \\ S^i = 2\displaystyle\int \varepsilon^i_{jk} x'^j \dfrac{T^{k0}}{c} d^3x', & \vec{B}_G = -4\frac{G}{c}\dfrac{3\vec{r}(\vec{r},\vec{S}) - \vec{S}r^2}{2r^5}, & \dfrac{T^{i0}}{c} = j^i. \end{cases}$$

The GEM theory describes the particular class of phenomena with a Lorentz type gravitational forces acting on a probe particle moving nearby the spinning point mass. The known Lense-Thirring effect [6] dealing with the precessions of the gyroscope and its orbit in the field of the rotating star predicted by the GEM theory was successfully measured (most recently by Gravity Probe B [2]). The properties of the spinning mass (black holes) are also described with the use of Kerr metric. In both cases the anisotropy of the space-time is in a sense already present though inconsistently.

The equations describing the inverse square force law for both theories – electromagnetism and gravitation – can be deduced from the geometrical identity

$$(1.2)\qquad \frac{\partial F_{\rho\sigma}}{\partial x^\tau} + \frac{\partial F_{\sigma\tau}}{\partial x^\rho} + \frac{\partial F_{\tau\rho}}{\partial x^\sigma} = 0$$

which is known in geometry as “Maxwell equations” and which is true for *any* ($F_{\rho\sigma} = A_{\sigma,\rho} - A_{\rho,\sigma}$)-type tensor. But evidently, there must be an essential difference between the electromagnetic and gravitational approaches to the interpretation of its consequences. When regarding the electromagnetic phenomena in the Riemannian space whose metric is isotropic, the (velocity dependent) Lorentz force is preserved as an external one, while for the gravitation such (velocity dependent) force *must enter the metric* due to the equivalence principle according to which one can not distinguish between inertia and gravitation. Thus, the metric and the space itself must become anisotropic, and this will change the dynamics equations. From the mathematical point of view, this can be done in electromagnetic case too (see [2], [14]]) – any corresponding 4-potential can either enter the metric and make it anisotropic or leave the metric as it is and remain external. But from the physical point of view, namely, from the relativistic one, in case of gravitation there is no such choice.

2 Anisotropic perturbation and generalized geodesics

Let us regard the manifold M whose tangent space TM is an 8-dimensional anisotropic space with the coordinates (x^i, y^i), $i = 1\dots 4$. Here along some curve (trajectory of the probe particle), $x^i = x^i(s)$, we always consider $y^i = \frac{dx^i}{ds}$. For physical interpretations one has to pass to the physical coordinates in the following way $(x^1, x^2, x^3, x^4, y^1, y^2, y^3, y^4) \leftrightarrow (ct, x, y, z, \frac{c}{H}, \frac{v_x}{H}, \frac{v_y}{H}, \frac{v_z}{H})$ in order to preserve the same units for all of them. Here $[c] = m/s$ and $[H] = 1/s$ are some fundamental constants demanded by the units choice.

Let us introduce the anisotropic metric of the following form

$$\tilde{g}_{ij}(x, u(x), y) \equiv g_{ij}(x, y) = \gamma_{ij} + \varepsilon_{ij}(x, y) \tag{2.1}$$

where γ_{ij} is x-independent metric (here we will choose Minkowski one), $\varepsilon_{ij}(x, y)$ is a small anisotropic perturbation, y belongs to the tangent space, and *u(x)* is the vector field related to the motion of the curvature sources, and it generates the anisotropy. This is a generalized Lagrange metric, and every point of the main manifold is supplied with *two* vectors belonging to a tangent space. The tangent bundle of a space with an anisotropic metric becomes an eight dimensional Riemannian manifold equivalent to the *phase space*. Euler-Lagrange equations can be obtained by varying the Lagrangian, $L = (\gamma_{hl} + \varepsilon_{hl}(x, y))y^h y^l$. In this case the expression for the generalized geodesics is obtained similarly to [12] and takes the form:

$$\frac{dy^i}{ds} + \left(\Gamma^i_{lk} + \frac{1}{2}\gamma^{it}\frac{\partial^2 \varepsilon_{kl}}{\partial x^j \partial y^t} y^j\right) y^k y^l = 0 \tag{2.2}$$

where $\Gamma^i_{jk} = \frac{1}{2}\gamma^{ih}(\frac{\partial \varepsilon_{hj}}{\partial x^k} + \frac{\partial \varepsilon_{hk}}{\partial x^j} - \frac{\partial \varepsilon_{jk}}{\partial x^h})$ are the y-dependent Christoffel symbols.

Let us use the generalized geodesics (2.2) to follow the classical Einstein approach [3] step by step. Particularly, first, make the simplifying assumptions. Two of them are just those introduced by Einstein when deriving Newton law, and the third assumption reproduces the second one with regard to the y-derivatives. This means that $\varepsilon(x, y)$ is considered small enough to use a linear approximation.

The assumptions are the following:

1. The velocities of the material objects are much less than the fundamental velocity. This means that the components y^2, y^3 and y^4 can be neglected in comparison with y^1 which is equal to unity within the accuracy of the second order;

2. Since the velocities are small, the "time" derivative of metric $\frac{\partial \varepsilon_{hj}}{\partial x^1}$ can be neglected in comparison to the "space" derivatives $\frac{\partial \varepsilon_{hj}}{\partial x^\alpha}; \alpha = 2, 3, 4$.

3. The same is taken true for the y-derivatives: $\frac{\partial \varepsilon_{hj}}{\partial y^1}$ can be neglected in comparison to the space derivatives $\frac{\partial \varepsilon_{hj}}{\partial y^\alpha}; \alpha = 2, 3, 4$.

As in [3], the assumptions make it possible to preserve only the terms with $k = l = 1$, which means that the only ε_{kl} remaining in the equation (2.2) is ε_{11}, while $y^k = y^l = 1$. Let us introduce the new notation for the y-derivative of the perturbation

$$\frac{1}{2}\frac{\partial \varepsilon_{11}}{\partial y^t} \equiv A_t \tag{2.3}$$

similar to a component of the Cartan tensor. Notice, that A_t are the components of the y-gradient of ε_{11}, i.e. $A_\alpha = \frac{1}{2}(\nabla_{(y)}\varepsilon_{11})_\alpha$ for $\alpha = 2, 3, 4$ (the same numeration 1 to 4 is used for both x- and y- variables). Then we get

$$\frac{dy^i}{ds} + \Gamma^i_{11} + \gamma^{it}\frac{\partial A_t}{\partial x^j} y^j = 0 \tag{2.4}$$

The third term in the (2.4) does not vanish since, though we assume $\frac{\partial A_i}{\partial x^1} << \frac{\partial A_i}{\partial x^\alpha}$, but $y^1 >> y^\alpha$ for $\alpha = 2, 3, 4$. In order to transform the geodesics into a convenient form, let us add and subtract the same value $\gamma^{it}\frac{\partial A_j}{\partial x^t}y^j$ to extract an anti-symmetric part of the tensor and obtain

$$\frac{dy^i}{ds} + \Gamma^i_{11} + \gamma^{it}\left[\left(\frac{\partial A_t}{\partial x^j} - \frac{\partial A_j}{\partial x^t}\right) + \frac{\partial A_j}{\partial x^t}\right] y^j = 0 \tag{2.5}$$

The expression $(\frac{\partial A_t}{\partial x^j} - \frac{\partial A_j}{\partial x^t})$ can be taken as a component of an anti-symmetric tensor, F_{jt}, and the (2.5) for the generalized geodesics yields

$$\frac{dy^i}{ds} + \Gamma^i_{11} - \gamma^{it}F_{tj}y^j + \gamma^{it}\frac{\partial A_j}{\partial x^t}y^j = 0 \tag{2.6}$$

For $\alpha, \beta = 2, 3, 4$ the expressions $(\frac{\partial A_\alpha}{\partial x^\beta} - \frac{\partial A_\beta}{\partial x^\alpha})$ are the components of the curl of the vector $\vec{A}$.

The two first terms of (2.6) present the Einstein result [3]. The three first terms make one think of an additional (electromagnetic?) field with a 4-potential. But no field but the ineradicable gravitation and no "potentials" to describe it were introduced up to now.

One can see that according to the assumption $\frac{\partial A_i}{\partial x^1} << \frac{\partial A_i}{\partial x^\alpha}$ and to the definition (2.3), vector $E^{(g)} = -\nabla_{(x)}A_1$ where A_1 is the value of the first component of the y-gradient of ε_{11}, i.e. $A_1 = \frac{1}{2}(\nabla_{(y)}\varepsilon_{11})_1$. Vectors $\vec{E}^{(g)}$ and $\vec{B}^{(g)} \equiv rot_{(x)}\vec{A}$ are *obtained out of the anisotropic metric* and are related to the vector field $u(x)$ in the expression of metric. It is only now that one could give names like vector potential of the gravitational field, mass density of the source of gravity, and density of the mass flow corresponding to the proper motion of the source and its parts. Notice the difference with the GEM approach, in which the gravitation field was endowed with these characteristics from the very beginning. Now their origin is rather geometrical than physical, and there is an additional term in the equation. After that, one still has an impressive analogy with electromagnetism, and all the formalism developed for it can be used in calculations.

3 Equation of motion and gravitation forces

As it was shown in [11], the expression for the gravitation force acting on a particle with mass m gets the form

$$m\frac{d\vec{v}}{dt} = \vec{F}^{(g)} = \frac{mc^2}{2}\left\{-\nabla_{(x)}\varepsilon_{11} + \left[\vec{v}, rot_{(x)}\frac{\partial\varepsilon_{11}}{\partial\vec{v}}\right] + \nabla_{(x)}\left(\frac{\partial\varepsilon_{11}}{\partial\vec{v}}, \vec{v}\right)\right\} \tag{3.1}$$

The first term is related to the expression for the usual gravity force, $F_N^{(g)}$, acting on a particle with mass, m. For the stationary spherically symmetric source of gravitation with mass M, the solution of Poisson equation suggests $\varepsilon_{11} \sim 1/r$, where r is the distance from the particle to the source, and in this case the expression $\varepsilon_{11} = \frac{2GM}{c^2}\frac{1}{r}$ in (3.1) for the point source at sufficient distances would give Newton law $F_N^{(g)} = G\frac{Mm}{r^2}$. The value $r_S = \frac{2GM}{c^2}$ corresponds to Schwarzschild radius.

Introducing the notation

$$\vec{u} \equiv \frac{c^2}{2}\frac{\partial\varepsilon_{11}}{\partial\vec{v}} \equiv [\vec{\omega}, \vec{r}] \tag{3.2}$$

we get the following expression for the gravitation force

$$\vec{F}^{(g)} = m\nabla_{(x)} \left\{ -\frac{c^2}{2}\varepsilon_{11} + (\vec{u}, \vec{v}) \right\} + m[\vec{v}, rot_{(x)}\vec{u}] \tag{3.3}$$

Here $\vec{v}$ is the velocity of the particle whose dynamics is described by (3.1), and the proper motion of the gravitation sources is described by vector $\vec{u}$. Thus, one can see that the actions produced by $\vec{F}_C^{(g)} \equiv m[\vec{v}, rot_x\vec{u}]$ on a body could be attraction, repulsion and tangent action depending on the angle between $\vec{v}$ and $\vec{\omega}$. Notice that the component of velocity $\vec{v}$, which is parallel to $\vec{\omega}$ is not affected by the second term in (3.1).

Introducing the specific vectors $\vec{\beta} = \vec{v}/c$ and $\vec{\Theta} = \vec{\omega}/H$, in which c and H represent the geometrically motivated constants mentioned above, one obtains

$$\vec{F}_C^{(g)} = 2m \cdot cH \cdot [\vec{\beta}, \vec{\Theta}] \tag{3.4}$$

If we interpret the fundamental velocity, c, as the speed of light (as it is usually done) and the measurement units factor, H, as Hubble constant, we find out that the origin of the value of numerical factor which was noticed and discussed many times in astrophysics and gravitation theory modifications stems from geometry. When the product, $\beta\Theta$ approaches unity, the value of additional acceleration approaches cH.

In order to understand the meaning of the scalar product in the first term in (3.3), let us recollect the expression for the inertial force acting on a body in a non-inertial reference frame, choosing the uniformly rotating one for simplicity:

$$\vec{F}_{N-I} = m\nabla \left(\Phi + \frac{u^2}{2} \right) + m[\vec{v}, rot\vec{u}] \tag{3.5}$$

where Φ is the force potential, v is the particle velocity, and $\vec{u} = [\vec{\omega}, \vec{r}]$ is the linear velocity of the frame at the distance r from the axis of rotation. The second term in (3.5) is Coriolis force and the second term in brackets under gradient is called potential centrifugal energy. As r approaches infinity, this energy also becomes infinite which shows the difference between the physical energies (e.g. the energy of gravitational interaction that vanishes at infinity) and formal ones originating from the formalism (choice of reference frame). We see that the second term under gradient in (3.3) has obtained the features of the physical and not formal energy. It does not become infinite at infinity but can be *finite* or zero there and can have *both* signs depending on the value of the scalar product. Moreover, the meaning of this "infinity" as it could appear now in the physical problems requires some clarification. There is a difference between the situation within the vicinity of a single gravitation source and the situation inside the *distribution* of the (moving) gravitation sources like a galaxy.

The term in question could correspond to the action produced on a moving particle by radial expansion (explosion) or by radial contraction (collapse) of the system of gravitating sources. The particle suffers an additional attraction to or repulsion from the center of mass distribution depending on the sign of scalar product. If the system of sources expands and the particle moves radially inwards, or if the system of sources contracts and the particle moves radially outwards, there is an additional attraction. If the system of sources expands and the particle moves radially outwards, or if the system of sources contracts and the particle moves radially inwards, the particle suffers a repulsion from the center of mass distribution.

Thus, the characteristic features of what can be called "anisotropic geometrodynamics" (AGD) approach given here and stemming from geometry are the following. The total acceleration of the probe particle can now depend not only on the location of distributed masses but also on their proper motion and on the motion of the particle itself. Notice, that in AGD the gravitational interaction ceases to be simple attraction as before, it depends on the motion of the particle and of the sources and can be attraction, repulsion and transversal action. The value of *cH,* which earlier had an empirical origin, may now be regarded as an intrinsic (geometrical) property of the theory. It goes without saying that all the GRT results remain valid for a planetary system scale.

4 Basic model of the gravitation source in AGD and its applications

In order to obtain the testable predictions and compare them to the astrophysical restrictions for any modification of the gravitation theory demanded in [1], we need a simple model. The basic model of the gravitation source in the GRT is a massive spherical body. It works well for a planetary system with a star in the center, but fails to explain flat rotation curves in spiral galaxies, Tully-Fisher law for the luminosity of spiral galaxies

$$L_{lum} \sim v_{orb}^4 \tag{4.1}$$

and the globular clusters problem. The last one has two sides. On the one hand, the globular clusters that don't belong to the galaxy plane obey the usual Einstein or Newton gravitation and, therefore, there is no need for the theory modification with regard to the motion in the direction orthogonal to the galaxy plane (anisotropy). On the other hand, too many of them are known to be located in the vicinity of the galaxy center instead of spending most of their time on the periphery in accordance with second Kepler law. In [1] it was argued that none of the known proposals suffices all of these restrictions. One could also mention the lensing effect, which confirms the GRT qualitatively but is sometimes 4-6 times larger than predicted.

In the AGD developed for the distributions of moving sources, the situation changes. A spiral galaxy mentioned above consists of billions of stars and has the natural preferential direction – the axis of its rotation. It can not be modeled by a point unless we are far enough from it. The basic (and the simplest) model in case of the AGD must be different.

Let a system consist of a central mass and an effective circular mass current, $J^{(m)}$ around it. For brevity, we will call such model a CPC (center plus current). Its parameters are the effective values of contour radius, R_{eff}, constant angular velocity, Ω_{eff}, and linear velocity of mass density motion along the contour, $V_{eff} = \Omega_{eff} R_{eff}$. Simple formulas for estimating these parameters from observations for concrete galaxies can be found in [10].

Due to the identity of the origin of Maxwell equations for electrodynamics and for gravitation, such model is quite similar to electromagnetic one that consists of a circular electric current and a charge at the center of the contour. Thus, if we regard the radially stationary system, the mathematical results already obtained in electrodynamics can be used in calculations dealing with velocity dependent gravitation.

4.1 Flat rotation curve

Let a spiral galaxy possess a bulge and an effective circular mass current. In the electromagnetic version of the CPC model, we regard a positive charge, a circular contour with current, J around it and an electron orbiting the system in the plane of the contour. Strictly speaking, an electron in such a system cannot be in a finite motion and has either to fly away or to fall on the center. But this system could be meta-stable, and the number of electron rotations could be large enough. The value of $B_z(r)$ component of the magnetic induction produced by the contour with radius, R_{eff}, can be found with the help of Bio-Savart law. According to [5] with $c = 1$ it is equal to

$$(4.2)\quad \begin{aligned} &B_z(r) = J\frac{2}{\sqrt{(R_{eff}+r)^2+z^2}}\left[K + \frac{R_{eff}^2 - r^2 - z^2}{(R_{eff}-r)^2+z^2}E\right] \\ &K = \int_0^{\pi/2}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}; \qquad E = \int_0^{\pi/2}\sqrt{1-k^2\sin^2\theta}\,d\theta \\ &k^2 = \frac{4R_{eff}r}{(R_{eff}+r)^2+z^2}, \end{aligned}$$

where K and E are the elliptic integrals. Introducing notation, $b = r/R_{eff}$, and taking $z = 0$, one gets

$$(4.3)\quad B_z(r) = J\frac{2}{R_{eff}(1+b)}\left[K + \frac{1-b^2}{(1-b)^2}E\right].$$

The internal region close to the charge corresponds to $b << 1$ and $B_z(r) \to J/2R_{eff}$, the far away region corresponds to $b >> 1$ and $B_z(r) \to 0$, and the intermediate region to which the contour also belongs corresponds to $b = O(1)$ and

$$(4.4)\quad B_z(r) \sim J/r.$$

The centrifugal force acting on the orbiting electron, $m\frac{v_{orb}^2}{r}$, is equal to the sum of the Coulomb attraction, $F_{Cl} = qC_1/r^2$, produced by the central charge and the Lorentz force, $F = qv_{orb}B_z(r)$. For the intermediate region corresponding to the periphery of a galaxy, the dynamics equation with $J \equiv C_2$ can be written as

$$(4.5)\quad \frac{mv_{orb}^2}{r} = \frac{qC_1}{r^2} \pm qv_{orb}\frac{C_2}{r}$$

where C_1 and C_2 are constants characterizing the system and the sign corresponds to the direction of current and the location of the electron inside or outside the contour. Applying this equation to the gravitational case, we have to substitute the electric charge by the gravitational one, $q \equiv m_g$, and use the equivalence principle, $m_g = m$. The smaller root of the square equation (4.5),

$$v_{orb_s} = \frac{C_2}{2}\left(1 - \sqrt{1 \pm \frac{4C_1}{rC_2^2}}\right)$$

corresponds to Newton's law, the sign depends on the direction of the electron motion. Neglecting the small term inside the square root in the larger root of the equation,

$$v_{orb_l} = \frac{C_2}{2}\left(1 + \sqrt{1 \pm \frac{4C_1}{rC_2^2}}\right),$$

one gets

$$v_{orb} \sim C_2. \tag{4.6}$$

This corresponds to the flat rotation curve on the periphery of a spiral galaxy.

4.2 Tully-Fisher law

Let us estimate $C_2(R_{eff}) = J^{(m)}(R_{eff})$. The mass current is given by $J^{(m)}(R_{eff}) \sim M/T$ with M proportional to the area of a spiral galaxy, R_{eff}^2, and the period $T \sim R_{eff}^{3/2}$ according to Kepler law. This gives $J^{(m)}(R_{eff}) \sim \sqrt{R_{eff}}$. Since the luminosity, L_{lum}, is also proportional to the galaxy area, we get $R_{eff} \sim \sqrt{L_{lum}}$. Therefore,

$$J^{(m)}(R_{eff}) \sim \sqrt{R_{eff}} \sim L_{lum}^{1/4},$$

and

$$v_{orb} \sim L_{lum}^{1/4}. \tag{4.7}$$

This corresponds to the observational Tully-Fisher law (4.1).

4.3 Applicability region

The results presented by (4.6) and (4.7) suggest to estimate the regions and regimes for which this or that term in (3.1) plays an essential role. With regard to definitions and (3.2, 3.3), we can take $\omega \sim \frac{G}{c^2 r}\frac{MV_{eff}}{R_{eff}}$, and the acceleration given by $a_C = 2[\vec{v}, \vec{\omega}]$ is proportional to

$$a_C \sim v\frac{GMV_{eff}}{c^2 r R_{eff}} \tag{4.8}$$

Then the ratio of this acceleration to the Newtonian one, $a_N = \frac{GM}{r^2}$ is

$$\frac{a_C}{a_N} \sim \frac{vV_{eff}}{c^2}\frac{r}{R_{eff}} = \frac{vr}{c^2}\Omega_{eff}, \tag{4.9}$$

and one can estimate the region where the specific features of AGD become significant. For a given particle moving with velocity, v, at the distance, r, from the center of spiral galaxy this ratio becomes

$$\frac{a_C}{a_N} \sim \frac{vr}{c^2} \frac{L_{eff}}{I_{eff}} \tag{4.10}$$

where L_{eff} and I_{eff} characterize the galaxy. Every concrete case must be considered with regard to (4.10).

Let us regard an illustrative limit case when the contour with mass current is close to the rim of the giant black hole in the center of a galaxy. Its mass is M, the effective radius is $R_{eff} = r_S = \frac{2GM}{c^2}$ and the effective velocity is equal to orbital velocity $V_{eff} = c$. Then the acceleration ratio (4.9) will be equal to

$$\frac{a_C}{a_N} \sim \frac{vV_{eff}}{c^2} \frac{r}{R_{eff}} = \frac{cvr}{2GM} \tag{4.11}$$

Let us find the distance at which both accelerations are equal and $M = 10^{11}$ Solar masses (e.g. as in Milky Way)

$$1 \sim \frac{cvr}{M_{Sol}} \sim 10^{-23} vr. \tag{4.12}$$

We see that the measured observable orbital velocity of stars at the periphery, $v \sim 10^5 [m/s]$, corresponds to the distance, $r \sim 10^{18} [m]$, which is in accordance with the estimation for the galaxy radius. This means that from the point of view of such gravitation theory as AGD, there is no reason to expect the behavior of the rotation curves to be Newtonian.

5 Classical GRT tests in the AGD

The CPC model makes it possible to discuss the classical GRT tests – orbit precession, light bending and gravitational red shift – as they appear on the galactic scale. In view of the AGD approach, some visual results can be obtained with the help of numerical calculations. The last is primarily due to the fact that the needed functions (known for the CPC model in electrodynamics) are expressed by elliptical integrals.

5.1 Orbit precession

In order to describe the motion of a star around a spiral galaxy with a bulge in the center, let us use the CPC model and regard a particle orbiting the system in the plane of the contour. As it was mentioned in section 4.1, a particle in such a system can not perform finite motion and has either to fly away or to fall to the center. Therefore, as can be seen on Fig.1a, a globular cluster can be found in the vicinity of the center and not at the periphery, where it should spend the majority of its lifetime when on Keplerian elliptical orbit corresponding to Newton gravity.

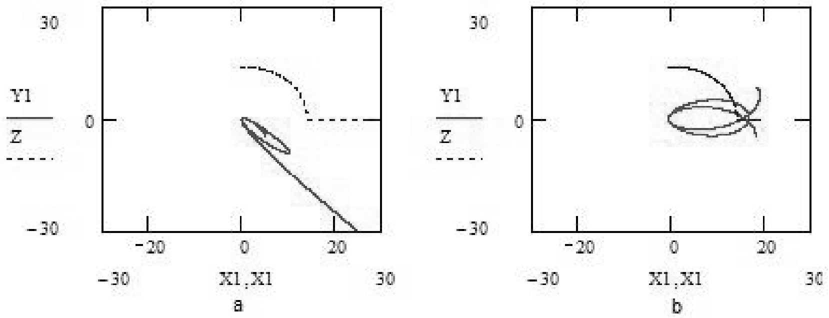

Fig.1.
AGD based trajectories mimicking various observable phenomena: a) non-Keplerian behavior of globular clusters; b) quasi-precession.

If the number of the particle's rotations characterizing the meta-stable situation when it orbits the CPC is large enough, one can see on Fig.1b what could be called the quasi-precession. This type of star's motion is a galactic scale AGD analogue of the GRT orbit precession. It is also this type of motion that is presumed in the explanation of the spiral's arms origination by the density wave theory [7].

5.2 Light bending

On Fig.2 there are the results of the numerical modeling of a probe body scattering on a Coulomb center and on a CPC. Since the metric now depends on the proper motion of the source, the light beams will give similar trajectories.

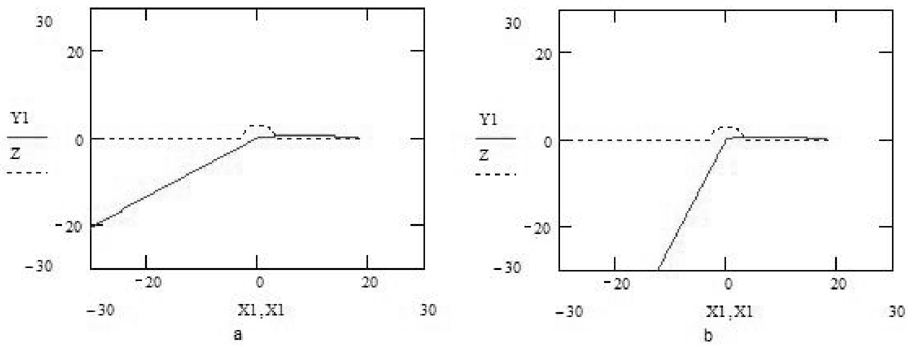

Fig.2. AGD based trajectories for a probe body scattering: a) on the Coulomb center; b) on the CPC.

This explains why the observable gravitational lenses can give larger refraction than they should according to the GRT in which another basic model is used to model them. Moreover, on Fig.3 there is a probe body trajectory with double bending which can appear with the change of the current direction. From the point of view of light refraction, it means that the AGD predicts the existence of the negative gravitational lenses that could diminish the angular size of the objects behind them.

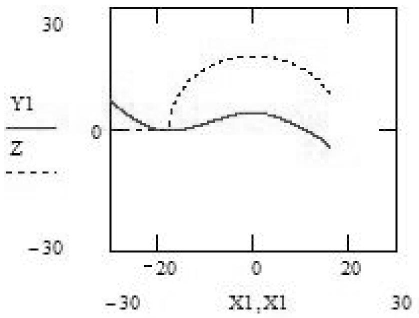

Fig.3. AGD based trajectory with double bending.

Therefore, if such a lens is located between the object and the observer on the Earth, the distance to the object might be considered larger than it really is. If this object is the astronomical standard candle, then this could be the reason of misinterpretation of the last decade supernovae 1a observations [9] as breaking the linear Hubble law. It is these observations that lead to the appearance of the notion of dark energy (of repulsion) that causes the acceleration of the Universe expansion, and to the revival of the cosmological constant in Einstein equations. Now it has to have the opposite sign with regard to that used by Einstein who also needed extra energy (of attraction) to preserve the observable stationary Universe consistent with the GRT. But interpreting these SN1a observations with regard to the possible negative gravitational lenses refraction predicted by the AGD, one has no need to demand this dark energy from the Universe, while all the results of the GRT remain still valid in the region of its applicability.

5.3 Gravitational red shift

Similarly to the GRT, the gravitation in the AGD also causes the increase of the time interval and consequently, causes the gravitational red shift, but as follows from (2.1) now it also depends on the motion of the source of gravitation. Taking into consideration the CPC model, this means that the light coming to the Earth from the far away can be affected by the masses moving tangentially to the line of sight and can acquire the gravitational red shift. This suggestion finds support in the observations of the tangent motions of distant quasars – they take place at amazingly high velocities [8]. Can this red shift explain the observable red shift discovered by Hubble, is an open question. If its value is comparable to the Hubble red shift, which is presumably caused by the Universe expansion, then the very expansion can be doubted. At any rate, this AGD mechanism should be included into the cosmological picture, which now becomes not stationary as in Einstein times but also not simply radially expanding as it was considered up to now. As follows from the AGD, the tangential motion and the corresponding gravitation at the periphery of the observed Universe provides a new insight on Mach's principle and on the border of the Universe problem.

6 Discussion

The modification of the gravitation theory must include mass currents produced by the distributed sources, while the geometrical identity (1.2) leads to Maxwell equations independently of the origin of physical field that could be used for interpretation. The difference with electromagnetism is that the equivalence principle demands to account for the interaction between the moving probe body and a (mass) current not by introduction of an extra (Lorentz type) force, but by the modification of the space-time metric which becomes anisotropic. This anisotropy is interpreted as the dependence of gravitation forces acting between the bodies not only on their position but also on their motion. The proper motion of the distributed sources of gravitation adds extra terms to the gravitation field, and the moving probe body interacts with it with account to its own velocity as shows (3.1).

It seems clear that all the results of the GRT remain valid when the proper motion of the sources can be neglected, for example, in a planetary system when a star gravitation dominates over other sources. But when many comparable gravitation sources start to move relative to each other and the scale of phenomena in question grows, the discovered anisotropy in geometrodynamics starts to play an essential role as shown by (4.9, 4.10). This is demonstrated by the explanation of the flat character of the rotation curves of spiral galaxies, (4.6), which can not be done in frames of classical GRT. It turns out that the AGD approach also explains the empirical Tully-Fisher law, (4.7), provides the fundamental (geometrical) origin to the cH acceleration value, (3.4), and suffices the astrophysical restrictions for the gravitation theory modifications with concern to the observed motion of globular clusters. The role of the third term in the expression for the gravitation force (3.1) might appear important for radial instabilities of mass distributions like explosions and collapses. The possible existence of negative gravitational lenses could influence the interpretation of the supernovas 1a observations in such a way that no acceleration of the Universe expansion takes place.

These ideas have far going perspectives in cosmology in general in which the following new direction of thought appears. First, we see that the flat rotation curves can be explained without introduction of *dark matter* notion for galaxies, and it makes one think that the gravitational binding in galaxy clusters could be also provided by additional gravitational force due to the relative motion of galaxies. Second, according to the AGD, the repulsive forces presumably acting on the cosmological scale could be provided by the velocity dependent gravitation, therefore, the notions of *dark energy* of repulsion and its sources (quintessence?) start to cause specific doubts. Third, the Hubble red shift the explanation of which nowadays refers to the (infinite) Universe expansion could be also caused by the tangent motion of huge masses on the periphery of the (finite) visible part of the Universe – such motion would cause the additional gravitation force and, consequently, the *gravitational* red shift with accord to fundamental GRT ideas. This suggestion is supported by the observations of the tangent motions of distant quasars – they move at very high velocities [8]. Finally, the AGD provides a new insight for Mach's principle and for the border of the Universe problem.

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