Eulerian distributions applied in the reliability

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Dedicated to the 70-th anniversary
of Professor Constantin Udriste

Abstract. The main objective of the paper is to prove that Eulerian functions are adaptive distributions for describing of the reliability of systems, because these sets of elements contain symmetrical and skewed functions. Numerical adequate examples from auto equipments were processed with specialised software. These proposed models allow simulating and prediction of the lifetime of the products too.

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1 Introduction

"The English biometrician Sir Francis Galton (1822 - 1911) fitted a straight line to the plot of the heights of sons versus the heights to their father in his studies of inheritance, in particular of the law of universal regression" [3].

In order to achieve these behaviors many new models were developed in the technical books [1], [2]. Eulerian distributions are widely used in reliability analysis [10], [12], because for many data sets they fit very well. Some lifetime laws, derived from the Eulerian distributions, have a bathtub-shaped hazard-rate function [5], [6].

The experimental researches show that the distributions can be symmetric or skewed, left or right. In many papers were presented some Weibullean models too [7], [11]. The present paper analyses some of these laws and focuses the study of new adequate Eulerian models, for the given experimental automotive industry data. These models are used at the prediction the reliability of the systems [8], [9].

The ranges of canonical distributions are usually unbounded, while the lifetime of the products is a bounded interval [12], [13]. The parameters of the proposed functions were estimated from the experimental data and the computations were carried out in CurveExpert.

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2 Methods and modelling data set

The following continuous probability laws were considered for presented experimental data:

$$f_1(t) = \exp\left(a + \frac{b}{t} + c\ln t\right), \tag{2.1}$$

(the vapor pressure model)

$$f_2(t) = a - b\exp(-ct^d), \tag{2.2}$$

(an adaptive Weibullean distribution)

$$f_3(t) = at^b(11 - t)^c, \tag{2.3}$$

(an Eulerian distribution of beta type)

$$f_4(t) = a(t-2)^b(11 - t)^c, \tag{2.4}$$

(a shifted Eulerian distribution of beta type)

$$f_5(t) = a(t-2)^b\exp[(11 - t)^c], \tag{2.5}$$

(a shifted Eulerian distribution of gamma type).

In order to apply the proposed models, experimental failure time data for automotive components (Table 1) are processed with the software CurveExpert.

Interval	Interval mean	Relative frequency
[2.75 − 3.25)	3	4
[3.25 − 3.75)	3.5	7
[3.75 − 4.25)	4	11
[4.25 − 4.75)	4.5	12
[4.75 − 5.25)	5	14
[5.25 − 5.75)	5.5	16
[5.75 − 6.25)	6	17
[6.25 − 6.75)	6.5	15
[6.75 − 7.25)	7	14
[7.25 − 7.75)	7.5	13
[7.75 − 8.25)	8	10
[8.25 − 8.75)	8.5	8
[8.75 − 9.25)	9	7
[9.25 − 9.75)	9.5	5
[9.75 − 10.25]	10	3

Table1. Experimental failure time data, grouped by intervals

In comparison with the canonical forms, it was choose adaptive models for the cases of bounded intervals. The unknown coefficients (a, b, c, d) are determined based on the experimental data using the nonlinear regression. In Figs. 1, 2, 3, 4, 5 are represented the experimental points and the obtained regression curves for each model. The computations were carried out with CurveExpert, based of the obtained distribution functions.

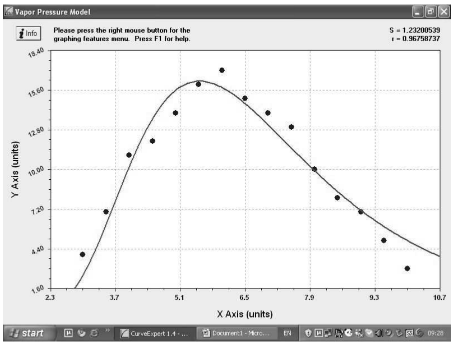

Fig.1. Vapor pressure model.

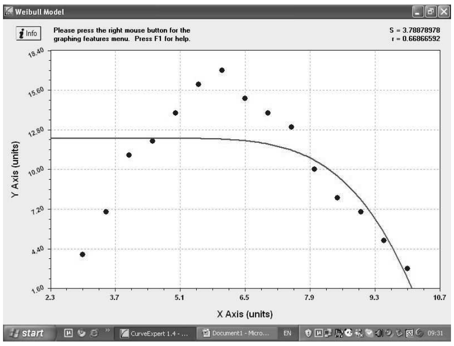

Fig.2. An adaptive Weibullean distribution

$$f_1(t) = \exp\left(24.5 - \frac{44.2}{t} - 8\ln t\right) \qquad f_2(t) = 1.22 - 10630\exp(-78t^{-1.048}).$$

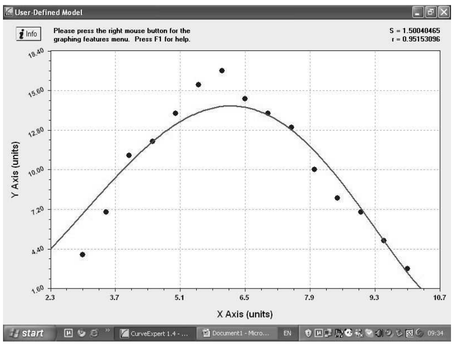

Fig.3. Eulerian distribution of β type.

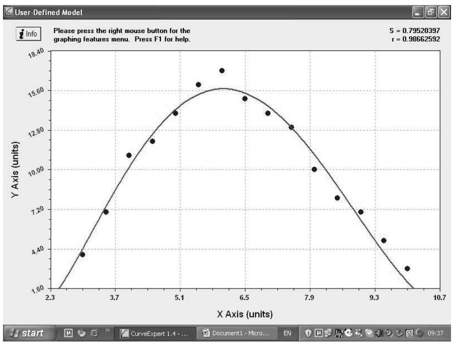

Fig.4. Shifted Eulerian distribution of β type

$$f_3(t) = 0.015t^{2.24}(11 - t)^{1.75} \qquad f_4(t) = 0.08(t - 2)^{1.56}(11 - t)^{1.93}.$$

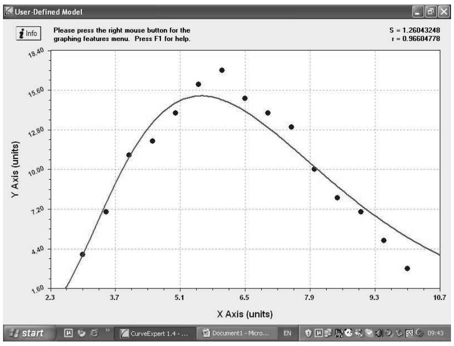

Fig.5. Shifted Eulerian distribution of γ type

$$f_5(t) = 0.02(t-2)^{2.41} \exp[(11-t)^{0.68}].$$

In some cases, the observed values are positively skewed, in other cases have a small variance and hence present a big degree of peakedness, compared to the normal distribution. An explication for this variability is the diversity of work conditions and the natural variability of the physical properties of the materials and of the manufacturing technology.

3 Results and discussions

In the table 2, are presented the standard errors and correlation coefficients $(S; r)$ for the analyzed functions.

Computed distribution	Standard error S	Correlation coefficient r
Vapor pressure model	1.2320	0.9675
Adaptive Weibullean distribution	3.7887	0.6686
Eulerian distribution of beta type	1.5004	0.9516
Shifted Eulerian distribution of beta type	0.7952	0.9866
Shifted Eulerian distribution of gamma type	1.2604	0.9660

Table 2. Standard errors and correlation coefficients

The skewed of the curves was better estimated with adaptive Eulerian laws [7], the graphs of these laws are much closer the analysis of values in table 2; it results that the Eulerian laws have a better goodness of fit with experimental data; the correlation coefficients have in most cases good values ($r \geq 0.95$).

The models vapor pressure model, Eulerian distribution of beta type, shifted Eulerian distribution of beta type and shifted Eulerian distribution of gamma type have a good fitting to experimental data as shown by standard errors S and correlation coefficients, r, but for these experimental data the adaptive Weibullean distribution offers a weaker approximation. The best result it is obtain with shifted Eulerian distribution of beta type, because this function has the smallest standard error and simultaneous with the biggest correlation coefficient.

These results can be useful both in design work and in the practice, allowing the prediction of the reliability of automotive components. The analysed adequate models can be easily adapted in the similar situations [4].

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