Some remarks about first order jet bundles over foliated manifolds

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Abstract. Let (E, π, M) be a bundle over the foliated manifold $(M, \mathcal{F})$ and $\mathcal{F}'$ the pull-back foliation of $\mathcal{F}$ by π. We consider the leafwise 1-jet manifold $J^{l,1}\pi$ and the fibre bundle $(J^{l,1}\pi, \pi^l_{1,0}, E)$. We prove that every first prolongation of a section of π induces a leafwise first prolongation of the same section. We prove that the pull-back bundle of the vertical bundle of π by $\pi^l_{1,0}$ is a direct summand of the pull-back bundle of the structural bundle of $\mathcal{F}'$ by $\pi^l_{1,0}$.

M.S.C. 2010: 55R10, 53C12, 58A20.
Key words: fiber bundles, foliated manifold, leafwise, first order jets.

1 Preliminaries

The geometry of jet bundles is an interest topic for many researchers in the last years. For an introduction in this subject we used [2], [6]. Some results in this domain are given in [1], [7]. The case of jets of bundles over foliated manifolds was studied in [5], [3]. Basic informations about geometry of foliated manifolds could be found in [4], [8].

Let (E, π, M) be a bundle, where $\pi : E \to M$ is a surjective submersion, M is a m-dimensional differentiable manifold and the fiber dimension is equal to n (so, E is a $(m+n)$-dimensional manifold). For a local chart $\left(V, \left(x^i\right)\right)$ in M, the adapted coordinate system in $\pi^{-1}(V) \subset E$ is $\left(x^i, y^\alpha\right)$, where $i = \overline{1,m}$, $\alpha = \overline{1,n}$. We shall use the same notation x^i for the coordinate functions x^i from M and $x^i \circ \pi$ from the manifold E. A local section of the bundle π in $x \in M$ is a map $\Phi : V \to E$, $x \in V \subset M$ such that $\pi \circ \Phi = 1_V$. The set of all local sections of π in x is denoted by $\Gamma_x(\pi)$. In [6] the 1-jet of a local section is defined as it follows:

Definition 1.1. We said that two local sections $\Phi, \Psi \in \Gamma_x(\pi)$ are *1-equivalent* at x if $\Phi(x) = \Psi(x)$ and if in some adapted coordinate system $\left(x^i, y^\alpha\right)$ around $\Phi(x)$,

$$\frac{\partial \left(y^\alpha \circ \Phi\right)}{\partial x^i}(x) = \frac{\partial \left(y^\alpha \circ \Psi\right)}{\partial x^i}(x), \tag{1.1}$$

BSG Proceedings 18. The Int. Conf. of Diff. Geom. and Dynamical Systems (DGDS-2010), October 8-11, 2010, Bucharest Romania, pp. 60-66.

for $i = \overline{1,m}$ and $\alpha = \overline{1,n}$. The equivalence class containing Φ is called the **1**-*jet* of the section Φ at x and is denoted $j^1_x\Phi$.

It is proved that the conditions (1) have geometrical meaning and that for $\Phi, \Psi \in \Gamma_x(\pi)$ which satisfy $\Phi(x) = \Psi(x)$, the equality $j^1_x\Phi = j^1_x\Psi$ is equivalent with $\Phi_{*|T_xM} = \Psi_{*|T_xM}$, where Φ_* is the linear tangent map of the map Φ.

The *1-jet manifold* of π is the set

$$J^1\pi = \left\{ j^1_x\Phi \mid x \in M, \Phi \in \Gamma_x(\pi) \right\}.$$

Given an atlas of adapted charts (U,u) on E, where $u = \left(x^i, y^\alpha\right)$, the collection of charts $\left(U^1, u^1\right)$ is a $(m+n+mn)$-dimensional C^∞-atlas on $J^1\pi$, where

$$U^1 = \left\{ j^1_x\Phi \in J^1\pi \mid \Phi(x) \in U \right\},$$

and the functions

$$u^1 = \left(x^i, y^\alpha, y^\alpha_i\right), \tag{1.2}$$

are defined by $x^i\left(j^1_x\Phi\right) = x^i(x)$, $y^\alpha\left(j^1_x\Phi\right) = (y^\alpha \circ \Phi)(x)$, $y^\alpha_i\left(j^1_x\Phi\right) = \frac{\partial(y^\alpha\circ\Phi)}{\partial x^i}(x)$. Moreover, $\left(J^1\pi, \pi_1, M\right)$ and $\left(J^1\pi, \pi_{1,0}, E\right)$ are bundles, where the surjection submersions $\pi_1 : J^1\pi \to M$, $\pi_{1,0} : J^1\pi \to E$ are defined by $\pi_1\left(j^1_x\Phi\right) = x$ and $\pi_{1,0}\left(j^1_x\Phi\right) = \Phi(x)$.

Let $W \subseteq M$ be an open domain. The first prolongation of a section $\Phi \in \Gamma_W(\pi)$ is the section $j^1\Phi \in \Gamma_W(\pi_1)$. Taking into account that $\pi_{1,0} \circ j^1\Phi = \Phi$, we have that a section $\psi \in \Gamma_W(\pi_1)$ is the first prolongation of a section in π if and only if $\psi = j^1(\pi_{1,0} \circ \psi)$, [6].

We consider now the vertical sub-bundle $(V\pi, \tau_V, E)$ of the tangent bundle (TE, τ_E, E), where $\tau_V = \tau_E|_{V\pi}$. It is known that the sub-bundle τ_V did not has distinguished complement in the absence of a connection on π. But, when these bundles are pulled-back to $J^1\pi$ by $\pi_{1,0}$, there is a complement of τ_V, called the bundle of holonomic tangent vectors, [6]. The pulled-back bundles are:

$$(\pi^*_{1,0}(TE), \pi^*_{1,0}(\tau_E), J^1\pi), \quad (\pi^*_{1,0}(V\pi), \pi^*_{1,0}(\tau_V), J^1\pi),$$

where

$$\pi^*_{1,0}(TE) = \{(X_w, j^1_x\phi) \in T_wE \times J^1\pi | w \in E, \tau_E(X_w) = \pi_{1,0}(j^1_x\phi)\},$$

$$\pi^*_{1,0}(\tau_E)((X_w, j^1_x\phi)) = j^1_x\phi,$$

$$\pi^*_{1,0}(V\pi) = \{(X_w, j^1_x\phi) \in V_w\pi \times J^1\pi | w \in E, \tau_V(X_w) = \pi_{1,0}(j^1_x\phi)\},$$

$$\pi^*_{1,0}(\tau_V)((X_w, j^1_x\phi)) = j^1_x\phi.$$

For every point $x \in M$, every section $\phi \in \Gamma_x(\pi)$ and $\zeta \in T_xM$, the holonomic lift of ζ by ϕ is defined to be the pair $(\phi_*(\zeta), j^1_x\phi) \in \pi^*_{1,0}(TE)$. It is proved, [6], that

Proposition 1.1. *We have the following decomposition*

$$(\pi^*_{1,0}(TE))_{j^1_x\phi} = (\pi^*_{1,0}(V\pi))_{j^1_x\phi} \oplus \phi_*(T_xM),$$

where $\phi_*(T_xM)$ denotes the collection of holonomic lifts of tangent vectors in T_xM by ϕ. Hence, the bundle $(\pi_{1,0}^*(TE), \pi_{1,0}^*(\tau_E), J^1\pi)$ may be written as the direct sum of two sub-bundles

$$(\pi_{1,0}^*(V\pi)) \oplus H\pi_{1,0}, \pi_{1,0}^*(\tau_E), J^1\pi),$$

where $H\pi_{1,0}$ is the union of the fibres $\phi_*(T_xM)$ for $x \in M$.

In this paper we study some similar properties for the pulled-back bundles when the base manifold M of the bundle π is a foliated manifold.

2 Leafwise 1-jet manifold

In the following we shall consider the bundles over foliated manifold.

A p-dimensional foliation $\mathcal{F}$ of a m-dimensional manifold M is a partition of M into p-dimensional submanifolds, called *leaves*. On the foliated manifold $(M, \mathcal{F})$ there is an adapted atlas whose coordinate system on the open set $V \subset M$ is $\left(x^i\right) = (x^a, x^u)$, where $a = \overline{1, m-p}$, $u = \overline{m-p+1, m}$, such that the points in the same leaf $\mathcal{L} \cap V$ have their first $m-p$ coordinates equal, and are distinguished by their last p coordinates.

In this paper, the indices will take the following values: $i, i_1, .. = \overline{1, m}$; $a, b, .. = \overline{1, m-p}$; $u, u_1, ... = \overline{m-p+1, m}$ and $\alpha, \alpha_1, .. = \overline{1, n}$.

For two adapted charts $(V, (x^a, x^u))$, $\left(\tilde{V}, (\tilde{x}^{a_1}, \tilde{x}^{u_1})\right)$ whose domains overlaps, there are the following changing rules:

$$\tilde{x}^{a_1} = \tilde{x}^{a_1}\left(x^a\right); \tilde{x}^{u_1} = \tilde{x}^{u_1}\left(x^a, x^u\right).$$

The set of all vector fields tangent to the leaves is a subbundle of the tangent bundle TM, called the *structural* bundle and denoted by $T\mathcal{F}$.

Let now $(M, \mathcal{F})$ be a foliated manifold and (E, π, M) a bundle over M, with $\dim M = m$, $\dim E = m+n$. For a local chart $(V, (x^a, x^u))$ in M, we have an adapted local chart $(U, (x^a, x^u, y^\alpha))$ in E and a local chart $\left(U^1, (x^a, x^u, y^\alpha, y^\alpha_a, y^\alpha_u)\right)$ in the 1-jet manifold $J^1\pi$. The last one is exactly the chart (2), where we replace the index i taking values from 1 to m with the indices $a = \overline{1, m-p}$, $u = \overline{m-p+1, m}$.

Definition 2.1. We say that two local sections $\Phi, \Psi \in \Gamma_x(\pi)$ are *leafwise 1-equivalent* at $x \in M$ if $\Phi(x) = \Psi(x)$ and if, in some adapted coordinate system (x^a, x^u, y^α) around $\Phi(x)$

$$\frac{\partial\left(y^\alpha \circ \Phi\right)}{\partial x^u}(x) = \frac{\partial\left(y^\alpha \circ \Psi\right)}{\partial x^u}(x), \tag{2.1}$$

for every $u = \overline{m-p+1, m}$. The equivalence class containing Φ is called the *leafwise 1-jet* of Φ and it is denoted by $j^{l,1}_x\Phi$.

Remark 2.1. The conditions (3) do not depend upon the choice of charts. Indeed, let $(\tilde{x}^{a_1}, \tilde{x}^{u_1}, \tilde{y}^{\alpha_1})$ be another coordinate system around $\Phi(x)$. Then we have

$$\frac{\partial\left(\tilde{y}^{\alpha_1} \circ \Phi\right)}{\partial \tilde{x}^{u_1}}(x) = \frac{\partial \tilde{y}^{\alpha_1}}{\partial x^u}(\Phi(x)) \cdot \frac{\partial x^u}{\partial \tilde{x}^{u_1}}(x) + \frac{\partial \tilde{y}^{\alpha_1}}{\partial y^\alpha}(\Phi(x)) \cdot \frac{\partial\left(y^\alpha \circ \Phi\right)}{\partial x^u}(x) \cdot \frac{\partial x^u}{\partial \tilde{x}^{u_1}}(x),$$

using the relationship $x^u \circ \Phi = x^u$ between similarly-named coordinate functions on E and M. The result follows now from $\Phi(x) = \Psi(x)$ and relations (3).

Remark 2.2. We can say that $\Phi, \Psi \in \Gamma_x(\pi)$ are leafwise 1-equivalent at x if they are 1-equivalent at x (definition 1.1) in the leaf which contains the point x.

Ones can see that:

Proposition 2.1. *Let $\Phi, \Psi \in \Gamma_x(\pi)$ be two local sections such that $\Phi(x) = \Psi(x)$. Then $j_x^{l,1}\Phi = j_x^{l,1}\Psi$ if and only if $\Phi_{*|T_x\mathcal{F}} = \Psi_{*|T_x\mathcal{F}}$.*

Remark 2.3. If $j_x^1\Phi = j_x^1\Psi$, then $j_x^{l,1}\Phi = j_x^{l,1}\Psi$. Indeed, if Φ and Ψ are local sections 1-equivalent at x, then $\Phi_{*|T_xM} = \Psi_{*|T_xM}$, which assures $\Phi_{*|T_x\mathcal{F}} = \Psi_{*|T_x\mathcal{F}}$, so $j_x^{l,1}\Phi = j_x^{l,1}\Psi$ by proposition 2.1. The reverse is not true: $j_x^{l,1}\Phi = j_x^{l,1}(\Phi + \Omega)$ for every local basic section $\Omega \in \Gamma_x(\pi)$ (that means $\frac{\partial(y^\alpha \circ \Omega)}{\partial x^u}(x) = 0$), but $j_x^1\Phi \neq j_x^1(\Phi + \Omega)$.

Let $\mathcal{A}_E = \{(U, u = (x^a, x^u, y^\alpha))\}$ be an adapted atlas on E. The induced coordinate system $\left(U^{l,1}, u^{l,1}\right)$ on the set

$$J^{l,1}\pi = \left\{ j_x^{l,1}\Phi \mid x \in M, \Phi \in \Gamma_x(\pi) \right\},$$

is defined by:

$$U^{l,1} = \left\{ j_x^{l,1}\Phi \in J^{l,1}\pi \mid \Phi(x) \in U \right\},$$

$$u^{l,1} = (x^a, x^u, y^\alpha, z_u^\alpha), \tag{2.2}$$

with $x^a\left(j_x^{l,1}\Phi\right) = x^a(x)$, $x^u\left(j_x^{l,1}\Phi\right) = x^u(x)$, $y^\alpha\left(j_x^{l,1}\Phi\right) = \Phi(x)$,

$$z_u^\alpha\left(j_x^{l,1}\Phi\right) = \frac{\partial(y^\alpha \circ \Phi)}{\partial x^u}(x).$$

It is easy to verify that the collection of all charts $\left(U^{l,1}, u^{l,1}\right)$ is a $(m + n + np)$-dimensional C^∞-atlas on $J^{l,1}\pi$. The maps

$$\pi_1^l : J^{l,1}\pi \to M; \quad \pi_1^l\left(j_x^{l,1}\Phi\right) = x,$$

$$\pi_{1,0}^l : J^{l,1}\pi \to E; \quad \pi_{1,0}^l\left(j_x^{l,1}\Phi\right) = \Phi(x),$$

for every $j_x^{l,1}\Phi \in J^{l,1}\pi$, are the correspondent of maps π_1, $\pi_{1,0}$ from the previous section. The remark 2.3 assures that the map

$$\pi^l : J^1\pi \to J^{l,1}\pi; \quad \pi^l\left(j_x^1\Phi\right) = j_x^{l,1}\Phi, \tag{2.3}$$

is well-defined. Moreover, there are satisfying the relations

$$\pi_{1,0}^l \circ \pi^l = \pi_{1,0}, \tag{2.4}$$

$$\pi_1^l \circ \pi^l = \pi_1; \quad \pi \circ \pi_{1,0}^l = \pi_1^l.$$

The functions $\pi_{1,0}^l$, π^l are surjective submersions.

Let $W \subseteq M$ be an open domain.

Definition 2.2. The *leafwise first prolongation* of the section $\Phi \in \Gamma_W(\pi)$ is the section $j^{l,1}\Phi \in \Gamma_W(\pi_1^l)$, defined by $j^{l,1}\Phi(x) = j_x^{l,1}\Phi$.

Since $\pi^l_{1,0} \circ j^{l,1}\Phi = \Phi$, for every section $\Phi \in \Gamma_W(\pi)$ we have

$$j^{l,1}(\pi^l_{1,0} \circ j^{l,1}\Phi) = j^{l,1}\Phi. \tag{2.5}$$

By the above relation it follows that:

Proposition 2.2. *A section $\psi^l \in \Gamma_W(\pi^l_1)$ is the leafwise first prolongation of a section in π if and only if $\psi^l = j^{l,1}(\pi^l_{1,0} \circ \psi^l)$.*

Considering now a section $\psi \in \Gamma_W(\pi_1)$, by relation $\pi^l_1 \circ \pi^l = \pi_1$ and $\pi_1 \circ \psi = \mathbf{1_W}$, it follows $\pi^l_1(\pi^l \circ \psi) = \mathbf{1_W}$, hence $\pi^l \circ \psi$ is a section over W in π^l_1.

Finally, we can prove that:

Proposition 2.3. *If the section $\psi \in \Gamma_W(\pi_1)$ is the first prolongation of a section $\Phi \in \Gamma_W(\pi)$, then $\pi^l \circ \psi$ is the leafwise first prolongation of the same section Φ. Conversely, every leafwise prolongation of a section $\Phi \in \Gamma_W(\pi)$ is the image by π^l of $j^1\Phi$.*

Proof: Since $\psi \in \Gamma_W(\pi_1)$ is the first prolongation of a section $\Phi \in \Gamma_W(\pi)$, we know by [6] that $\psi = j^1(\pi_{1,0} \circ \psi)$. Taking into account also relations (5) and (6), we obtain

$$\pi^l \circ \psi = \pi^l(j^1(\pi_{1,0} \circ \psi)) = \pi^l(j^1(\pi^l_{1,0}(\pi^l \circ \psi)) = j^{l,1}(\pi^l_{1,0}(\pi^l \circ \psi)),$$

and by Proposition 2.2 it follows that $\pi^l \circ \psi$ is the leafwise first prolongation of the section Φ. The second assertion follows from the relation (5).

3 The leafwise holonomic tangent vectors

Let (E, π, M) be a fiber bundle over the foliated manifold $(M, \mathcal{F})$ and let $\mathcal{F}'$ be the pull-back foliation of $\mathcal{F}$ by π. The leaves of $\mathcal{F}'$ are connected components of the inverse images by π of the leaves of $\mathcal{F}$, [4]. We denote by $(T\mathcal{F}, \tau_F, M)$ and $(T\mathcal{F}', \tau_{F'}, E)$ the structural bundles of $\mathcal{F}$ and $\mathcal{F}'$, respectively. We have to remark that the vertical distribution $V\pi$ define a subfoliation of $\mathcal{F}'$, since for every $X_w \in V\pi$, $\pi_*(X_w) = 0$, so $X_w \in \pi_*^{-1}(T\mathcal{F})$. We consider now the pull-back bundles of $\tau_{F'}$ and τ_V by $\pi^l_{1,0}$:

$$(\pi^{l*}_{1,0}(T\mathcal{F}'), \pi^{l*}_{1,0}(\tau_{F'}), J^{l,1}\pi), \quad (\pi^{l*}_{1,0}(V\pi), \pi^{l*}_{1,0}(\tau_V), J^{l,1}\pi),$$

where

$$\pi^{l*}_{1,0}(T\mathcal{F}') = \{(X_w, j^{l,1}_x\phi) \in T_w\mathcal{F}' \times J^{l,1}\pi | w \in E, \tau_{F'}(X_w) = \pi^l_{1,0}(j^{l,1}_x\phi)\},$$

$$\pi^{l*}_{1,0}(\tau_{F'})((X_w, j^{l,1}_x\phi)) = j^{l,1}_x\phi,$$

$$\pi^{l*}_{1,0}(V\pi) = \{(X_w, j^{l,1}_x\phi) \in V_w\pi \times J^{l,1}\pi | w \in E, \tau_V(X_w) = \pi^l_{1,0}(j^{l,1}_x\phi)\},$$

$$\pi^{l*}_{1,0}(\tau_V)((X_w, j^{l,1}_x\phi)) = j^{l,1}_x\phi.$$

Definition 3.1. Let (E, π, M) be a bundle over the foliated manifold M and $x \in M$, $\phi \in \Gamma_x(\pi)$, $\zeta \in T_x\mathcal{F}$. The *leafwise holonomic lift* of ζ by ϕ is defined to be

$$(\phi_*(\zeta), j^{l,1}_x\phi) \in \pi^{l*}_{1,0}(T\mathcal{F}').$$

Remark 3.1. For any $\zeta \in T\mathcal{F}$ we have $\pi_*(\phi_*(\zeta)) = \zeta \in T\mathcal{F}$, so $\phi_*(\zeta) \in T\mathcal{F}'$. Moreover, taking into account Proposition 2.1, we obtain that the leafwise holonomic lift from the above definition depends only by the leafwise 1-jet of ϕ at x (it not depends by the section ϕ)). We denote by $\phi_*(T_x\mathcal{F})$ the collection of holonomic lifts of tangent vectors from $T_x\mathcal{F}$ by ϕ.

Proposition 3.1. *The vector space* $(\pi^{l*}_{1,0}(T\mathcal{F}'))_{j^{l,1}_x\phi}$ *allows the decomposition as the direct sum*

$$(\pi^{l*}_{1,0}(V\pi))_{j^{1,1}_x\phi} \oplus \phi_*(T_x\mathcal{F}).$$

Proof: Let $(\xi, j^{l,1}_x\phi)$ be an arbitrary element of $(\pi^{l*}_{1,0}(T\mathcal{F}'))_{j^{l,1}_x\phi}$.That means $\xi \in T\mathcal{F}'$, which implies $\pi_*(\xi) \in T\mathcal{F}$, so $\phi_*(\pi_*(\xi)) \in T\mathcal{F}'$. It follows that

$$(\phi_*(\pi_*(\xi)), j^{l,1}_x\phi) \in \pi^{l*}_{1,0}(T\mathcal{F}').$$

We also have

$$\pi_*(\phi_*(\pi_*(\xi))) = \pi_*(\xi) \Rightarrow \pi_*(\xi - \phi_*(\pi_*(\xi))) = 0 \Rightarrow \xi - \phi_*(\pi_*(\xi)) \in V\pi.$$

We obtain that $(\xi - \phi_*(\pi_*(\xi)), j^{l,1}_x\phi) \in \pi^{l*}_{1,0}(V\pi)$. On the other hand, if $(\xi, j^{l,1}_x\phi) \in \pi^{l*}_{1,0}(V\pi) \cap \phi_*(T_x\mathcal{F})$, then there is $\zeta \in T_x\mathcal{F}$ such that $\xi = \phi_*(\zeta)$ and $\pi_*(\xi) = 0$. It results $\zeta = 0$, which ends the proof.

As a consequence of the last Proposition, it results:

Proposition 3.2. *The bundle* $(\pi^{l*}_{1,0}(T\mathcal{F}'), \pi^{l*}_{1,0}(\tau_{F'}), J^{l,1}\pi)$ *may be written as the direct sum of two sub-bundles*

$$(\pi^{l*}_{1,0}(V\pi) \oplus H\pi^{l*}_{1,0}, \pi^{l*}_{1,0}(\tau_{F'}), J^{l,1}\pi),$$

where $H\pi^{l*}_{1,0}$ *is the union of the fibres* $\phi_*(T_x\mathcal{F})$*, for* $x \in M$*.*

To obtain the coordinate representation of a leafwise holonomic lift of a tangent vector, suppose that

$$\zeta = \zeta^u \frac{\partial}{\partial x^u}|_x \in T_x\mathcal{F}.$$

Then,

$$\phi_*(\zeta) = \zeta^u \phi_*(\frac{\partial}{\partial x^u}|_x) = \zeta^u(\frac{\partial}{\partial x^u}|_{\phi(x)} + z^\alpha_u(j^{l,1}_x\phi)\frac{\partial}{\partial y^\alpha}|_{\phi(x)}).$$

It follows that the decomposition of $(\xi, j^{l,1}_x\phi) \in (\pi^{l*}_{1,0}(T\mathcal{F}'))_{j^{l,1}_x\phi}$ may be found by letting

$$\xi = \xi^u \frac{\partial}{\partial x^u}|_{\phi(x)} + \xi^\alpha \frac{\partial}{\partial y^\alpha}|_{\phi(x)},$$

so that

$$\xi = (\xi^\alpha - \xi^u z^\alpha_u(j^{l,1}_x\phi))\frac{\partial}{\partial y^\alpha}|_{\phi(x)} + \xi^u(\frac{\partial}{\partial x^u}|_{\phi(x)} + z^\alpha_u(j^{l,1}_x\phi)\frac{\partial}{\partial y^\alpha}|_{\phi(x)}).$$

Let $\Gamma(\pi^{l*}_{1,0})$ be the module of sections in the bundle $\pi^{l*}_{1,0}(T\mathcal{F}')$, also called vector fields of $T\mathcal{F}'$ along $\pi^{l*}_{1,0}$. The submodule corresponding to sections of the bundle

$\pi^{l*}_{1,0}(T\mathcal{F}')|_{\pi^{l*}_{1,0}(V\pi)}$ will be denoted by $\Gamma^v(\pi^{l*}_{1,0})$ and the submodule corresponding to sections of the bundle $\pi^{l*}_{1,0}(T\mathcal{F}')|_{H\pi^{l*}_{1,0}}$ will be denoted by $\Gamma^h(\pi^{l*}_{1,0})$. An element of $\Gamma^h(\pi^{l*}_{1,0})$ will be called a leafwise total derivative. By the Proposition 3.2, the module $\Gamma(\pi^{l*}_{1,0})$ may be written as the direct sum

$$\Gamma^v(\pi^{l*}_{1,0}) \oplus \Gamma^h(\pi^{l*}_{1,0}).$$

Considering now the natural projection $f : TE \to T\mathcal{F}'$, it is easy to see that the pair (f, π^l) is a bundle morphism between $(\pi^*_{1,0}(TE), \pi^*_{1,0}(\tau_E), J^1\pi)$ and $(\pi^{l*}_{1,0}(T\mathcal{F}'), \pi^{l*}_{1,0}(\tau_{F'}), J^{l,1}\pi)$.

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