# Mathematical modelling of semi-active control with application to building seismic protection

Ana Maria Mitu, Iuliana Popescu and Tudor Sireteanu

**Abstract.** In this paper the semi-active control is achieved by a base isolation system. The magnetorheological dampers (MRD) hysteretic force is described by models based on nonlinear differential equation Bouc-Wen. The Bouc-Wen model is able to capture, in an analytical form, a range of shapes of hysteretic cycles which match the behavior of a wide class of nonlinear systems. In order to reduce the seismic response of protected structure, the "balance logic" control strategy is proposed. The benefits of using semi-active base isolation systems are illustrated by a comparative numerical analysis of passive and semi-active control strategies, applied to a structure acted on by a synthetic seismic excitation derived from a given design response spectrum. For numerical simulation a Matlab-Simulink computer program was developed.

**M.S.C. 2010**: : 34H05, 49K15, 93C15, 93C80
**Key words**: : dynamical system; semi-active control; magnetorheological damper; base isolation.

## 1 Introduction

A memory-dependent relation between force and deformation, i.e., hysteresis, is often observed in structural materials and elements, such as steel, base isolators and dampers. Many mathematical models have been developed to efficiently describe such behavior for use in time history and random vibration analyses. One of the most popular is the Bouc-Wen class of hysteresis models and has been extensively used in the current literature to describe mathematically components and devices with hysteretic behaviors. The model essentially consists in a first-order non-linear differential equation that relates the input displacement to the output restoring force in a hysteretic way. By choosing a set of parameters appropriately, it is possible to accommodate the response of the model to the real hysteresis loops. This problem has stirred a lot of research effort due to its difficulty being nonlinear and nondifferentiable. Some identification methods have been proposed with a rigorous analysis of the convergence of the

BSG Proceedings, Vol. 19, 2012, pp. 88-99.

parameters to their true values, while others relied on numerical simulations and experimentation: least-squares based identification, Kalman Filter based identification, genetic algorithm based identification, Gauss-Newton iterative based identification, Bootstrap Filter based identification, Simplex method based identification The model has the advantage of computational simplicity, because only one auxiliary nonlinear differential equation is needed to describe the hysteretic behavior [8]. Likewise a variety of control algorithms has been proposed over the years. These include sliding mode control [13] reliability-based control [4], fuzzy control [1] and other non linear control techniques. A survey can be found in Spencer et al. [17, 18]. A spring cancelation control algorithm using semi-active control of friction dampers has been applied by Stammers and Sireteanu [19,20 and 21]. The principle of this control strategy is to reduce the total force transmitted through the base isolation system by totally or partially balancing the elastic and dissipative forces when these forces act in opposite direction and by maximally reducing the dissipative force when these forces have same direction. This approach the authors call "balance logic" [11, 22]. The objective of seismic isolation systems is to decouple the building structure from the damaging components of the earthquake input motion, i.e. to prevent the superstructure of the building from absorbing the earthquake energy. A variety of base isolation semi-active techniques have been investigated over the years in order to control structural seismic response. Controlled friction forces can be used to limit building response to an earthquake. Fujita studied a system in which a friction damper is positioned between the base and the ground [5]. Feng [2, 3] used sliding bearings to achieve base isolation. Nishitani, Nitta and Ishibashi [10] carried out scale model experiments using variable friction dampers. This work investigates the feasibility of using MRD for base isolation of buildings by applying "balance logic" control strategy. It should be noted that the damping level achieved by the conventional system must be minimized so as not to limit the possibilities of controlling the energy dissipation through MRD. A comparative analysis of base isolation systems with controllable damping and passive lead-rubber bearing systems showed a notable decrease in base drifts over comparable passive systems with no accompanying increase in base shears or in floor accelerations [15].

## 2 The mathematical model of a structure with semi-active base isolation

The mechanical model used in this paper to analyze the seismic response of a shear building with semi-active base isolation is the discrete oscillating system with $n$ degrees of freedom, represented in Fig.1 [9]. The elastic and dissipative characteristics of the conventional base isolation system are assumed linear, with total stiffness $k_1$ and total damping constant $c_1$. The total force developed by the MR-dampers, placed in semi-active base isolation system, is denoted by $F_1$. The relation between the lateral displacements $x_i$ of the building floors and their relative displacements $y_i$ (inter-storey drifts) can be written as

$$y_i = x_i - x_{i-1}, \quad x_i = \sum_{j=1}^{i} y_j + x_0, \quad i = 1, \ldots, n \tag{2.1}$$

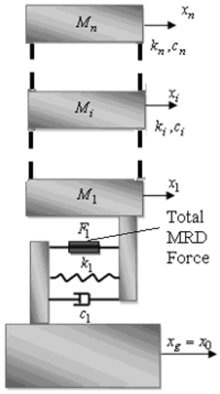



Fig.1 - The mechanical model of the base isolated structure.

The structure response to the ground seismic motion $x_g(t)$ is described by the following system of differential equations

$$(2.2)\quad \begin{cases} M_1\ddot{x}_1 + c_1 y_1 + k_1 y_1 + F_1(y_1, \dot{y}_1, u) - c_2\dot{y}_2 - k_2 y_2 = 0, \\ \dots\dots\dots\dots \\ M_i\ddot{x}_i + c_i\dot{y}_i + k_i y_i - c_{i+1}\dot{y}_{i+1} - k_{i+1}y_{i+1} = 0,\ i = 2, \dots, n-1 \\ \dots\dots\dots\dots \\ M_n\ddot{x}_n + c_n\dot{y}_n + k_n y_n = 0. \end{cases}$$

Introducing the following notations

$$(2.3)\quad \begin{array}{l} \omega_i = \sqrt{\dfrac{k_i}{M_i}},\ \zeta_i = \dfrac{c_i}{2M_i\omega_i},\ \omega_i^2 = \dfrac{k_i}{M_i},\ 2\zeta_i\omega_i = \dfrac{c_i}{M_i}\ i = 1, 2, \dots, n \\ \mu_i = \dfrac{M_{i+1}}{M_i},\ \dfrac{k_{i+1}}{M_i} = \mu_i\omega_{i+1}^2,\ \dfrac{c_{i+1}}{M_i} = 2\mu_i\omega_{i+1},\ i = 1, 2, \dots, n-1 \end{array}$$

and using notations (2.1), the equations of motion (2.2) can be written in the following form

$$(2.4)\quad \begin{cases} \ddot{y}_1 = & -\omega_1^2 y_1 + \mu_1\omega_2^2 y_2 - 2\zeta_1\omega_1\dot{y}_1 + 2\mu_1\zeta_2\dot{y}_2 - aF_{mrd}(y, \dot{y}, u) - \ddot{x}_0 \\ \ddot{y}_2 = & \omega_1^2 y_1 - (1+\mu_1)\omega_2^2 y_2 + \mu_2\omega_3^2 y_3 + 2\zeta_1\omega_1\dot{y}_1 - 2\mu_1\zeta_2\omega_2\dot{y}_2 \\ & +2\mu_2\zeta_3\omega_3\dot{y}_3 + aF_{mrd}(y, \dot{y}, u) \\ \dots & \\ \ddot{y}_i = & \omega_{i-1}^2 y_{i-1} - (1+\mu_{i-1})\omega_i^2 y_i + \mu_i\omega_{i+1}^2 y_{i+1} + 2\zeta_{i-1}\omega_{i-1}\dot{y}_{i-1} \\ & -2(1+\mu_{i-1})\zeta_i\omega_i\dot{y}_i + 2\mu_i\zeta_{i+1}\omega_{i+1}\dot{y}_{i+1},\ i = 3, \dots, n-1 \\ \dots & \\ \ddot{y}_n = & \omega_{n-1}^2 y_{n-1} - (1+\mu_{n-1})\omega_n^2 y_n + 2\zeta_{n-1}\omega_{n-1}\dot{y}_{n-1} - 2(1+\mu_{n-1})\zeta_n\omega_n\dot{y}_n, \end{cases}$$

where $F_{mrd}(y_1, \dot{y}_1, u)$ is the force developed by a single magnetorheological damper and $a = F_1/(M_1 F_{mrd})$ is a coefficient that depends on the number of employed dampers and on the concentrated mass of the first level.

In the mathematical modeling of the seismic behavior of structure, this coefficient acts as a weighting factor, its value being chosen according to the seismic safety criteria, formulated in terms of inter-storey drift and floor absolute acceleration. In case of an uniform structure, the following parameters are introduced in the equations of motion (2.4)

$$\mu_1 = \mu,\ \mu_2 = \mu_3 = \ldots = \mu_{n-1} = 1,\ \omega_1 = \omega_0,\ \omega_2 = \ldots = \omega_n = \omega,\ \zeta_1 = \zeta_0,\ \zeta_2 = \ldots = \zeta_n = \zeta, \tag{2.5}$$

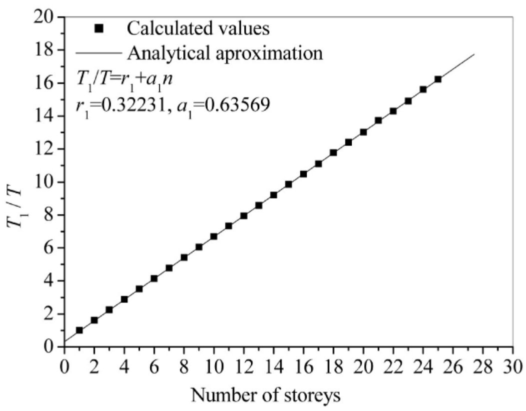



Fig.2. The variation of period ratio $T_1/T$ versus number of building storeys

Denoting by $T = 2\pi/\omega$ the natural period of the SDOF system corresponding to one storey and by $T_k$ the period of $k^{th}$ vibration mode of the uniform structure without base isolation, one can establish relationships of almost linear dependence between the ratio $T_k/T$ and the total number of building storeys. Fig.2 shows the linear fitting curve for the first vibration mode. This diagram is very useful for numerical simulations because it allows the determination of the parameter $\omega$ of the uniform structure model, for an imposed value of the first vibration mode of unprotected structure.

## 3 The mathematical model of the dynamic response of magnetorheological dampers

A MRD is a device consisting of a fixed-orifice damper filled with a MR fluid having magnetic coils mounted on the piston head. A MR fluid is a particular type of oil which contains in suspension micron-sized particles of ferromagnetic material [13]. By applying a magnetic field (via a current supplied to the coils) to the fluid, ferromagnetic particles form chains, thus changing the value of yield stress. As a consequence the rheological properties of the MR fluid change and the fluid passes from liquid state to semi-solid state. Hence, by varying the current supplied to the coils it is possible to produce variations of the damping force according to the applied control strategy. This creates a controllable damping action, without taking recourse to valves or any moving mechanical part. MRD are fail-safe since damping is generated even in the passive mode. The mechanical system is highly reliable. The MRD requires only a battery and not mains power (which is often lost in an earthquake).

The mechanical model of an MR-damper, shown in Fig.3, consists of binding in parallel a linear viscous damper of damping constant with a hysteretic element described by the Bouc-Wen model [15].

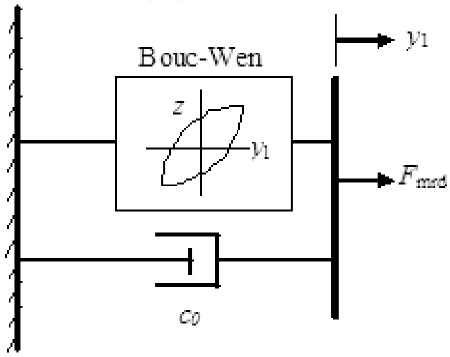



Fig.3. The mechanical model of an MR-damper

The force developed by the MR-damper is function of the relative displacement and velocity $y_1$ and $\dot{y}_1$ and the control voltage $u$ is given by the equation

$$F_{amr}(y_1, \dot{y}_1, u) = c_0(u)\dot{y}_1 + \alpha(u)z(y_1, \dot{y}_1) \tag{3.1}$$

where the internal variable $z$, describing the hysteretic behavior, satisfies the Bouc-Wen differential model

$$\dot{z} = [A - |z|^n(\beta + \gamma \operatorname{sgn}(z\dot{y}_1))]\dot{y}_1 \tag{3.2}$$

and the coefficients $c_0(u)$ and $\alpha(u)$ have a linear variation with respect to the control voltage $u$:

$$c_0(u) = c_{0a} + c_{0b}u, \quad \alpha_0(u) = \alpha_{0a} + \alpha_{0b}u. \tag{3.3}$$

The delay introduced by the control system is described as the response to a first order filter modeled by equation

$$\dot{u} = -\eta(u - v), \tag{3.4}$$

where $v(t)$ is the voltage applied to the control circuit, and $1/\eta$ is the filter time constant.

The values of the model parameters are determined by inverse methods based on experimental data obtained by recording the force-displacement and/or force-velocity characteristics for different types of relative motions imposed between the mounting ends of MRD and different types of control voltage signals [6]. In the present work were employed the values given in Table1, obtained by Yoshida and Dyke through the identification of the analytical model of a MR-damper prototype for seismic applications, built at Washington University [22]. The maximum force of this damper is 1000kN for a maximum value of the control voltage of 10V.

Table 1

| | | | |
|---|---|---|---|
| $c_{0a}$ | $0.44kNs/m$ | $A$ | 1.2 |
| $c_{0b}$ | $4.4kNs/(mV)$ | $\beta$ | $300m^{-1}$ |
| $\alpha_{0a}$ | $10872kN/m$ | $\gamma$ | $300m^{-1}$ |
| $\alpha_{0b}$ | $49616kN/(mV)$ | $n$ | 1 |
| $\eta$ | $50s^{-1}$ | | |

In Figs. 5 and 6 are shown the force-displacement and force-velocity characteristics of magnetorheological damper, obtained by using equations (3.1)-(3.4) for an imposed harmonic motion with frequency 1Hz and amplitude 0.1 m, and for different constant values of control voltage.

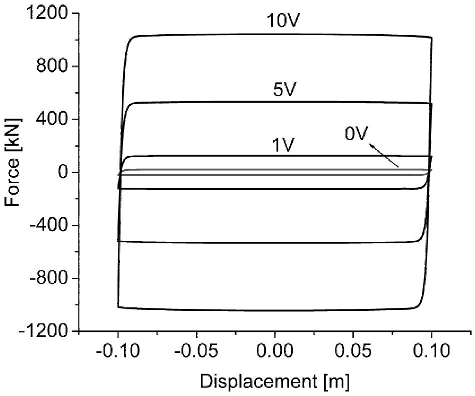



Fig. 4 Force-displacement characteristics

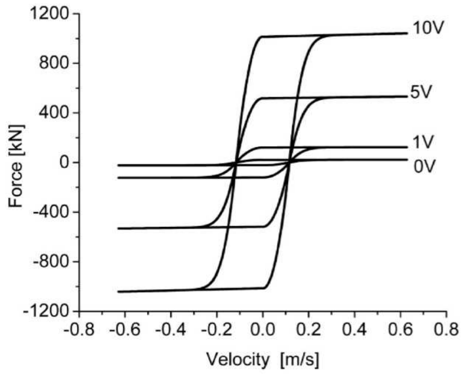



Fig. 5 Force-velocity characteristics

# 4 The mathematical model of the "balance logic" semi-active control strategy

In order to reduce the seismic response of semi-active base isolation, the present paper considers the "balance logic" control strategy which aims to reduce the dynamic forces transmitted from the ground seismic motion to the first floor of building. The principle of this semi-active control strategy is to reduce the total force acting on the first floor by totally or partially balancing the elastic forces, developed by passive devices or structural elements with the dissipative forces developed by the MRD, in those sequences of the ground-first floor drift motion in which these forces act in the opposite direction and by maximally reducing the force developed by MRD, when these forces have the same direction [21]. Absolute acceleration of the base structure is obtained from the first equation of system (2.4) as

$$\ddot{x}_1 = -\omega_1^2 y_1 + \mu\omega_2^2 y_2 - 2\zeta_1\omega_1\dot{y}_1 + 2\mu\zeta_2\omega_2\dot{y}_2 - aF_{amr}(y, \dot{y}, u) \tag{4.1}$$

Therefore, the "balance logic" semi-active control strategy can be modeled by the following variation law of control voltage during seismic motion

$$v(t) = \begin{cases} \lambda|-\omega_1^2 y_1 + \mu\omega_2^2 y_2| & \text{if} \quad (-\omega_1^2 y_1 + \mu\omega_2^2 y_2)\dot{y}_1(t) \geq 0 \\ 0 & \text{if} \quad (-\omega_1^2 y_1 + \mu\omega_2^2 y_2)\dot{y}_1(t) \leq 0 \end{cases} \tag{4.2}$$

where $\lambda$ is a weighting coefficient to be determined so that the peak or r.m.s. values of the first floor acceleration $\ddot{x}_1$ are as low as possible and the absolute accelerations of the other floors and the inter-storey drifts displacements between levels are within the acceptable limits of buildings seismic response (floor accelerations less than $3-4m/s^2$ and inter-storey drifts below 0.5 % storey height). Equations (4.2) can be written in a compact form as

$$v(t) = 0.5\lambda|(-\omega_1^2 y_1 + \mu\omega_2^2 y_2)|[1 + \text{sgn}\,((-\omega_1^2 y_1 + \mu\omega_2^2 y_2)\dot{y}_1(t))] \tag{4.3}$$

which is more convenient for numerical simulation.

The control strategy, expressed by the above relations, involves real-time measurement of relative displacements $y_1(t)$, $y_2(t)$ and determination of sign of the relative velocity speed $\dot{y}_1(t)$. The entire control scheme can be implemented very simply with resistive displacement transducers and with DSP modules to generate the voltage command signals.

# 5 Numerical simulation results

The complete mathematical model for response analysis of structures with semi-active base isolation with magnetorheological damper and control strategy of "balance logic" type is described by the set of equations (2.4), (3.1)-(3.4) and (4.3). In general, the seismic actions for which the structures are designed are represented by seismic acceleration time histories compatible with an imposed design response spectrum. To illustrate the application of the previously developed model for the study the seismic response of structures with semi-active base isolation, a uniform structure with four floors (first floor and three storeys) was considered. In this case, by using relations (2.4) and (2.5), the equations of motion of the considered structure are obtained as

$$\begin{cases} \ddot{y}_1 = -\omega_0^2 y_1 + \mu\omega^2 y_2 - 2\zeta_0\omega_0\dot{y}_1 + 2\mu\zeta\omega\dot{y}_2 - aF_{amr}(y,\dot{y},u) - \ddot{x}_0 \\ \ddot{y}_2 = \omega_0^2 y_1 - (1+\mu)\omega^2 y_2 + \omega^2 y_3 + 2\zeta_0\omega_0\dot{y}_1 - 2(1+\mu)\zeta\omega\dot{y}_2 + 2\zeta_3\omega\dot{y}_3 + \\ +aF_{amr}(y,\dot{y},u) \\ \ddot{y}_3 = \omega^2 y_2 - 2\omega^2 y_3 + \omega^2 y_4 + 2\zeta\omega\dot{y}_2 - 4\zeta\omega\dot{y}_3 + 2\zeta_4\omega\dot{y}_4 \\ \ddot{y}_4 = \omega^2 y_3 - 2\omega^2 y_4 + 2\zeta\omega^2\dot{y}_3 - 4\zeta\omega_4\dot{y}_4. \end{cases} \tag{5.1}$$

The seismic ground acceleration was numerically synthesized such as to be compatible with a given velocity response design spectrum [14]. The synthetic acceleration time history is plotted in Fig.6. The imposed and simulated design response spectra are comparatively shown in Fig.7.

For numerical simulation a Matlab-Simulink computer program was developed such as to easily allow modifications of model parameters and of semi-active control system, as well as to quickly evaluate the effects of these changes on the building seismic response. For the parameters of structure and of base isolation system, the following set of values was considered :

$$\mu = 1,\ \omega_0 = 2\pi rad/s,\ \omega = 2\pi \cdot 3.37 rad/s,\ \zeta_0 = \zeta = 0.02,\ a = 0,01Kg^{-1}. \tag{5.2}$$

The value of parameter $\omega$ was chosen from the diagram given in Fig.2, such as the period of the structure first vibration mode to have the value $T_1 = 0.67s$ ($f_1 = 1.5Hz$).

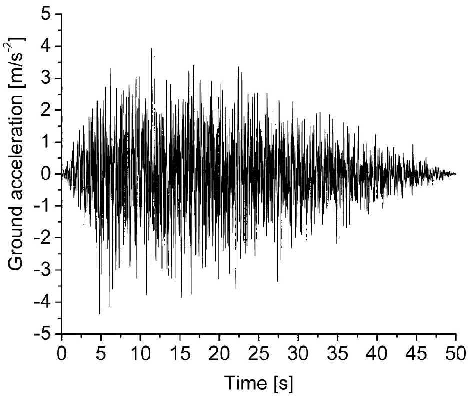



Fig. 6 The synthetic seismic ground notion acceleration

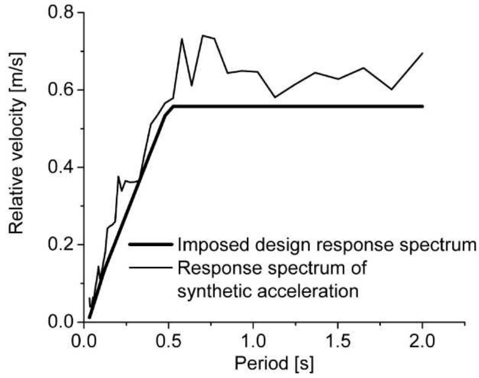



Fig. 7 The target and simulated design spectra

As expected, the maximum lateral accelerations of the structure were obtained on the last floor of the building, while the maximum inter-storey drift occurred between first and second floor. In Fig. 8 and 9 are represented the variation of r.m.s. and maximum values of these outputs versus the gain coefficient $\lambda$.

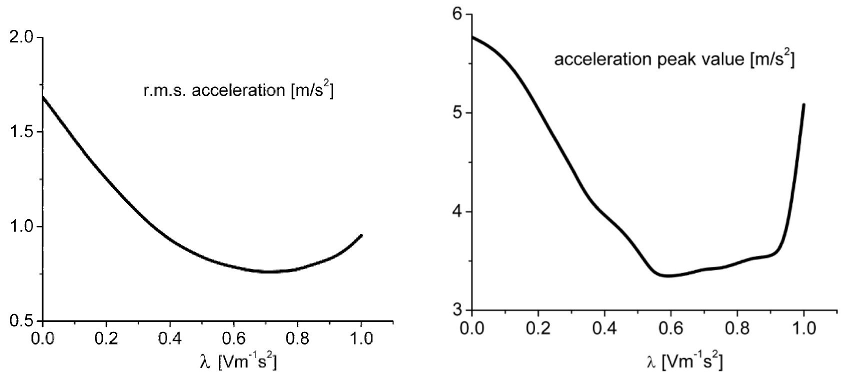



Fig. 8 Variation of r.m.s. and peak values of absolute acceleration of $4^{th}$ floor versus gain $\lambda$

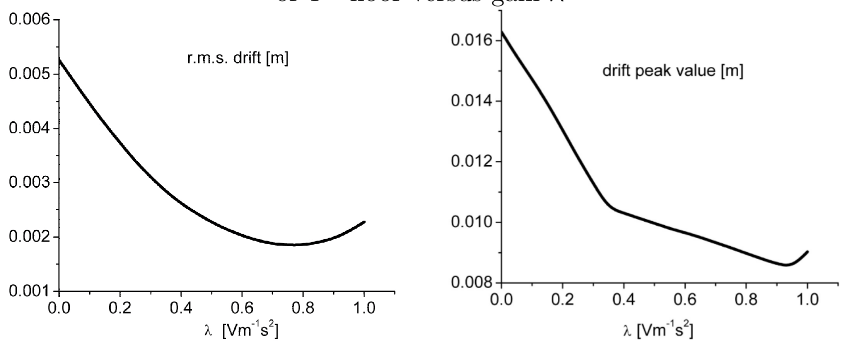
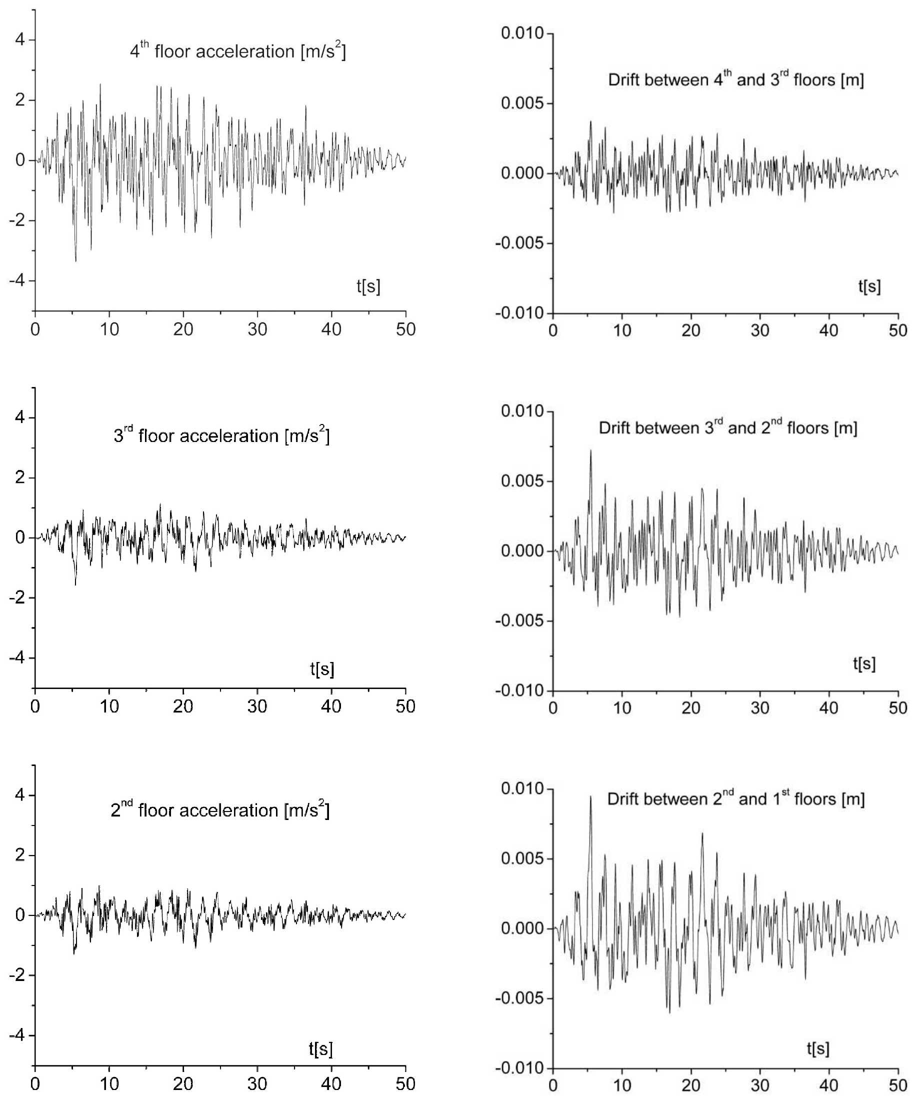



Fig. 9 Variation of r.m.s. and peak values of drift between $1^{st}$ and $2^{nd}$ floors versus gain $\lambda$

It follows from these diagrams that the values of coefficient $\lambda$ can be chosen in the range 0.55-0.85 $Vm^{-1}s^2$, so that maximum values of the acceleration are below 3.5 $ms^{-2}$ and those of the relative displacements between levels do not exceed 10mm, meaning less than 0.5% of the usual floor height of a building. To illustrate the seismic response of structure and the demanded control voltage for semi-active base isolation, the results obtained for $\lambda = 0.65Vm^{-1}s^2$ are presented in Fig. 10.

To assess the efficiency of proposed semi-active control strategy, the seismic output of same structure with hysteretic BIS passive devices [7], used for base isolation system, was numerically simulated. Fig.11 shows comparatively the maximum floor acceleration and inter-story drift, obtained for optimal configurations of semi-active and passive base isolation systems.



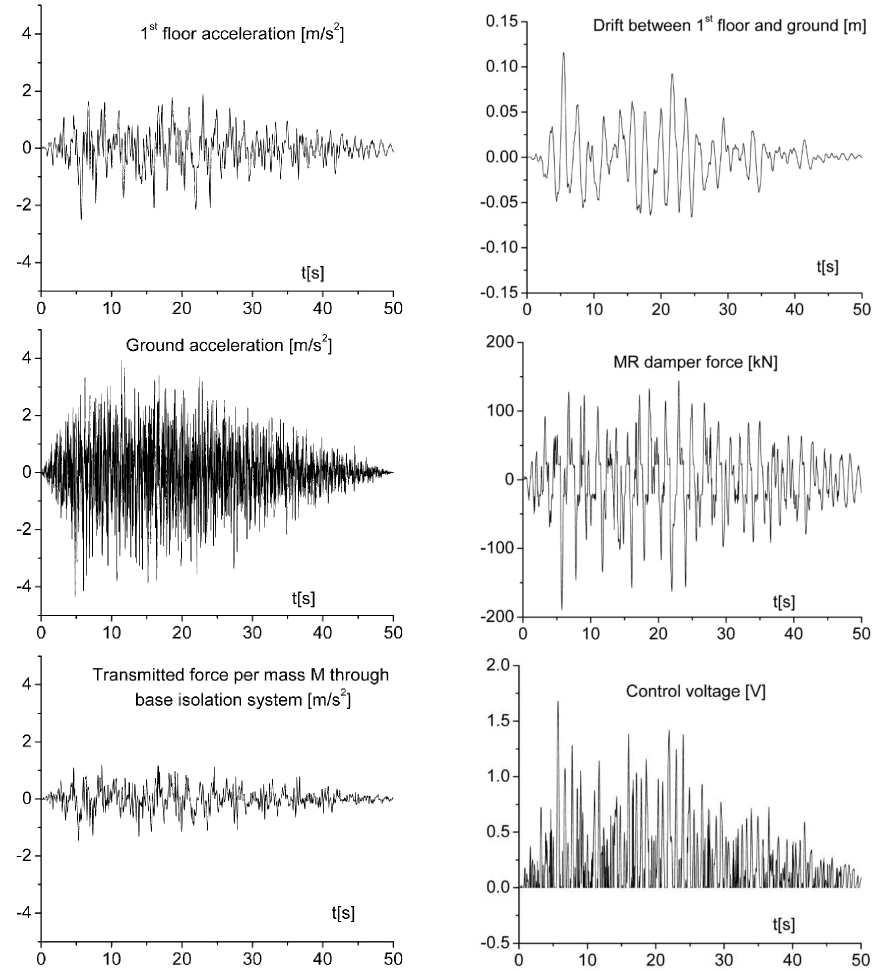



Fig. 10 The time histories of seismic input, output and of control voltage

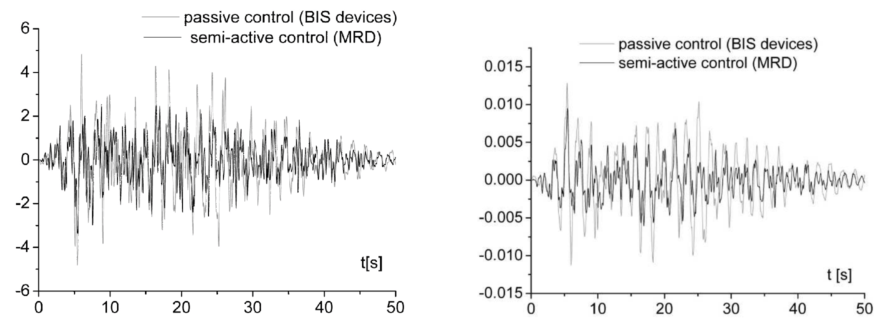



Fig. 11 The time histories of $4^{th}$ floor acceleration $m/s^2$ and of inter-story drift $[m]$ between $2^{nd}$ and $1^{st}$ floor, simulated for optimum semi-active and passive base isolation systems

## 6 Conclusions

The results presented in this paper prove the effectiveness of base isolations with semi-active control, realized with passive devices, mounted in parallel with controllable magnetorheological dampers.

The "balance logic" control strategy, proposed in this paper, is convenient for practical applications since only a few relative displacement transducers are required to measure the building response, regardless of its number of floors. The processing of measured signal can be easily achieved by employing specialized DSP modules in order to supply the voltage signal for practically real time control the forces that must be developed by magnetorheological dampers.

The comparative study of semi-active and passive control of base isolation system showed that the semi-active base isolation system can provide a better seismic protection than the passive one. Moreover, the semi-active control with MRD can be easily adapted to specific seismic inputs by changing the gain factor $\lambda$ in the "balance logic" equation. This implies only a suitable modification of the control software and there is no need to modify any mechanical parts of the base isolation system as in the case of passive isolation solutions.

## References

[1] F. Casciati, L. Faravelli and T. Yao, *Control of nonlinear structure using the fuzzy control approach*, Nonlinear Dynamics, 11 (1997), 171–187.

[2] M. Q. Feng, *Application of hybrid sliding isolation system to buildings*, Jour. of Engineering Mechanics, 119, 10 (1993), 2090–2108.

[3] M. Q. Feng, *Innovations in earthquake isolation, The Active Control of Vibration*, IUTAM Symposium, Bath UK, 1994, 307–314.

[4] R. V. Field, P. G. Voulgaris and L. A. Bergman, *Methods to compute probabilistic measures of robustness for structural systems, J. Vibration and Control* 2, 4 (1996), 447–464.

[5] T. Fujita, *The active control of vibration. Semi-active control of base isolated structures*, IUTAM Symposium, Bath UK, 1994, 289–295.

[6] M. Giuclea, T. Sireteanu, D. Stancioiu and C.W. Stammers, *Model parameter identification for vehicle vibration control with magnetorheological dampers using computational intelligence methods*, Pro. Instn. Mech. Engers. 218 PartI: J.Systems and Control Engineering, 17 (2004), 569–581.

[7] L. Guerreiro, J.P. Ferreira, G.P. Colato, G. Castellano and P. Baldo, *Base isolation for seismic protection-the new hospital in Lisbon*, Proc. of IABSE Symp. on Structures and Extreme Events, Lisbon, Portugal, 14-16 Sept. 2005.

[8] M. Ismail, F. Ikhouane and J. Rodellar, *The hysteresis Bouc-Wen, a survey*, Arch. Comput Methods Eng. 16 (2009), 161-188.

[9] A. M. Mitu, *Contributions to the study of dynamic behaviour of systems with hysteretic characteristics* (in Romanian), PhD Thesis, U.P.B. 2010.

[10] A. Nishitani, Y. Nitta and Y. Ishibashi, *Structural control based on semi-active variable friction dampers*, 12-th World Conference on Earthquake Engineering, Auckland, New Zealand, 2000, paper 1909 (CD-ROM).

[11] I. Popescu, A.M. Mitu and T. Sireteanu, *A comparative study of active and semi-active control of building seismic response*, Diff. Geom. Dyn. Syst., BGS Proceedings 17, Geometry Balkan Press, Bucharest 2010, 172–174.

[12] J. C. Ramallo, E. A. Johnson, Spencer Jr., *"Smart" base iosolation systems*, Jour. of Eng. Mechanics, October (2002), 1088–1099.

[13] M. P. Singh, E. Matheu and C. Beattie, *Output feedback sliding mode control for civil engineering structures*, Proc. 2nd Int Workshop on Structural Control, Hong-Kong, 1996, 609–620.

[14] T. Sireteanu, G. Ghita and D. Stancioiu, *Magnetorheological Fluids and Dampers* (in Romanian), Bren Printing House, Bucharest, 2005.

[15] T. Sireteanu and M. Giuclea, *A genetic algorithm method for synthesizing seismic accelerograms*, Proc. Ro. Acad., Series A, 1, 1 (2000), 37–40.

[16] B. F. Spencer, S. J. Dyke, M. K. Sain and J. D. Carlson, *Phenomenological model for magnetorheological dampers"*. Jour. of Eng. Mechanics, 123, (1997), 230–238.

[17] B. F. Spencer, E. A. Jr., Johnson and J. C. Ramallo, *"Smart" isolation for seismic control*, JSME Int. J., Ser. C, 43, 3 (2000), 704–711.

[18] B. F. Spencer and M. K. Sain, *Controlling buildings: a new frontier in feedback*, Shock and Vibration Digest, 30, 4 (1998), 267–281.

[19] C. W. Stammers and T. Sireteanu, *Semi-active control of ground induced building vibrations using friction dampers*, MOVIC Conf., Switzerland, 1 (1998), 51–54.

[20] C. W. Stammers and T. Sireteanu, *Dual semi-active friction dampers to reduce building response to seismic inputs*, 12th World Conference on Earthquake Engineering, Auckland, New Zealand, 2000, paper 2561 (CD-ROM).

[21] C. W. Stammers and T. Sireteanu, *Control of building seismic response by means of three semi-active friction dampers*, Jour. Sound Vib., 237, 5 (2000), 745–759.

[22] C. W. Stammers and T. Sireteanu, *Vibration control of machines by use of semi-active dry friction damping*, Jour. Sound Vib. 209, 4 (1998), 671–684.

[23] O. Yoshida and S. J. Dyke, *Response control in full scale irregular buildings using MR dampers*, ASCE Jour. of Structural Engineering, 131 (2005), 734–742.

*Authors' addresses:*

Ana Maria Mitu and Tudor Sireteanu
Institute of Solid Mechanics, Romanian Academy,
15 Constantin Mille Str., RO-010141 Bucharest, Romania.
E-mail: anamariamitu@yahoo.com, siretimsar@yahoo.com

Iuliana Popescu
Technical University of Civil Engineering,
Department of Mathematics and Computers Science,
38 Lacul Tei Blvd., RO-020396 Bucharest, Romania.
E-mail: iulianapopescu1@gmail.com