Loss functions and Taguchi Theory

C. Târcolea, A.S. Paris and D. Sylvan

Abstract. Robust Design is one of the most important tools in manufacturing science. The applications of Taguchi's robust parameter and tolerance choices for the manufacturing engineering are mainly allocated through designed experiments, related to the classical (fractional) factorial designs introduced by Fisher [8]. This paper is organized as follows: the first section is a brief overview of the Taguchi theory and its applications to the statistical process control. The key concept of quality loss function and some examples are presented in section 2. Section 3 proposes some new symmetrical and asymmetrical loss functions, and a practical application based on the proposed models is showed in Section 4, focusing on an ANOVA model with interactions. Last section is dedicated to discussion and concluding remarks.

M.S.C. 2010: 62H12, 62P30.
Key words: uni- and multivariate quality loss functions; symmetrical and asymmetrical distributions; software applications.

1 Introduction

In statistical decision theory, the Taguchi philosophy is a recent point of view that improves classical dichotomous- type decision rule (good/bad), with the main goal to increase products' quality in the industrial practice. The quality loss function gives a financial value for customers' increasing dissatisfaction as the product performance goes below the desired target performance. Determining the target performance is an educated guess, often based on customer surveys and feedback [9]. The applications of Taguchi's robust parameter and tolerance choices for the manufacturing engineering are mainly allocated through designed experiments. Much of the discussion about the methods of Taguchi pursues the optimization of a single quality attribute. For multiple quality attributes, the objective is to determine the best factor settings, which will at the same time optimize all the quality characteristics of interest to the experimenter.

Dr. Genichi Taguchi bases his method on conventional statistical tools and provides guidelines for laying out design of experiments and for analyzing the results of

BSG Proceedings 21. The International Conference "Differential Geometry - Dynamical Systems" DGDS-2013, October 10-13, 2013, Bucharest-Romania, pp. 175-180.

these experiments. Taguchi's approach to quality control applies to the entire process of developing and manufacturing a product - from the initial concept, through design and engineering, to manufacturing and production [7].

Some loss functions developments are introduced in the next sections, with applications in the quality and reliability theory. Typically, the canonical distributions have unbounded range, while the loss function can be non-zero only on bounded intervals. The distribution analysis of experimental data points indicated that curve profiles can be more adequately described by allowing more flexibility in the choice of the skewness and peakedness of the curves.

2 Univariate loss functions

A quality loss function gives an evaluation of the quality in financial terms.

Definition 1. A univariate loss function $L(u) : \mathbb{R} \to \mathbb{R}_+$ is any continuous function which has a minimum at the null point, increases for $u > 0$, and decreases for $u < 0$.

Examples.

• The linex loss function [16]: $L(u) = e^{au} - au - 1$, defined for $a \in \mathbb{R}$.

• An asymmetric loss function [3]: $L(u) = [a + (1 - 2a).\ 1(u < 0)]|u|^p$, $a \in (0, 1)$, $p \in \mathbf{N}$, where 1 is the indicator function.

Loss functions can be viewed graphically as a way to describe a phenomenon affecting the value of products. We give below some types of models using the exponential and generalized gamma pattern [13]; [15]. Below are examples of some new laws used as univariate loss functions.

• An adapted hyperbolic cosine model

$$y_1 = a \exp(b(x - e)) + a \exp(c(e - x)) - 2a.$$

• A generalized of hyperbolic cosine model

$$y_2 = a \exp(b(x - e)) + d \exp(c(e - x)) - a - d.$$

• Sum of shifted generalized gamma

$$y_3 = ax^m \exp(b(x - e)) + dx^n \exp(c(e - x)) - a - d.$$

As can be seen, these newly proposed loss laws can be symmetrical as well as asymmetrical, thus being more suitable models and giving better approximations for the real phenomena. In our applications we used software developed for curve fitting which applies nonlinear regression (least squares method), including the Levenberg-Marquardt algorithm [10].

3 Multivariate loss functions

A multivariate loss function must extend the properties of the univariate case in a reasonable way. A multivariate loss function must be globally minimized at zero, and on the other hand, domains of the lower loss have to lie closer to the origin.

Definition 2. Let $D(l) = \{u : u\mathbb{R}^N, L(u) \leq l\}$ be a closed set bounded by the level curve of loss l.

Any continuous function $L(u) : \mathbb{R}^N \to \mathbb{R}_+$ satisfying

1. $L(u)$ is globally minimized at $u = 0$, and
2. $\forall\, l_1 < l_2 \in L(\mathbb{R}^N) \Rightarrow D(l_1) \subset D(l_2)$

is called *multivariate loss function*.

Since cost interactions are direction-specific, one might be better off by requiring the loss to be larger for larger forecast errors in a given direction. This leads to the characterization of multivariate loss functions as follows: any continuous function $L(u) : \mathbb{R}^N \to \mathbb{R}_+$, such that $L(u) = L(u.u_0)$ is a univariate loss function for any $u_0 \in \mathbb{R}^N$, is a multivariate loss function [4].

Below are some models of loss functions in the multidimensional case:

Model 1. Loss functions without interactions [7], [12]

$$L(u_1, u_2, \ldots, u_k) = \sum_{i=1}^{k} k_i(u_i - m_i)^2$$

and for a sample: $\bar{L}(u) = \sum_{i=1}^{k} k_i[s_i^2 + (\bar{u}_i - m_i)^2]$

Model 2. Loss functions with interactions

$$L(u_1, u_2, \ldots, u_p) = \sum_{i=1}^{p} k_i(u_i - m_i)^2 + \sum_{1\leq i<j\leq p} k_{ij}(u_i - m_i)(u_j - m_j)$$

and for a sample:

$$\bar{L}(u_1, u_2, \ldots, u_p) = \sum_{i=1}^{p} k_i[s_i^2 + (\bar{u}_i - m_i)^2] + \sum_{1\leq i<j\leq p} k_{ij}[s_{ij}^2 + (\bar{u}_i - m_i)(\bar{u}_j - m_j)].$$

4 Applications

The use of asymmetric loss functions in quality control is illustrated by the following example. The aim of the study is to improve the overall quality of a printing machine in applying coloring inks on package labels [5]. The data [3] are given in Table 1.

Table 1. Experimental Data [3]

y_1	-2.5	-2.1	-1.58	-6.34	-0.71	1.25	-9.8
y_2	-0.14	-0.03	0.34	-0.038	-0.27	-0.18	-0.06
L	0.146	1.301	6.713	0.577	2.0	10.88	1.19

y_1	-11.9	-28.3	0.75	0.41	2.76	-3.26	-5.04
y_2	0.12	0.43	0.145	0.04	-0.14	0.3	0.153
L	4.987	23.17	0.073	0.784	1.98	0.21	1.767

y_1	1.66	-5.49	-5.48	-3.1	2.28	2.56	0.68
y_2	-0.19	0.079	0.025	-0.114	0.17	-0.067	0.138
L	5.279	0.496	3.355	11.05	0.073	0.255	0.945

y_1	-0.01	-0.71	1.8	-3.2	-0.16	0.9
y_2	-0.0022	0.108	-0.1325	-0.0344	-0.1445	-0.79
L	0.043	0.716	3.332	0.201	0.89	6.51

The first analysis uses linear regression in Excel. The coefficient of determination was $R^2 = 0.757$, meaning that 75.7% of the variation of L around its mean is explained by the model. Regressors y_1^2, y_2^2 and interaction $y_1 y_2$ were found to be significant. However, Excel standard errors, t-statistics and p-values are based on the hypotheses that the errors are independent with constant variance (homoskedastic), assumptions that are realistically not met. Since Excel does not provide alternatives, such as heteroskedastic-robust or autocorrelation-robust standard errors and t-statistics and p-values, we need more specialized software, for instance STATA, EVIEWS, SAS, LIMDEP, PC-TSP [1].

Table 2. Regression Results with Minitab 15

Predictor	Coef	SE Coef	T	P
(y_1^2)	0,02556	0,01891	1,35	0,189
(y_2^2)	55,80	20,05	2,78	0,010
$(y_1 y_2)$	0,600	1,343	0,45	0,659

From a simple summary of the above output it results the positive definite quadratic form:

$$L(y_1^2, y_2^2, y_1 y_2) = 0,0256 y_1^2 + 55,8 y_2^2 + 0,60 y_1 y_2,$$

because $\Delta_1 = 0,025 > 0$ and $\Delta_2 = 1,305 > 0$.

Table 3. ANOVA Results with Minitab

Analysis of Variance					
Source	DF	SS	MS	F	P
Regression	3	673,81	224,60	18,99	0,000
Residual Error	24	283,80	11,83		
Total	27	957,61			

The computed F is 18.99, much more than the critical theoretical F_c, and it rejects H_0 at significance level 0.05.

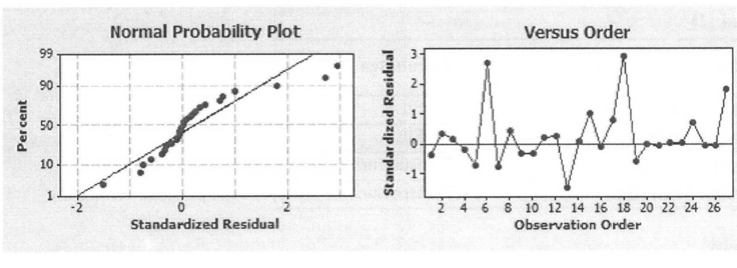

Fig.1 Residual Plots for the Quality Loss Function

In the Fig.1 (left) it can be seen that the normal distribution is adequate, based on the values of the standardized residuals. In the right side of the Fig.1 it pinpoints the outliers of the experimental data.

The asymmetrical quality loss function of the experimental data is given in Fig. 2 for a case with interactions.

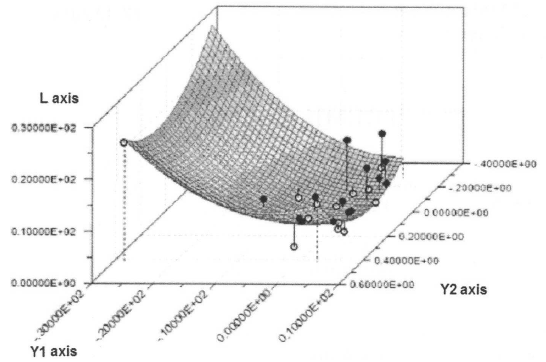

Fig.2 A quadratic loss function with interactions

5 Conclusions

The purpose of this research is to give and overview of practical models that are useful in applied quality analysis and Taguchi techniques. The presentation integrates theory and practice covering formal uni- and multivariate models and exploratory data analysis. We pursued a comparative study of the proposed models for experimental data and identified adequate stochastic laws. By using quality loss functions accounting for symmetrical/asymmetrical cases [2], we found better performance when using truncated models on a given interval. The analyzed laws prove their efficiency if applied in the design stage. The results were extended in the n-dimensional space, taking into consideration multidimensional variables with or without the potential interactions, which can cause a fraction of quality loss too [10], [11]. It seems that it may be possible to adapt this methodology in the area of reliability [15].

To conclude, in this paper we proposed adaptive stochastic distributions for describing the loss functions in the usual manufacturing models [9]. The results that we obtained show that our approach has practical application in the design of actual strategies, allowing, moreover, for prediction of production costs.

References

[1] C. Cameron, EXCEL 2007, *Multiple Regression*, http://cameron.econ.ucdavis.edu /excel/ex61multipleregression.html

[2] C-H. Chen, *A Note on the Appropriateness of Taguchi's Loss Function*, Heldermann Verlag Economic Quality Control 20, 2 (2005), 271–275.

[3] M. Demetrescu, *Some Statistical Methods with Relevance for Robust Design under General Loss Functions*, Doctoral Thesis, 2009.

[4] M. Demetrescu, A.S. Paris and C. Târcolea, *Uni-and multivariate loss functions and the Taguchi theory*, BSG Proc. 17, Geometry Balkan Press, Bucharest 2010, 79–83.

[5] M. Demetrescu, A.S. Paris and C. Târcolea, *Loss Functions with Interactions and Practical Application*, 296–299. Proceedings of the 10th WSEAS, MACMESE'08, Bucharest, Romania.

[6] G. Drăgoi, C. Târcolea, D. Drăghicescu and S. Tichkiewitch, *Multidimensional Taguchi's model with dependent variables in quality system*, Proceedings of the 2nd IDMME Conference, Compiegne, France. Kluwer Academic Publishers, 1999, 635–642.

[7] Y. Fathi, C. Poonthanomsook, *A quartic quality loss function and its properties*, Journal of Industrial and Systems Engineering, 1, 1(2007), 8–22.

[8] R.A. Fisher, *The arrangement of field experiments*, J. Min. Agric. G. Br., 33 (1926), 503–513.

[9] A. Paris and C. Tarcolea, *Taguchi applications on manufacturing systems*, ICMaS 2004 08-09.10.2004 UPB, Romanian Journal of Technical Sciences Applied Mechanics, ICMAS'2004, 49 (2004), 459–462.

[10] C.B. Rae, Sittichai Kaewkuekool, Shannon R. Bowling, and Mohammad T. Khasawneh, *The optimum target value for a process based on a quadratic loss function*, Quality Engineering, 17, 2 (2005), 227–241.

[11] L. Swinton, *Management Gurus - The Quick and Easy Guide*, http://www.mftrou.com/support-files/management-gurus.pdf

[12] G. Taguchi, *Subir Chowdhury, Yuin Wu -Taguchi's Quality Engineering Handbook*, John Wiley, 2005.

[13] C. Târcolea, A.S. Paris, A. Târcolea, *Statistical models applied to manufacturing systems*, BSG Proc. 12, Geometry Balkan Press, Bucharest 2005, 259–264.

[14] C. Târcolea, A.S. Paris, *Bidimensional models of Taguchi type* (in Romanian), Management Industrial, 1 (1995), 29–32.

[15] C. Târcolea, A.S. Paris, *Loss functions used in the quality theory*, U.P.B. Sci. Bull. Series A, 73, 1, 2011, 45–54.

[16] H. Varian, *A Bayesian Approach to Real Estate Assessment*, in Studies in Bayesian Econometrics and Statistics in Amsterdam, North-Holland, 1974.

Authors' addresses:

Constantin Târcolea, Adrian Stere Paris
Faculty of Applied Sciences, University Politehnica of Bucharest,
Splaiul Independenţei 313, 060042 Bucharest, Romania.
E-mail: constantin_tarcolea@yahoo.com , sparis@rdslink.ro

Dana Sylvan
Hunter College of the City University of New York, U.S.
E-mail: dana.sylvan@gmail.com