

On the group of isometries of the space

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Abstract. In this paper we consider the group of spatial rotations and translations in the universe, in case of a bounded universe and some other assumptions. This compact group is studied and assuming that the universe is simply connected, this group is shown to be isomorphic to $Spin(4)$, i.e. $S^3 \times S^3$, while the observable universe is homeomorphic to S^3 .

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1 Introduction

In this paper we consider a group of all translations and rotations in the space or globally in the universe. It is well known that if the space is \mathbb{R}^3 , this group is given by $\left\{ \begin{bmatrix} M & h^T \\ 0 & 1 \end{bmatrix} \right\}$, where $M \in SO(3, \mathbb{R}^3)$ and h is the vector of translation. But this affine group must be changed, because the space is bounded, such that this 6-dimensional group of transformations is at least a compact Lie group. Each translation will be considered as a rotation for angle φ , such that the corresponding translation should be $R\varphi$, where R is extremely large constant, which may be called radius of the universe. In this paper we give a description of this group.

In order the space and time to be of the same dimension, many authors ([1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12]) have considered the time as 3-dimensional. In order to preserve the Lorentz transformations (up to isomorphism) it is convenient to replace the group of Lorentz transformations $O_+^\uparrow(1, 3)$ with its isomorphic Lie group $SO(3, \mathbb{C})$. In [9] the Lorentz transformation is converted into complex orthogonal transformation on \mathbb{C}^3 in the frame of the 3-dimensional temporal coordinates. While the Lie algebra of $SO(3, \mathbb{C})$ is given by

$$\begin{bmatrix} C & A \\ -A & C \end{bmatrix}$$

where A and C are antisymmetric 3×3 matrices, the Lie algebra of the required group G_{rt} of rotations and translations in [11] is introduced by

$$\begin{bmatrix} C & B \\ B & C \end{bmatrix}$$

where C and B are antisymmetric 3×3 matrices. The matrix C is the Lie algebra which corresponds to the spatial rotations, i.e. to the Lie algebra of $SO(3, \mathbb{R})$ and the group of unit quaternions S^3 , while B is an antisymmetric matrix which corresponds to the set of "translations". So, if we put

$$C = \begin{bmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix},$$

(c_1, c_2, c_3) determines the direction of 3-vector for a spatial rotation, while (b_1, b_2, b_3) determines the direction of 3-vector of a spatial translation. If $b_1, b_2, b_3 \sim 1/R$, the vector $R(b_1, b_2, b_3)$ can be considered approximately as a vector of translation \vec{r} , i.e. $(b_1, b_2, b_3) \approx \frac{\vec{r}}{R}$. But the coefficient of proportionality $1/R$ is not necessary to be so small constant ([11]), and some important conclusions are given in [11]. For example, it leads to consideration of the motion of a spinning body on a horizontal plane in a gravitational field ([11]), which can be experimentally verified. Analogously as there is not a strict separation between the spatial coordinates and the 1-dimensional temporal coordinate with respect to the Special Relativity, there is not strict separation between the rotations and translations. Analogous to the invariant $I = -ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$ from the Spacial Relativity, according to the considered 3+3+3-model of space-time (3 classic spatial coordinates, 3 coordinates about the rotation of the body and 3-temporal coordinates) there appear 4 invariants ([11]):

$$I_1 = d\xi_1^2 + d\xi_2^2 + d\xi_3^2 + d\eta_1^2 + d\eta_2^2 + d\eta_3^2 - c^2 d\theta_1^2 - c^2 d\theta_2^2 - c^2 d\theta_3^2,$$

$$I_2 = d\xi_1 d\eta_1 + d\xi_2 d\eta_2 + d\xi_3 d\eta_3,$$

$$I_3 = d\xi_1 d\theta_1 + d\xi_2 d\theta_2 + d\xi_3 d\theta_3,$$

$$I_4 = d\eta_1 d\theta_1 + d\eta_2 d\theta_2 + d\eta_3 d\theta_3,$$

where $d\eta_1, d\eta_2, d\eta_3$ correspond to the space displacement, $d\xi_1, d\xi_2, d\xi_3$ are induced by the rotation and $d\theta_1, d\theta_2, d\theta_3$ correspond to the temporal displacement. The last two invariants I_3 and I_4 are equal to 0. The geometrical/physical interpretations of these invariants are presented in [11]. While in the paper [11] it is shown that I_1, I_2, I_3, I_4 are invariants and some applications are given, in this paper we consider in more details the group G_{rt} and show that its universal covering is isomorphic to $Spin(4)$.

2 On the compact group of all translations and rotations

After the previous introduction we study now the mentioned group G_{rt} of rotations and translations. According to the Lie algebra of G_{rt} , the 6-dimensional Lie group of rotations and translations G_{rt} locally is generated by the following 6 subgroups of matrices:

$$R_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha & 0 & 0 & 0 \\ 0 & \sin \alpha & \cos \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos \alpha & -\sin \alpha \\ 0 & 0 & 0 & 0 & \sin \alpha & \cos \alpha \end{bmatrix},$$

$$\begin{aligned}
 R_{y,\beta} &= \begin{bmatrix} \cos \beta & 0 & \sin \beta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin \beta & 0 & \cos \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \beta & 0 & \sin \beta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin \beta & 0 & \cos \beta \end{bmatrix}, \\
 R_{z,\gamma} &= \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 & 0 & 0 & 0 \\ \sin \gamma & \cos \gamma & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \gamma & -\sin \gamma & 0 \\ 0 & 0 & 0 & \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \\
 T_{x,\alpha} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos \alpha & 0 & 0 & 0 & \sin \alpha \\ 0 & 0 & \cos \alpha & 0 & -\sin \alpha & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \sin \alpha & 0 & \cos \alpha & 0 \\ 0 & -\sin \alpha & 0 & 0 & 0 & \cos \alpha \end{bmatrix}, \\
 T_{y,\beta} &= \begin{bmatrix} \cos \beta & 0 & 0 & 0 & 0 & -\sin \beta \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos \beta & \sin \beta & 0 & 0 \\ 0 & 0 & -\sin \beta & \cos \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \sin \beta & 0 & 0 & 0 & 0 & \cos \beta \end{bmatrix}, \\
 T_{z,\gamma} &= \begin{bmatrix} \cos \gamma & 0 & 0 & 0 & \sin \gamma & 0 \\ 0 & \cos \gamma & 0 & -\sin \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \sin \gamma & 0 & \cos \gamma & 0 & 0 \\ -\sin \gamma & 0 & 0 & 0 & \cos \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},
 \end{aligned}$$

where the first three are rotations around x , y and z axes for angles α , β and γ respectively, while the last three are "translations" along the x , y and z axes for lengths $R\alpha$, $R\beta$ and $R\gamma$. All these matrices are orthogonal 6×6 matrices of type $\begin{bmatrix} A & B \\ B & A \end{bmatrix}$, and if $1/R^2$ is neglected, then this group becomes the classical group of rotations and translations, analogously as the Special Relativity reduces to the classical Newtonian mechanics if we neglect c^{-2} .

Theorem 2.1. (a) *The mappings*

$$\begin{aligned}
 R_{x,\alpha} &\mapsto \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}, & R_{y,\beta} &\mapsto \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}, \\
 R_{z,\gamma} &\mapsto \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}, & T_{x,\alpha} &\mapsto \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix},
 \end{aligned}$$

$$T_{y,\beta} \mapsto \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix}, \quad T_{z,\gamma} \mapsto \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

generate locally an epimorphism of groups $\varphi : G_{rt} \rightarrow SO(3, \mathbb{R})$;

(b) The mappings

$$\begin{aligned} R_{x,\alpha} &\mapsto \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}, & R_{y,\beta} &\mapsto \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}, \\ R_{z,\gamma} &\mapsto \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}, & T_{x,\alpha} &\mapsto \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}, \\ T_{y,\beta} &\mapsto \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}, & T_{z,\gamma} &\mapsto \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}, \end{aligned}$$

generate locally an epimorphism of groups $\psi : G_{rt} \rightarrow SO(3, \mathbb{R})$;

(c) There is a local isomorphism $\theta : G_{rt} \rightarrow SO(3, \mathbb{R}) \times SO(3, \mathbb{R})$, which is defined by $\theta(M) = \varphi(M) \times \psi(M)$.

Proof. Let us consider the matrices of type $\begin{bmatrix} A & B \\ B & A \end{bmatrix}$ which belong to $SO(6, \mathbb{R})$ and which are path connected with $\text{diag}(1,1,1,1,1,1)$. This implies

$$\begin{bmatrix} A & B \\ B & A \end{bmatrix} \begin{bmatrix} A^T & B^T \\ B^T & A^T \end{bmatrix} = \begin{bmatrix} I & O \\ O & I \end{bmatrix}$$

and consequently

$$(2.1) \quad AA^T + BB^T = I \quad \text{and} \quad AB^T + BA^T = O.$$

It is easy to verify that these matrices give a connected subgroup G of $SO(6, \mathbb{R})$. Its Lie algebra coincides with the Lie algebra of G_{rt} , which was previously described. So, locally the Lie group G_{rt} can be considered that is given by the matrices $\begin{bmatrix} A & B \\ B & A \end{bmatrix}$, where A and B are 3×3 matrices which satisfy the conditions (2.1).

(a) It is sufficient to prove that the given 6 mappings generate an epimorphism of groups $\varphi : G \rightarrow SO(3, \mathbb{R})$. If there exists such a homomorphism $\varphi : G \rightarrow SO(3, \mathbb{R})$ with the required conditions, then it is unique. So we will prove that such a homomorphism exists and it is given by

$$(2.2) \quad \varphi\left(\begin{bmatrix} A & B \\ B & A \end{bmatrix}\right) = A + B.$$

Using that the conditions (2.1) are satisfied, we obtain

$$\begin{aligned} (A + B)(A + B)^T &= (A + B)(A^T + B^T) = \\ &= (AA^T + BB^T) + (AB^T + BA^T) = I + O = I, \end{aligned}$$

and so $A + B \in O(3, \mathbb{R})$. Moreover, since $\varphi : G \rightarrow O(3, \mathbb{R})$ is a continuous mapping, G is a connected manifold and $\varphi\left(\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}\right) = I \in SO(3, \mathbb{R})$, we notice that $\varphi : G \rightarrow SO(3, \mathbb{R})$. This mapping is homomorphism because

$$\begin{aligned} \varphi\left(\begin{bmatrix} A & B \\ B & A \end{bmatrix}\right) \varphi\left(\begin{bmatrix} C & D \\ D & C \end{bmatrix}\right) &= \varphi\left(\begin{bmatrix} AC + BD & AD + BC \\ AD + BC & AC + BD \end{bmatrix}\right) = \\ &= AC + BD + AD + BC = (A + B)(C + D) = \varphi\left(\begin{bmatrix} A & B \\ B & A \end{bmatrix}\right) \cdot \varphi\left(\begin{bmatrix} C & D \\ D & C \end{bmatrix}\right). \end{aligned}$$

This mapping is also surjective, because for any matrix $A \in SO(3, \mathbb{R})$, $A = \varphi\left(\begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}\right)$ and the matrix $\begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}$ is obviously path connected with $\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$. Finally, the required 6 conditions are trivially satisfied.

(b) In this case instead of (2.2) we have

$$(2.3) \quad \psi\left(\begin{bmatrix} A & B \\ B & A \end{bmatrix}\right) = A - B.$$

and then proof is analogous to the proof in case (a).

(c) Firstly it is sufficient to prove that $\theta : G \rightarrow \varphi(G) \times \psi(G)$ defined by $\theta(M) = \varphi(M) \times \psi(M)$ is isomorphism.

θ is injection: Assume that

$$\varphi\left(\begin{bmatrix} A & B \\ B & A \end{bmatrix}\right) = \varphi\left(\begin{bmatrix} C & D \\ D & C \end{bmatrix}\right), \quad \text{i.e.} \quad (A + B, A - B) = (C + D, C - D).$$

Hence $A = C$ and $B = D$ and thus $\begin{bmatrix} A & B \\ B & A \end{bmatrix} = \begin{bmatrix} C & D \\ D & C \end{bmatrix}$.

θ is surjection: Assume that $(A, B) \in SO(3, \mathbb{R}) \times SO(3, \mathbb{R})$. Then

$$\theta\left(\begin{bmatrix} (A + B)/2 & (A - B)/2 \\ (A - B)/2 & (A + B)/2 \end{bmatrix}\right) = (A, B).$$

Now, using that $AA^T = BB^T = I$, it is easy to prove that

$$\frac{A + B}{2} \left(\frac{A + B}{2}\right)^T + \frac{A - B}{2} \left(\frac{A - B}{2}\right)^T = I$$

and

$$\frac{A + B}{2} \left(\frac{A - B}{2}\right)^T + \frac{A - B}{2} \left(\frac{A + B}{2}\right)^T = 0.$$

Moreover, since the mappings (2.2) and (2.3) are homomorphisms, we obtain that θ is isomorphism.

We should only to prove that $\begin{bmatrix} (A + B)/2 & (A - B)/2 \\ (A - B)/2 & (A + B)/2 \end{bmatrix}$ is path connected to $\begin{bmatrix} I & O \\ O & I \end{bmatrix}$ in G . This is obvious because $\begin{bmatrix} (A + B)/2 & (A - B)/2 \\ (A - B)/2 & (A + B)/2 \end{bmatrix}$ is path connected to $\begin{bmatrix} (A + I)/2 & (A - I)/2 \\ (A - I)/2 & (A + I)/2 \end{bmatrix}$ in G and

$\begin{bmatrix} (A+I)/2 & (A-I)/2 \\ (A-I)/2 & (A+I)/2 \end{bmatrix}$ is path connected to $\begin{bmatrix} I & O \\ O & I \end{bmatrix}$ in G . \square

Since the group G_{rt} was defined via its Lie algebra, it is determined only locally. If $G_{rt} = G$, then G_{rt} is isomorphic to $SO(3, \mathbb{R}) \times SO(3, \mathbb{R})$. But if we require for the universe to be simply connected, and also for the group of rotations to be simply connected because of symmetry between them, then G_{rt} is also simply connected Lie group. In this case G_{rt} is homeomorphic to $S^3 \times S^3$ as universal covering of $SO(3, \mathbb{R}) \times SO(3, \mathbb{R})$.

3 Connection with the group $Spin(4)$

Finally we come to the following question: What is the shape of the universe? First we state the following proposition, which is a consequence of the second equality in (2.1).

Proposition 3.1. *If $\begin{bmatrix} A & B \\ B & A \end{bmatrix}$ belongs to the group G and A is non-singular, then $A^{-1}B$ is an antisymmetric matrix.*

If we neglect the rotations in the Lie group G_{rt} , we obtain a subset U of G_{rt} which is the spatial part of the universe, but it is not a subgroup of G_{rt} , similarly as the set of Lorentz boosts is not a subgroup of the Lorentz group of transformations. The products of two matrices in U as well as two Lorentz boosts, contain small spatial rotations. The set U of the spatial part of the universe locally is given by the 3-dimensional antisymmetric matrix $A^{-1}B$ from the Proposition 3.1. Using the representations of $T_{x,\alpha}$, $T_{y,\beta}$ and $T_{z,\gamma}$, the corresponding antisymmetric matrices $A^{-1}B$ are

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \tan \alpha \\ 0 & -\tan \alpha & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & -\tan \beta \\ 0 & 0 & 0 \\ \tan \beta & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & \tan \gamma & 0 \\ -\tan \gamma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Hence, if we consider the group G_{rt} to be isomorphic to $SO(3, \mathbb{R}) \times SO(3, \mathbb{R})$, then the subset U is homeomorphic to $SO(3, \mathbb{R})$, i.e. RP^3 and then the function $\tan \varphi$ from the previous matrices is considered as a function with period π . But it is more convenient to consider the group G_{rt} to be isomorphic to $S^3 \times S^3$ and then the subset U is homeomorphic to S^3 . In this case the function $\tan \varphi$ from the previous matrices is considered as a function with period 2π , because for a given angle φ and given direction correspond two different points from U which lead to the same matrix representation $A^{-1}B$.

Now if we consider the group of isometries of the sphere S^3 which preserve orientation, we obtain again the same group $S^3 \times S^3$. Indeed, this group can be obtained if we consider S^3 embedded into \mathbb{R}^4 . Locally this group is $SO(4, \mathbb{R})$, but its universal covering group $Spin(4)$ is isomorphic to $S^3 \times S^3$. We skip the proof of the following theorem, because by straight verification it can be proved.

Theorem 3.2. *The mapping*

$$\begin{bmatrix} A & b^T \\ -b & 0 \end{bmatrix} \mapsto \begin{bmatrix} A & B \\ B & A \end{bmatrix},$$

where B is antisymmetric matrix such that $B_{12} = b_3$, $B_{23} = b_1$, $B_{31} = b_2$, defines an isomorphism between the Lie algebras of $SO(4, \mathbb{R})$ and G_{rt} .

According to Theorem 3.2 we obtain the isomorphism between the universal coverings of the considered two groups, which is indeed $Spin(4)$. This alternative representation of the group G_{rt} as isometries (which preserve orientation) of S^3 embedded into \mathbb{R}^4 leads to the following invariant element. Assuming that $d\theta_1 = d\theta_2 = d\theta_3 = 0$, then the invariant I_1 in 4 dimensions reduces to $I_1 = d\eta_1^2 + d\eta_2^2 + d\eta_3^2 + d\xi^2 = dx^2 + dy^2 + dz^2 + d\xi^2$, where $d\xi^2 = d\xi_1^2 + d\xi_2^2 + d\xi_3^2$, but we do not know the direction of $(d\xi_1, d\xi_2, d\xi_3)$. This invariant is analogous to the well know invariant from the Special Relativity $-ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$, which is also a special case of the invariant I_1 .

4 Conclusion

Although the subset of translations, which is homeomorphic to the visible part of the universe from a chosen point, is homeomorphic to S^3 , it is not convenient to consider it as embedded in \mathbb{R}^4 . Also, it is not convenient to endow it with linear connection as a submanifold of \mathbb{R}^4 . It is much more convenient to endow the total space G_{rt} with a flat linear connection and non-zero torsion tensor arising from the Lie group structure, such that it admits 6 parallel vector fields. The Lie group G_{rt} of isometries acts on itself, but not on a different space. Analogously the group $SO(3, \mathbb{C})$ acts on itself, instead of the Lorentz group which acts on the 4-dimensional space-time points.

References

- [1] V. S. Barashenkov, *Multitime generalization of Maxwell electrodynamics gravity*, Tr. J. Phys. 23 (1999), 831-838.
- [2] V. S. Barashenkov, *Quantum field theory with three-dimensional vector time*, Particles and Nuclei Letters 2 (2004), 54-63.
- [3] E. A. B. Cole, *Particle decay in six-dimensional relativity*, J. Phys. A: Math. Gen. (1980), 109-115.
- [4] R. Franco A. Jorge, *Vectorial Lorentz transformations*, Elec. J. Theor. Phys. 9 (2006), 35-64.
- [5] H. Kitada, *Theory of local times*, Nuovo Cim. B 109 (1994), 281-302.
- [6] D.G. Pavlov, *Chronometry of the three-dimensional time*, Space-Time Structure. Algebra and Geometry, Russian Hypercomplex Society, Lilia Print, Moscow 2007, 32-44.
- [7] J. Strnad, *Once more on multi-dimensional time*, J. Phys. A: Math. Gen. 14 (1981), L433-L435.
- [8] K. Trenčevski, *Duality in the special relativity based on the isomorphic structural groups $SO(3, \mathbb{C})$ and $O_+^\uparrow(1, 3)$* , Tensor 72 (2010), 32-46.
- [9] K. Trenčevski, *Representation of the Lorentz transformations in 6-dimensional space-time*, Kragujevac J. of Math. 35 (2011), 327-340.

- [10] K. Trenčevski, *Special relativity based on $SO(3,C)$ structural group and 3-dimensional time*, Math. Balkanica 25(1-2) (2011), 193-201.
- [11] K. Trenčevski, *On the geometry of space-time and motion of spinning bodies*, Central European J. of Phys. 11(3) (2013), 296-316.
- [12] A.P. Yefremov, *Six-dimensional "Rotational Relativity"*, Acta Phys. Hung. N.S. (Heavy Ion Physics) 11 (2000), 147-153.

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