Errata and corrigenda:
Efficiency for multitime variational problems with geodesic quasiinvex functionals on Riemannian manifolds

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Abstract

This errata and corrigenda are given for a part of the paper "Ştefan Mititelu, Mădălina Constantinescu, Constantin Udrişte, Efficiency for multitime variational problems on Riemannian manifolds", published in *BSG Proceedings 22, The International Conference "Differential Geometry - Dynamical Systems" DGDS-2014, September 1-4, 2014, Mangalia-Romania, pp. 38-50.*

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We are grateful to colleagues who have flagged inaccuracies in our work. As a result, we consider that it is necessary to appropriately correct the existing drawbacks.

1 Elementary setting

We can use the spaces $T = \mathbb{R}^\ell$, $M = \mathbb{R}^n$ and $\Omega \subset T$ as hyper-parallelepiped, fixed by opposite diagonal points. Then everything works as it is.

2 Errata and corrigenda

When we pass to Riemannian manifolds (T, λ) and (M, χ) of dimensions ℓ and n, $\ell < n$, we introduce the next corrigenda.

Suppose M is a complete manifold. Denote $t = (t^1, ..., t^\ell) = (t^\upsilon)$ the points of T, $x = (x^1, ..., x^n) = (x^i)$ the points of M. We need $\Omega \subset T$ as an open, relative compact subset, with boundary $\partial\Omega$ consisting in a finite number of C^1 hypersurfaces. The sense of smoothness for functions is those in differential geometry.

Definition 2.1. [1] Let (M, χ) be a complete Riemannian manifold. Let $\eta\colon M \times M \to TM$, $\eta(x^0, x) \in T_{x^0}M$, $x^0, x \in M$, with $\eta(x^0, x) = 0 \Leftrightarrow x^0 = x$, a vectorial function and $S \subset M$ a non-void set.

(i) The set S is called $\eta-$geodesic invex, if, for any x^0, $x \in S$, the geodesic

$$\gamma : [0,1] \to M,\ \gamma(t) = \exp_{x^0}(t\eta(x^0, x))$$

is included in S.

(ii) Let $S \subset M$ be an open set, $\eta-$geodesic invex and $f\colon S \to \mathbb{R}$ be a function of class C^1. The function f is called $\eta-$geodesic invex on S, if

$$f(x) - f(x^0) \geq df_{x^0}(\eta(x^0, x)),\ \forall x^0, x \in S.$$

To further develop our theory, we need the set of functions

$$\mathcal{F}(\Omega, M) = \{x : \bar{\Omega} \to M \ \big|\ x \text{ is of class } C^1 \text{ on } \Omega \text{ and of class } C^0 \text{ on } \bar{\Omega}\}.$$

Definition 2.2. [2] Let $x^0(\cdot)$, $x(\cdot) \in \mathcal{F}(\Omega, M)$. A function $\varphi : \Omega \times [0,1] \to M$, is called geodesic deformation of the pair of functions $(x^0(\cdot), x(\cdot))$, if it satisfies the properties: (1) the function $\tau \to \varphi(t, \tau)$ is geodesic; (2) $\varphi(t, 0) = x^0(t)$, $\varphi(t, 1) = x(t)$.

For a given function $x^0(\cdot) : \Omega \to M$, we define the η-geodesic deformation $x(t)$ of $x^0(\cdot)$, by $\varphi_{x^0,x}(t, \tau) = \exp_{x^0(t)}(\tau\eta(x^0(t), x(t))$.

Definition 2.3. The set $\mathcal{S} = \mathcal{F}(\Omega, S) \subset \mathcal{F}(\Omega, M)$ is called $\eta-$geodesic invex, if for each pair $x^0(\cdot)$, $x(\cdot) \in \mathcal{S}$, where $x(\cdot)$ is a $\eta-$ geodesic deformation of $x^0(\cdot)$ and $\tau \in [0, 1]$, the function $\varphi_{x^0,x}(\cdot, \tau)$ belongs to $\mathcal{S}$.

For efficiency and optimality sufficient conditions, we shall introduce the notion of $(\rho, b)-$geodesic quasi-invex functional.

Let us introduce a distance on the set of functions $\mathcal{F}(\Omega, M)$. If $d_\chi(x^0, x)$ is the geodesic distance in (M, χ), we define

$$d(x^0(\cdot), x(\cdot)) = \sup_{t \in \Omega} d_\chi(x^0(t), x(t)).$$

In this sense, $(\mathcal{F}(\Omega, M), d)$ is a metric space.

We fix the number $\rho \in \mathbb{R}$, a symmetric functional $b\colon \mathcal{F}(\Omega, M) \times \mathcal{F}(\Omega, M) \to [0, \infty)$ and the distance function $d(x^0(\cdot), x(\cdot))$, previously introduced on $\mathcal{F}(\Omega, M)$.

We consider the functional

$$E\colon \mathcal{F}(\Omega, M) \to \mathbb{R},\ \ E(x(\cdot)) = \int_\Omega X(j_t^1 x) dv.$$

Definition 2.4. Let (M, χ) be a complete Riemannian manifold. Let $\mathcal{S}$ be an open subset and $\eta-$geodesic invex of $\mathcal{F}(\Omega, M)$.

(i) The functional E is called (strictly) $(\rho, b)-$geodesic quasi-invex in $x^0(\cdot) \in \mathcal{S}$, with respect to η, if $E(x(\cdot)) \leq E(x^0(\cdot))$ implies

$$b(x, x^0) \int_\Omega \left(\xi^i \frac{\partial X}{\partial x^i}(j_t^1 x^0) + \frac{\partial \xi^i}{\partial t^\upsilon} \frac{\partial X}{\partial x_\upsilon^i}(j_t^1 x^0) \right) dv \ (<) \leq -\rho b(x, x^0) d^2(x, x^0),$$

for any $x(\cdot) \in \mathcal{S}$, where $\xi(t) = \eta(x^0(t), x(t))$.

(ii) The functional E is called monotonic $(\rho, b)-$geodesic quasi-invex in $x^0(t) \in S$, with respect to η, if $E(x(\cdot)) \leq E(x^0(\cdot))$ implies

$$b(x, x^0) \int_\Omega \left(\xi^i \frac{\partial X}{\partial x^i}(j_t^1 x^0) + \frac{\partial \xi^i}{\partial t^\upsilon} \frac{\partial X}{\partial x_\upsilon^i}(j_t^1 x^0) \right) dv = -\rho b(x, x^0) d^2(x, x^0),$$

for any $x(\cdot) \in \mathcal{S}$, where $\xi(t) = \eta(x^0(t), x(t))$.

Example 2.1. *We fix $\Omega = (0,1)^m$, the set of functions*

$$\mathcal{S} = \{x \colon \bar{\Omega} \subset \mathbb{R}^m \to S = \mathbb{R}_+ \mid x(\cdot) \ \textit{de clasă}\ C^1 \ \textit{pe}\ \Omega \ \textit{şi de clasă}\ C^0 \ \textit{pe}\ \bar{\Omega}\}$$

and the "negative" Boltzmann-Shannon functional

$$E \colon \mathcal{S} \to \mathbb{R}, \ E(x(\cdot)) = \int_\Omega x(t) \ln x(t) dv.$$

This functional is $(\rho \leq 0, b = 1)$ - geodesic quasi-invex with respect to

$$\eta(x^0, x) = -(\ln x - \ln x^0)\, d^2(x^0, x).$$

We can obtain the previous frame, by assuming $x(\cdot)$ is just continuous.

References

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