Black hole effect and gravitational redshift in the scaled Maneff's field

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Abstract. Considering the scaled Maneff gravitational field, we provide new insights on the Maneff problem (i.e., the two body problem associated to a post-Newtonian potential of the form $-A/r - B_0/r^2$, where r is the distance between a photon and the center of a star, A, B_0 well-known constants). In the general relativistic framework, we identify the black hole effect corresponding to the scaled Maneff field (sMF). In this regard, we obtain $\rho_{M_S} = (1+\sqrt{7})\alpha$ the gravitational radius of the sMF (where $\alpha = A/c^2$) which is larger than $\rho_M = 3\alpha$ the gravitational radius associated to non-scaled Maneff field. Further, we calculate the difference between the gravitational redshift of the scaled Maneff potential and the one associated to Newton's potential. We consider this difference both ΔS - in special approximation and ΔG - in general relativistic approximation. We obtain $\Delta S = \Delta G = 3R_S^2/4R^2$. Also, we obtain a value of three for the relative difference, which is two times greater than the one obtained in the non-scaled Maneff field.

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1 Introduction

Taking into account a non-relativistic law of gravitation, Mioc and Ureche studied post-Newtonian potentials and their implications in various astronomic experiences. Ureche computed free fall collapse time [5] and gravitational redshift [6]in the non-sacaled Maneff's field. Ureche has poined out why it is important to scale by factor 2 the Maneff field. Constantin [1] estimated the collapse time in the sMF and analyzed further cosmological effects in the Schwarzschild problem [2].

Definition 1.1. In the two body Maneff's problem in the non-scaled case, the force function is [4]:

$$U = m\left(\frac{A}{r} + \frac{B_0}{r^2}\right) = G\frac{Mm}{r}\left(1 + \frac{3G(m+M)}{2c^2 r}\right), \tag{1.1}$$

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where $A = GM$, non-scaled constant $B_0 = 3G(m+M)/2c^2$ and G is the Newtonian gravitational constant; M and m are the masses of two interacting bodies in the field (e.g. a massive cosmic object and a test particle); r is the distance between M and m; c is the speed of light.

In this paper, we extend the Ureche's work [6] and analyze the gravitational redshift in the two body problem associated to the scaled Maneff gravitational field.

Remark 1.2. In this certain case and for $m << M$ the associated Φ potential to (1.1) is:

$$\Phi(r) = -\frac{GM}{r} - \frac{3G^2M^2}{c^2r^2} = -\frac{A}{r} - \frac{B}{r^2}, \tag{1.2}$$

where B is scaled ($B = 2B_0$).

In this way, we identify the black hole effect (BHe) associated to the field described by the potential Φ given by (1.2). Also, we calculate the gravitational redshift for this potential in two frameworks: (i) in the special relativistic approximation (SRA) and (ii) in the general relativistic approximation (GRA). We discuss our results with respect to the ones obtained for the non-scaled Maneff gravitational field.

2 The black hole effect

Definition 2.1. The general relativistic metric in the spherical coordinates and associated to the potential Φ is [3]:

$$ds^2 = (1 + \frac{2\Phi}{c^2})c^2dt^2 - \frac{dr^2}{1+\frac{2\Phi}{c^2}} - r^2(d\theta^2 + sin^2\theta d\varphi^2).$$

Remark 2.2. We observe that in the terms of the first order approximation we find the Schwarzschild metric. Forwards, we use the index 0 for a distant observer.

We recall here the following well-known results:

The relation between the proper time τ and the time t of distant observer is:

$$\tau = \sqrt{1 + \frac{2\Phi}{c^2}}t. \tag{2.1}$$

Lemma 2.1. *The relation between corresponding wavelengths λ and λ_0 (λ_0 is the wavelength measured by distant observer) is [6]:*

$$\lambda = \sqrt{1 + \frac{2\Phi}{c^2}}\lambda_0. \tag{2.2}$$

Remark 2.3. We note that for Newtonian gravitational field the existence condition is [3]:

$$r > R_S = 2\alpha, \tag{2.3}$$

where R_S is the Schwarzschild gravitational radius.

Theorem 2.2. *The gravitational radius of the scaled Maneff gravitational field is:*

$$\rho_{M_S} = (1+\sqrt{7})\alpha.$$

Proof. Using the existence condition (2.3) in (2.1) for our potential Φ (see (1.2)), we obtain the black hole effect in the scaled Maneff gravitational field as following:

$$r > \rho_{M_S} = (1+\sqrt{7})\alpha > \rho_M > R_S,$$

where ρ_{M_S} is the gravitational radius of the sMF, $\rho_M = 3\alpha$ is the gravitational radius of non-scaled Maneff gravitational field ($\alpha = A/c^2$). □

3 Gravitational redshift

3.1 Special relativistic approximation

In the framework of the two body problem associated to the scaled Maneff potential (1.2), we consider a celestial body with the mass M and the radius R, a photon at the surface of the body, with the relativistic mass m_f, the wavelength λ (or the frequency ν). Also we use the index 0 for a distant observer.

Remark 3.1. Taking into account the conservation of energy law for the considered photon, Φ the potential at the surface of the body, we have [6]:

$$m_f c^2 + m_f \Phi = m_{f_0} c^2 + m_{f_0} \Phi_0. \tag{3.1}$$

Also, $m_f c^2 = \hbar\nu$, and $\nu = \frac{c}{\lambda}$, where $\hbar$ is the Planck's constant.

We use the following notations: $\lambda - \lambda_0 = \Delta\lambda$, $z_g = \Delta\lambda/\lambda$, where z_g is the gravitational redshift.

Further, computing ΔS the difference between the two redshifts, namely in the scaled Maneff gravitational field z_{gM_S} and respectively in the Newtonian gravitational field z_{gN}, both in the SRA, we obtain the next result:

Proposition 3.1. *As R_S is the Schwarzschild gravitational radius of Newtonian gravitational field then:*

$$\Delta S = z_{gM_S} - z_{gN} = \frac{3R_S^2}{4R^2}. \tag{3.2}$$

Proof. Neglecting Φ_0 in the relation (3.1) and developing this equation until the second order term, we obtain the gravitational redshift in the SRA, as follows: considering the scaled Maneff gravitational field and $r = R$, we obtain:

$$z_{gM_S} = \frac{R_S}{2R} + \frac{R_S^2}{R^2}.$$

At the same time, considering the Newtonian gravitational field we obtain:

$$z_{gN} = \frac{R_S}{2R} + \frac{R_S^2}{4R^2}.$$

□

3.2 General relativistic approximation

Proposition 3.2. *Computing the difference between* $z_g^{M_S}$ *and* z_g^N *in the GRA, we obtain:*

$$\Delta G = z_g^{M_S} - z_g^N = \frac{3R_S^2}{4R^2}. \tag{3.3}$$

Proof. In (2.2) considering the radial coordinate $r = R$ and developing z_g until to the second order, we obtain the gravitational redshift of GRA, as follows: we obtain the *scaled Maneff gravitational redshift*:

$$z_g^{M_S} = \frac{R_S}{2R} + \frac{9R_S^2}{8R^2}.$$

Also, we obtain the *Newtonian gravitational redshift*: $z_g^N = \frac{R_S}{2R} + \frac{3R_S^2}{8R^2}$. □

4 Conclusions

We have obtained in the scaled Maneff field as black hole effect

$$\rho_{M_S} = (1 + \sqrt{7})\alpha > \rho_M,$$

and some interesting results about redshift differences:

Corollary 4.1. *We have found that the two gravitational redshift differences are the same (see (3.2) and (3.3)) in the both frameworks SRA and GRA, namely:*

$$\Delta S = \Delta G = \frac{3R_S^2}{4R^2}.$$

Corollary 4.2. *Moreover, if we compute the relative difference, we obtain:*

$$\frac{\Delta G}{(z_g^N)^2} = 3.$$

So, taking in account the scaled constant $B = 2B_0$ an aspect imposed by the relativistic considerations in the Maneff problem, we obtain the values of the gravitational redshift differences: ΔS, ΔG, and the relative difference (see above) to be two times greater than the ones computed in the frame of non-scaled Maneff field [6]. These results need to be validated through their applications at some certain astronomical situations. This will be done in forthcoming papers.

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