On the matrix equation for a spin 2 particle in pseudo-Riemannian space-time, tetrad method

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Abstract. The theory of spin 2 field in Minkowski space-time is commonly formulated using the second order Pauli-Fierz formalism. However, less known in this respect is the Fedorov-Regge framework, which is applied for spin 2 particles described by the system of 1-st order equations for the tensor set - including scalars and vectors, the symmetric 2-rank tensors, and the skew symmetric in two indices 3-rank tensors.

In the present paper, we extend the first order system written in matrix form to pseudo-Riemannian space-time models, applying by the Tetrode-Weyl-Fock-Ivanenko tetrad method. The basic matrices of the equation and the Lorentzian generators for tensors are presented in block form. They are explicitly found, with the symmetries taken into account, and the set of tensors $\Psi(x) = \{\Phi, \Phi_c, \Phi_{(ab)}, \Phi_{[ab]c}\}$ is presented as a multi-component function. All the intrinsic constraints on tensors are contained in the structure of the basic matrices. It is shown that the relativistic invariance requirement provides 144 constraints on blocs and generators, which become identities when taking into account the explicit expressions for all the block matrices. The introduced tetrad matrix equation is specified in cylindrical and spherical coordinates for flat Minkowski space. The case of massless field is separately addressed, and the matrix representation of the gauge symmetry is particularly detailed in this theory.

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1 Introduction

After the study by Pauli and Fierz ([11],[23]), the theory of massive and massless fields with spin 2 has always attracted much attention ([1],[3]-[10],[12]-[16], [19]-[21],[25]-[26]; also see [2], [18]). Most of the studies were performed in the framework of 2-nd order differential equations. It is known that many specific difficulties may be avoided

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if from the very beginning we start with 1-st order systems. Apparently, the first systematic study of the theory of spin 2 fields within the first order formalism was done by F.I. Fedorov [7]. It turns out that this description requires a field function with 3 independent components. This theory was re-discovered and improved by Regee [25]. The first order approach is based from the very beginning on the general theory of relativistic wave equations by the Gel'fand-Yaglom and the Lagrangian formalisms.

In the present paper we develop the theory of the spin 2 field, in both massive and massless variants, starting from the matrix equation in Minkowski space-time and extending it to the generally covariant theory within the Tetrode-Weyl-Fock-Ivanenko tetrad method [22].

2 The spin 2 particle in Minkowski space

We start with the known system of the first order equations for a massive spin 2 particle ([7],[25]):

$$
\begin{gathered}
\partial^a \Phi_a = m\Phi, \quad \frac{1}{2}\partial_a \Phi - \frac{1}{3}\partial^b \Phi_{(ab)} = m\Phi_a, \\
\frac{1}{2}\left(\partial^k \Phi_{[ka]b} + \partial^k \Phi_{[kb]a} - \frac{1}{2} g_{ab} \partial^k \Phi_{[kn]}{}^{n}\right) \\
+ \left(\partial_a \Phi_b + \partial_b \Phi_a - \frac{1}{2} g_{ab} \partial^k \Phi_k\right) = m\Phi_{(ab)}, \\
\partial_a \Phi_{(bc)} - \partial_b \Phi_{(ac)} + \frac{1}{3}\left(g_{bc}\partial^k \Phi_{(ak)} - g_{ac}\partial^k \Phi_{(bk)}\right) = m\Phi_{[ab]c},
\end{gathered}
\tag{2.1}
$$

where the field variables are scalar, vector, symmetric 2-rank tensor, and 3-rank skew-symmetric in two first indices tensor, $m = \mathrm{i}\, M$. By excluding the vector and the 3-rank tensor, we obtain the 2-nd order equations with respect to the scalar and symmetric tensor ([11],[23]):

$$
\Phi = 0, (\Box + M^2)\Phi_{(ab)} = 0, \Phi_{(ab)} = \Phi_{(ba)}, \Phi^a{}_a = 0, \partial^k \Phi_{(ka)} = 0. \tag{2.2}
$$

It should be stressed out that the scalar field entering the systems (2.1) and (2.2) is all-important, because it vanishes only in absence of external fields. Moreover, this scalar variable is of crucial importance for the massless case, when the first order systems read

$$
\begin{gathered}
\partial^a \Phi_a = 0, \frac{1}{2}\partial_a \Phi - \frac{1}{3}\partial^b \Phi_{(ab)} = \Phi_a, \\
\frac{1}{2}\left(\partial^k \Phi_{[ka]b} + \partial^k \Phi_{[kb]a} - \frac{1}{2} g_{ab} \partial^k \Phi_{[kn]}{}^{n}\right) \\
+ \left(\partial_a \Phi_b + \partial_b \Phi_a - \frac{1}{2} g_{ab} \partial^k \Phi_k\right) = 0, \\
\partial_a \Phi_{(bc)} - \partial_b \Phi_{(ac)} + \frac{1}{3}\left(g_{bc}\partial^k \Phi_{(ak)} - g_{ac}\partial^k \Phi_{(bk)}\right) = \Phi_{[ab]c}.
\end{gathered}
\tag{2.3}
$$

By excluding from (2.3) the subsidiary variables, we derive the 2-nd order equations for the massless field ([11],[23]):

$$\begin{gathered}\frac{1}{2}\Box\Phi - \frac{1}{3}\partial^a\partial^b\Phi_{(ab)} = 0,\\ (\partial_a\partial_b + \frac{1}{2}g_{ab}\Box)\Phi - \frac{1}{4}g_{ab}\Box\Phi^n_n + \Box\Phi_{(ab)} - \partial_a\partial^n\Phi_{(nb)} - \partial_b\partial^n\Phi_{(na)} = 0.\end{gathered} \tag{2.4}$$

As it was shown by Pauli and Fierz, these equations have a class of so called *gauge solutions*:

$$\bar{\Phi} = \partial^l L_l, \quad \bar{\Phi}_{(ab)} = \partial_a L_b + \partial_b L_a - \frac{1}{2}g_{ab}\partial^l L_l, \tag{2.5}$$

where $L_l(x)$ stands for an arbitrary 4-vector. These special states do not contribute to physically observable quantities, for instance to the energy-momentum tensor. It is a matter of simple calculation to find the concomitant tensor components:

$$\begin{gathered}\bar{\Phi}_a = \frac{1}{3}\partial_a\partial^l L_l - \frac{1}{3}\Box L_a, \bar{\Phi}_{[ab]c} = \partial_c(\partial_a L_b - \partial_b L_a)\\ -\frac{1}{3}(g_{cb}\partial_a - g_{ca}\partial_b)\partial^l L_l + \frac{1}{3}(g_{cb}\Box L_a - g_{ca}\Box L_b).\end{gathered} \tag{2.6}$$

3 On the structure of the system matrices

The system (2.1) can be re-written in equivalent form

$$\begin{gathered}\partial_a (G^a)_{(0)}{}^k \Phi_k = m\Phi_{(0)},\\ \partial_a \left\{ \frac{1}{2}(\Delta^a)_k{}^{(0)}\Phi_{(0)} - \frac{1}{3}(K^a)_k{}^{(mn)}\Phi_{mn} \right\} = m\ \Phi_k\ ,\\ \partial_a \left\{ \frac{1}{2}\ (B^a)_{(cd)}{}^{[mn]l}\Phi_{mnl} + (\Lambda^a)_{(dc)}{}^k\Phi_k \right\} = m\Phi_{dc}\ ,\\ \partial_a \left\{ (F^a)_{[kb]c}{}^{(mn)}\Phi_{mn} \right\} = m\ \Phi_{kbc},\end{gathered} \tag{3.1}$$

where we use the block matrices

$$\begin{gathered}(G^a)_{(0)}{}^k = g^{ak}\ , \quad (\Delta^a)_k{}^{(0)} = \delta^a_k,\\ (K^a)_k{}^{(mn)} = g^{al}\frac{1}{2}\ \delta^{(mn)}_{(lk)}\ , \quad (\Lambda^a)_{(dc)}{}^k = \delta^{(ak)}_{(dc)} - \frac{1}{2}g_{dc}g^{ak}\ ,\\ (B^a)_{(dc)}{}^{[mn]l} = g^{ak}\left(\frac{1}{2}\delta^{[mn]}_{[kd]}\delta^l_c + \frac{1}{2}\delta^{[mn]}_{[kc]}\delta^l_d\right) - \frac{1}{2}g_{dc}\frac{1}{2}g^{[mn],[al]}\ ,\\ (F^a)_{[kb]c}{}^{(mn)} = \left(\delta^a_k\frac{1}{2}\delta^{(mn)}_{(bc)} - \delta^a_b\frac{1}{2}\delta^{(mn)}_{(kc)}\right) + \frac{1}{3}g^{ad}\left(\frac{1}{2}\delta^{(mn)}_{(dk)}g_{bc} - \frac{1}{2}\delta^{(mn)}_{(db)}g_{kc}\right);\end{gathered} \tag{3.2}$$

the generalized Kronecker symbols are defined by the formulas

$$\begin{gathered}\delta^{(mn)}_{(lk)} = \delta^m_l\ \delta^n_k + \delta^n_l\ \delta^m_k\ , \quad \delta^{[mn]}_{[lk]} = \delta^m_l\ \delta^n_k - \delta^n_l\ \delta^m_k\ ,\\ g^{[mn],[al]} = g^{ma}g^{nl} - g^{ml}g^{na}\ , \quad g^{(mn),(al)} = g^{ma}g^{nl} + g^{ml}g^{na}\ .\end{gathered} \tag{3.3}$$

Further, by introducing the matrices

$$\Gamma^a = \left(\begin{array}{cccc} 0 & G^a & 0 & 0 \\ \frac{1}{2}\Delta^a & 0 & -\frac{1}{3}K^a & 0 \\ 0 & \Lambda^a & 0 & \frac{1}{2}B^a \\ 0 & 0 & F^a & 0 \end{array} \right), \tag{3.4}$$

we re-write the system (3.1) as follows

$$(\Gamma^a \partial_a - m)\,\Psi = 0\,, \quad \Psi = \left(\begin{array}{l} \Phi \\ \Phi_k \\ \Phi_{(mn)} \\ \Phi_{[mn]l} \end{array} \right). \tag{3.5}$$

It is a matter of straightforward calculation to get the explicit expressions for these block matrices. At this we will follow only the independent components of the tensors:

$$\Psi = \{\ \Phi;\ \Phi_l;\ \vec{f},\ \vec{c},\ \vec{d},\ f_0;\ \varphi_0,\ \varphi_1,\ \varphi_2,\ \varphi_3\ \}\,, \quad (\Phi_{(ab)}) = \left(\begin{array}{cccc} f_0 & d_1 & d_2 & d_3 \\ d_1 & f_1 & c_3 & c_2 \\ d_2 & c_3 & f_2 & c_1 \\ d_3 & c_2 & c_1 & f_3 \end{array} \right),$$

while the components of tensor $\Phi_{[mn]k}$ are written as four skew-symmetric matrices:

$$\left(\begin{array}{c} \Phi_{[01]k} \\ \Phi_{[02]k} \\ \Phi_{[03]k} \\ \Phi_{[23]k} \\ \Phi_{[31]k} \\ \Phi_{[12]k} \end{array} \right) = \left(\begin{array}{c} E_{10} \\ E_{20} \\ E_{30} \\ B_{10} \\ B_{20} \\ B_{30} \end{array} \right) = \varphi_0, \quad \left(\begin{array}{c} E_{11} \\ E_{21} \\ E_{31} \\ B_{11} \\ B_{21} \\ B_{31} \end{array} \right) = \varphi_1, \quad \left(\begin{array}{c} E_{12} \\ E_{22} \\ E_{32} \\ B_{12} \\ B_{22} \\ B_{32} \end{array} \right) = \varphi_2, \quad \left(\begin{array}{c} E_{13} \\ E_{23} \\ E_{3k} \\ B_{13} \\ B_{23} \\ B_{33} \end{array} \right) = \varphi_3.$$

For the involved block matrices, we find the explicit expressions, which we include in the Appendix.

4 Extension to Riemannian space-time geometry

The matrix equation

$$(\Gamma^a \frac{\partial}{\partial x^a} - m)\,\Psi(x) = 0, \quad \Psi = \{H; H_1; H_2; H_3\} \tag{4.1}$$

is extended to the Riemannian space-time in accordance with the tetrad method [22]. In a space-time with given metric let, we fix a tetrad:

$$dS^2 = g_{\alpha\beta}(x)dx^\alpha dx^\beta, \quad g_{\alpha\beta}(x) \to e_{(a)\alpha}(x),$$
$$g_{\alpha\beta}(x) = \eta^{ab} e_{(a)\alpha}(x) e_{(b)\beta}(x), \quad \eta^{ab} = \text{diag}(+1,-1,-1,-1), \tag{4.2}$$

and then the generalized form gets written as follows

$$\left[\Gamma^\alpha(x) \left(\frac{\partial}{\partial x^\alpha} + \Sigma_\alpha(x) \right) - m \right] \Psi(x) = 0\,, \tag{4.3}$$

where the local matrices $\Gamma^{\alpha}(x)$ are determined with the use of the tetrad

$$\Gamma^{\alpha}(x) = e^{\alpha}_{(a)}(x)\Gamma^{a} = \begin{pmatrix} 0 & G^{\alpha}(x) & 0 & 0 \\ \frac{1}{2}\Delta^{\alpha}(x) & 0 & -\frac{1}{3}K^{\alpha}(x) & 0 \\ 0 & \Lambda^{\alpha}(x) & 0 & \frac{1}{2}B^{\alpha}(x) \\ 0 & 0 & F^{\alpha}(x) & 0 \end{pmatrix}, \tag{4.4}$$

and connection $\Sigma_{\alpha}(x)$ is defined by relations

$$J^{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & J_1^{ab} & 0 & 0 \\ 0 & 0 & J_2^{ab} & 0 \\ 0 & 0 & 0 & J_3^{ab} \end{pmatrix},$$

$$\Sigma_{\alpha}(x) = J^{ab} e^{\beta}_{(a)}(x)\, e_{(b)\beta;\,\alpha}(x) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & (\Sigma_1)_{\alpha} & 0 & 0 \\ 0 & 0 & (\Sigma_2)_{\alpha} & 0 \\ 0 & 0 & 0 & (\Sigma_3)_{\alpha} \end{pmatrix}, \tag{4.5}$$

where

$$\Sigma_i(x) = J_i^{ab}\, e^{\beta}_{(a)}(x)\, e_{(b)\beta;\,\alpha}(x)\,, \quad i = 1, 2, 3;$$

and $J_1^{ab}, J_2^{ab}, J_3^{ab}$ stand for the generators for the tensors $\Phi_k, \Phi_{(mn)}, \Phi_{[mn]l}$. The equation (4.3) can be presented by using the Ricci rotation coefficients

$$\left[\Gamma^{c}\left(e^{\alpha}_{(c)}(x)\frac{\partial}{\partial x^{\alpha}} + \frac{1}{2}J^{ab}\gamma_{abc}\right) - m\right]\Psi(x) = 0\,; \tag{4.6}$$

while we recall the definition, $\gamma_{[ab]c} = -\gamma_{[ba]c} = e_{(b)\rho;\,\sigma}e^{\rho}_{(a)}e^{\sigma}_{(c)}$. In block form, (4.3) reads

$$\begin{gathered} G^{\alpha}(x)[\partial_{\alpha} + (\Sigma_1)_{\alpha}]H_1 = mH\,, \\ \frac{1}{2}\Delta^{\alpha}(x)\partial_{\alpha}H - \frac{1}{3}K^{\alpha}(x)[\partial_{\alpha} + (\Sigma_2)_{\alpha}]H_2 = mH_1\,, \\ \Lambda^{\alpha}(x)[\partial_{\alpha} + (\Sigma_1)_{\alpha}]H_1 + \frac{1}{2}[\partial_{\alpha} + (\Sigma_3)_{\alpha}]H_3 = mH_2\,, \\ F^{\alpha}(x)[\partial_{\alpha} + (\Sigma_2)_{\alpha}]H_2 = mH_3\,. \end{gathered} \tag{4.7}$$

In the massless case, the system slightly changes:

$$\begin{gathered} G^{\alpha}(x)[\partial_{\alpha} + (\Sigma_1)_{\alpha}]H_1 = 0\,, \\ \frac{1}{2}\Delta^{\alpha}(x)\partial_{\alpha}H - \frac{1}{3}K^{\alpha}(x)[\partial_{\alpha} + (\Sigma_2)_{\alpha}]H_2 = H_1\,, \\ \Lambda^{\alpha}(x)[\partial_{\alpha} + (\Sigma_1)_{\alpha}]H_1 + \frac{1}{2}[\partial_{\alpha} + (\Sigma_3)_{\alpha}]H_3 = 0\,, \\ F^{\alpha}(x)[\partial_{\alpha} + (\Sigma_2)_{\alpha}]H_2 = H_3\,, \end{gathered} \tag{4.8}$$

but its physical content is completely different. In particular, let us detail tetrad representation for the gauge solutions:

$$\bar{\Phi} = \nabla_{\alpha}L^{\alpha}(x) \quad \Longrightarrow \quad \bar{\Phi} = e^{(c)\alpha}\partial_{\alpha}L_{(c)} + e^{\alpha}_{(c);\alpha}L^{(c)}\,, \tag{4.9}$$

$$\bar{\Phi}_{(\alpha\beta)} = \nabla_\alpha L_\beta + \nabla_\beta L_\alpha - \frac{1}{2} g_{\alpha\beta}(x) \nabla_\rho \Lambda^\rho \quad \Longrightarrow$$
$$\bar{\Phi}_{(ab)} = -\left(\gamma_{[ca]b} + \gamma_{[cb]a}\right) L^{(c)} + e^\alpha_{(a)} \partial_\alpha \Lambda_{(b)} + e^\alpha_{(b)} \partial_\alpha \Lambda_{(a)} - \frac{1}{2} g_{ab} \bar{\Phi} \,. \tag{4.10}$$

The concomitant components are determined by the relations

$$\bar{H}_1 = \frac{1}{2} \Delta^\alpha(x) \partial_\alpha \bar{H} - \frac{1}{3} K^\alpha(x) [\partial_\alpha + (\Sigma_2)_\alpha] \bar{H}_2 \,,$$
$$\bar{H}_3 = F^\alpha(x) \left[\partial_\alpha + (\Sigma_2)_\alpha\right] \bar{H}_2 \,. \tag{4.11}$$

The generalized matrix equation possess an important structural property: it is symmetric under the local Lorentz group, in accordance with the following relations

$$\Psi'(x) = S(x) \Psi(x), \quad S(x)\, \Gamma^\alpha(x) S^{-1}(x) = \Gamma'^{\alpha}(x) \,,$$
$$S(x)\, \Sigma_\alpha(x)\, S^{-1}(x) + S(x) \, \frac{\partial}{\partial x^\alpha} \, S^{-1}(x) = \Sigma'_\alpha \,, \tag{4.12}$$

where the primed indicate that these quantities are determined by the formulas (4.4)-(4.5), but with the use of the primed tetrad related to initial one by the local Lorentz transformation

$$e^\sigma_{(a')}(x) = L_a{}^b(x)\, e^\sigma_{(b)}(x) \,. \tag{4.13}$$

This symmetry proves the correctness of the constructed equation, because for a given metric one can use many different tetrads, and all the corresponding equations are equivalent due to relations (4.12). With respect to the coordinate transformation, the field function Ψ behaves as a scalar,

$$x^\alpha \to x'^{\alpha}, \quad \Psi(x) = \Psi'(x') \,. \tag{4.14}$$

5 The spin 2 field in cylindrical coordinates

Let us specify the main equation in cylindrical coordinates $x^\alpha = (t, r, \phi, z)$ of the Minkowski space

$$dS^2 = dt^2 - dr^2 - r^2 d\phi^2 - dz^2, \quad e^\beta_{(a)}(x) = \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/r & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) . \tag{5.1}$$

The non-vanishing Ricci rotation coefficients are

$$\gamma_{ab0} = 0 \,, \quad \gamma_{ab1} = 0 \,, \quad \gamma_{122} = -\gamma_{212} = +\frac{1}{r} \,, \quad \gamma_{ab3} = 0 \,. \tag{5.2}$$

Equation (4.3) takes the form

$$\left[\Gamma^0 \frac{\partial}{\partial t} + \Gamma^1 \frac{\partial}{\partial r} + \frac{\Gamma^2}{r} (\frac{\partial}{\partial \phi} + J^{12}) + \Gamma^3 \frac{\partial}{\partial z} - m \right] \Psi = 0 \,. \tag{5.3}$$

Taking in mind the expressions for separate terms:

$$\Psi = \begin{pmatrix} H \\ H_1 \\ H_2 \\ H_3 \end{pmatrix} \begin{matrix} 1 \\ 4 \\ 10 \\ 24 \end{matrix} , \quad \Gamma^0 \partial_t \Psi = \begin{pmatrix} G^0 \partial_t H_1 \\ \frac{1}{2}\Delta^0 \partial_t H - \frac{1}{3} K^0 \partial_t H_2| \\ \Lambda^0 \partial_t H_1 + \frac{1}{2} B^0 \partial_t H_3 \\ F^0 \partial_t H_2 \end{pmatrix},$$

$$\Gamma^1 \frac{\partial}{\partial r} \Psi = \begin{pmatrix} G^1 \partial_r H_1 \\ \frac{1}{2}\Delta^1 \partial_r H - \frac{1}{3} K^1 \partial_r H_2 \\ \Lambda^1 \partial_r H_1 + \frac{1}{2} B^1 \partial_r H_3 \\ F^1 \partial_r H_2 \end{pmatrix}, \Gamma^3 \partial_z \Psi = \begin{pmatrix} G^3 \partial_z H_1 \\ \frac{1}{2}\Delta^3 \partial_z H - \frac{1}{3} K^3 \partial_x H_2| \\ \Lambda^3 \partial_z H_1 + \frac{1}{2} B^3 \partial_z H_3 \\ F^3 \partial_z H_2 \end{pmatrix},$$

$$\frac{\Gamma^2}{r}(\partial_\phi + J^{12})\Psi = \frac{1}{r} \begin{pmatrix} G^2(\partial_\phi + J_1^{12}) H_1 \\ \frac{1}{2}\Delta^2 \partial_\phi H - \frac{1}{3} K^2 (\partial_\phi + J_2^{12}) H_2 \\ \Lambda^2(\partial_\phi + J_1^{12}) H_1 + \frac{1}{2} B^2 (\partial_\phi + J_3^{12}) H_3 \\ F^2 (\partial_\phi + J_2^{12}) H_2 \end{pmatrix},$$

we derive the system of equations in block form

$$G^0 \partial_t H_1 + G^1 \partial_r H_1 + \frac{1}{r} G^2 (\partial_\phi + J_1^{12}) H_1 + G^3 \partial_z H_1 = mH \,,$$

$$\frac{1}{2}\Delta^0 \partial_t H - \frac{1}{3} K^0 \partial_t H_2 + \frac{1}{2}\Delta^1 \partial_r H - \frac{1}{3} K^1 \partial_r H_2$$

$$+\frac{1}{r}\left[\frac{1}{2}\Delta^2 \partial_\phi H - \frac{1}{3} K^2 (\partial_\phi + J_2^{12}) H_2\right] + \left[\frac{1}{2}\Delta^3 \partial_z H - \frac{1}{3} K^3 \partial_x H_2\right] = mH_1 \,,$$

$$\Lambda^0 \partial_t H_1 + \frac{1}{2} B^0 \partial_t H_3 + \Lambda^1 \partial_r H_1 + \frac{1}{2} B^1 \partial_r H_3$$

$$+\frac{1}{r}\left[\Lambda^2 (\partial_\phi + J_1^{12}) H_1 + \frac{1}{2} B^2 (\partial_\phi + J_3^{12}) H_3\right] + \left[\Lambda^3 \partial_z H_1 + \frac{1}{2} B^3 \partial_z H_3\right] = mH_2 \,,$$

$$F^0 \partial_t H_2 + F^1 \partial_r H_2 + \frac{1}{r} F^2 (\partial_\phi + J_2^{12}) H_2 + F^3 \partial_z H_2 = mH_3 \,.$$

The transition in these equations to the massless case is evident. Let us detail the tetrad representation for gauge solutions:

$$\bar{\Phi} = \partial_t L_{(0)} - (\partial_r + \frac{1}{r}) L_{(1)} - \frac{1}{r} \partial_\phi L_{(2)} - \partial_z L_{(3)};$$

$$\bar{\Phi}_{(ab)} = -\left(\gamma_{[ca]b} + \gamma_{[cb]a}\right) L^{(c)} + e^\alpha_{(a)} \partial_\alpha L_{(b)} + e^\alpha_{(b)} \partial_\alpha L_{(a)} - \frac{1}{2} g_{ab} \bar{\Phi},$$

whence it follows

$$\bar{\Phi}_{00} = 2\partial_t L_{(0)} - \frac{1}{2}[\partial_t L_{(0)} - (\partial_r + \frac{1}{r}) L_{(1)} - \frac{1}{r} \partial_\phi L_{(2)} - \partial_z L_{(3)}],$$

$$\bar{\Phi}_{11} = 2\partial_r L_{(1)} + \frac{1}{2}[\partial_t L_{(0)} - (\partial_r + \frac{1}{r}) L_{(1)} - \frac{1}{r} \partial_\phi L_{(2)} - \partial_z L_{(3)}],$$

$$\bar{\Phi}_{22} = \frac{2}{r} L_{(1)} + \frac{2}{r} \partial_\phi L_{(2)} + \frac{1}{2}[\partial_t L_{(0)} - (\partial_r + \frac{1}{r}) L_{(1)} - \frac{1}{r} \partial_\phi L_{(2)} - \partial_z L_{(3)}],$$

$$\bar{\Phi}_{33} = 2\partial_z L_{(3)} + \frac{1}{2}[\partial_t L_{(0)} - (\partial_r + \frac{1}{r})L_{(1)} - \frac{1}{r}\partial_\phi L_{(2)} - \partial_z L_{(3)}],$$

$$\bar{\Phi}_{01} = \partial_t L_{(1)} + \partial_r L_{(0)}, \bar{\Phi}_{02} = \partial_t L_{(2)} + \frac{1}{r}\partial_\phi L_{(0)}, \bar{\Phi}_{03} = \partial_t L_{(3)} + \partial_z L_{(0)},$$

$$\bar{\Phi}_{23} = \frac{1}{r}\partial_\phi L_{(3)} + \partial_z L_{(2)}, \bar{\Phi}_{31} = \partial_z L_{(1)} + \partial_r L_{(3)}, \bar{\Phi}_{12} = -\frac{1}{r}L_{(2)} + \partial_r L_{(2)} + \frac{1}{r}\partial_\phi L_{(1)}.$$

The concomitant gauge components are determined by the formulas

$$\bar{H}_1 = \frac{1}{2}\Delta^\alpha(x)\partial_\alpha \bar{H} - \frac{1}{3}K^\alpha(x)[\partial_\alpha + (\Sigma_2)_\alpha]\bar{H}_2$$

$$= \frac{1}{2}\left[\Delta^0\partial_t + \Delta^1\partial_3 + \Delta^2\frac{\partial_\phi}{r} + \Delta^3\partial_z\right]\bar{H} - \frac{1}{3}\left[K^0\partial_t + K^1\partial_r + K^2\frac{\partial_\phi + J_2^{12}}{r} + K^3\partial_z\right]\bar{H}_2,$$

$$\bar{H}_3 = F^\alpha(x)[\partial_\alpha + (\Sigma_2)_\alpha]\bar{H}_2 = \left[F^0\partial_t + F^1\partial_r + F^2\frac{\partial_t + J_2^{12}}{r} + F^3\partial_z\right]\bar{H}_2\,.$$

6 The equation in spherical coordinates

Let us consider the structure of the matrix equation in spherical coordinates, $x^\alpha = (t, r, \theta, \phi)$:

$$dS^2 = dt^2 - dr^2 - r^2 d\theta^2 - r^2\sin^2\theta d\phi^2, e^\alpha_{(0)} = (1,0,0,0),$$
(6.1)
$$e^\alpha_{(3)} = (0,1,0,0), e^\alpha_{(1)} = (0,0,\frac{1}{r},0), e^\alpha_{(2)} = (1,0,0,\frac{1}{r\sin\theta}).$$

The corresponding Ricci rotation coefficients are

$$\gamma_{ab0} = 0, \gamma_{ab3} = 0, \gamma_{ab1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{r} \\ 0 & 0 & 0 & 0 \\ 0 & +\frac{1}{r} & 0 & 0 \end{pmatrix}, \gamma_{ab2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +\frac{\cos\theta}{r\sin\theta} & 0 \\ 0 & -\frac{\cos\theta}{r\sin\theta} & 0 & -\frac{1}{r} \\ 0 & 0 & +\frac{1}{r} & 0 \end{pmatrix}.$$

The main equation takes on the form

$$\left\{\Gamma^0\partial_t + \left(\Gamma^3\partial_r + \frac{\Gamma^1 J^{31} + \Gamma^2 J^{32}}{r}\right) + \frac{1}{r}\Sigma_{\theta,\phi} - m\right\}\Psi = 0\,, \tag{6.2}$$

where we separate the angular operator

$$\Sigma_{\theta,\phi} = \left(\Gamma^1\partial_\theta + \Gamma^2\frac{\partial_\phi + J^{12}\cos\theta}{\sin\theta}\right)\,. \tag{6.3}$$

Using the appropriate expressions for all the involved terms

$$\Gamma^0\partial_t\Psi = \begin{pmatrix} G^0\partial_t H_1 \\ \frac{1}{2}\Delta^0\partial_t H - \frac{1}{3}K^0\partial_t H_2| \\ \Lambda^0\partial_t H_1 + \frac{1}{2}B^0\partial_t H_3 \\ F^0\partial_t H_2 \end{pmatrix}, \quad \Gamma^3\partial_r\Psi = \begin{pmatrix} G^3\partial_r H_1 \\ \frac{1}{2}\Delta^3\partial_r H - \frac{1}{3}K^3\partial_r H_2 \\ \Lambda^3\partial_r H_1 + \frac{1}{2}B^3\partial_r H_3 \\ F^3\partial_r H_2 \end{pmatrix},$$

$$\frac{1}{r}(\Gamma^1 J^{31} + \Gamma^2 J^{32})\Psi = \frac{1}{r}\left(\begin{array}{c} (G^1 J_1^{31} + G^2 J_1^{32})H_1 \\ \frac{1}{2}(\Delta^1 + \Delta^2)H - \frac{1}{3}(K^1 J_2^{31} + K^2 J_2^{32})H_2 \\ (\Lambda^1 J_1^{31} + \Lambda^2 J_1^{32})H_1 + \frac{1}{2}(B^1 J_3^{31} + B^2 J_3^{32})H_3 \\ (F^1 J_2^{31} + F^2 J_2^{32})H_2 \end{array}\right),$$

$$\Sigma_{\theta\phi}\Psi = \left(\begin{array}{c} G^1\partial_\theta H_1 + G^2\frac{\partial_\phi + \cos\theta J_1^{12}}{\sin\theta}H_1 \\ \frac{1}{2}\Delta^1\partial_\theta H + \frac{1}{2}\Delta^2\frac{\partial_\phi + \cos\theta}{\sin\theta}H - \frac{1}{3}K^1\partial_\theta H_2 - \frac{1}{3}K^2\frac{\partial_\phi + \cos\theta J_2^{12}}{\sin\theta}H_2 \\ \Lambda^1\partial_\theta H_1 + \Lambda^2\frac{\partial_\phi + \cos\theta J_1^{12}}{\sin\theta}H_1 + \frac{1}{2}B^1\partial_\theta H_3 + \frac{1}{2}B^2\frac{\partial_\phi + \cos\theta J_3^{12}}{\sin\theta}H_3 \\ F^1\partial_\theta H_2 + F^2\frac{\partial_\phi + \cos\theta J_2^{12}}{\sin\theta}H_2 \end{array}\right),$$

we obtain the system of equations in block form

$$G^0\frac{\partial}{\partial t}H_1 + G^3\frac{\partial}{\partial r}H_1 + \frac{G^1 J_1^{31} + G^2 J_1^{32}}{r}H_1$$
$$+\frac{1}{r}\left(G^1\partial_\theta + G^2\frac{\partial_\phi + \cos\theta J_1^{12}}{\sin\theta}\right)H_1 = mH\,,$$

$$\frac{1}{2}\Delta^0\frac{\partial}{\partial t}H - \frac{1}{3}K^0\frac{\partial}{\partial t}H_2$$
$$+\frac{1}{2}\left(\Delta^3\frac{\partial}{\partial r} + \frac{\Delta^1 + \Delta^2}{r}\right)H - \frac{1}{3}\left(K^3\frac{\partial}{\partial r} + \frac{K^1 J_2^{31} + K^2 J_2^{32}}{r}\right)H_2$$
$$+\frac{1}{r}\left[\frac{1}{2}\left(\Delta^1\partial_\theta + \Delta^2\frac{\partial_\phi + \cos\theta}{\sin\theta}\right)H - \frac{1}{3}\left(K^1\partial_\theta + K^2\frac{\partial_\phi + \cos\theta J_2^{12}}{\sin\theta}\right)H_2\right] = mH_1\,,$$

$$\Lambda^0\frac{\partial}{\partial t}H_1 + \frac{1}{2}B^0\frac{\partial}{\partial t}H_3 +$$
$$+\left(\Lambda^3\frac{\partial}{\partial r} + \frac{\Lambda^1 J_1^{31} + \Lambda^2 J_1^{32}}{r}\right)H_1 + \frac{1}{2}\left(B^3\frac{\partial}{\partial r} + \frac{B^1 J_3^{31} + B^2 J_3^{32}}{r}\right)H_3 +$$
$$+\frac{1}{r}\left[\left(\Lambda^1\partial_\theta + \Lambda^2\frac{\partial_\phi + \cos\theta J_1^{12}}{\sin\theta}\right)H_1 + \frac{1}{2}\left(B^1\partial_\theta + B^2\frac{\partial_\phi + \cos\theta J_3^{12}}{\sin\theta}\right)H_3\right] = mH_2\,,$$

$$F^0\frac{\partial}{\partial t}H_2 + F^3\frac{\partial}{\partial r}H_2 + \frac{F^1 J_2^{31} + F^2 J_2^{32}}{r}H_2 + \frac{1}{r}\left(F^1\partial_\theta + F^2\frac{\partial_\phi + \cos\theta J_2^{12}}{\sin\theta}\right)H_2 = mH_3\,.$$

The transition in these equations to the massless case is evident. Let us detail the tetrad representation for the gauge solution:

$$\bar{\Phi} = \left[\partial_t L_{(0)} - (\partial_r + \frac{2}{r})L_{(3)} - \frac{1}{r}(\partial_\theta + \frac{\cos\theta}{\sin\theta})L_{(1)} - \frac{\partial_\phi}{r\sin\theta}L_{(2)}\right]; \tag{6.4}$$

$$\bar{\Phi}_{(ab)} = -\left(\gamma_{[ca]b} + \gamma_{[cb]a}\right)L^{(c)} + \left[e^\alpha_{(a)}\partial_\alpha L_{(b)} + e^\alpha_{(b)}\partial_\alpha L_{(a)}\right] - \frac{1}{2}g_{ab}\bar{\Phi}\,,$$

whence it follows

$$\bar{\Phi}_{00} = 2\partial_t L_{(0)} - \frac{1}{2}\left[\partial_t L_{(0)} - (\partial_r + \frac{2}{r})L_{(3)} - \frac{1}{r}(\partial_\theta + \frac{\cos\theta}{\sin\theta})L_{(1)} - \frac{\partial_\phi}{r\sin\theta}L_{(2)}\right],$$

$$\bar{\Phi}_{33} = 2\partial_r L_{(3)} + \frac{1}{2}\left[\partial_t L_{(0)} - (\partial_r + \frac{2}{r})L_{(3)} - \frac{1}{r}(\partial_\theta + \frac{\cos\theta}{\sin\theta})L_{(1)} - \frac{\partial_\phi}{r\sin\theta}L_{(2)}\right],$$

$$\bar{\Phi}_{11} = \frac{2}{r}L_{(3)} + \frac{2}{r}\partial_\theta L_{(1)} + \frac{1}{2}\left[\partial_t L_{(0)} - (\partial_r + \frac{2}{r})L_{(3)} - \frac{1}{r}(\partial_\theta + \frac{\cos\theta}{\sin\theta})L_{(1)} - \frac{\partial_\phi}{r\sin\theta}L_{(2)}\right],$$

$$\bar{\Phi}_{22} = 2\frac{\cos\theta}{r\sin\theta}L_{(1)} + \frac{2}{r}L_{(3)} + 2\frac{\partial_\phi}{r\sin\theta}L_{(2)}$$

$$+\frac{1}{2}\left[\partial_t L_{(0)} - (\partial_r + \frac{2}{r})L_{(3)} - \frac{1}{r}(\partial_\theta + \frac{\cos\theta}{\sin\theta})L_{(1)} - \frac{\partial_\phi}{r\sin\theta}L_{(2)}\right],$$

$$\bar{\Phi}_{01} = \partial_t L_{(1)} + \frac{1}{r}\partial_\theta L_{(0)}, \bar{\Phi}_{02} = \partial_t L_{(2)} + \frac{1}{r\sin\theta}\partial_\phi L_{(0)},$$

$$\bar{\Phi}_{03} = \partial_t L_{(3)} + \partial_r L_{(0)}, \bar{\Phi}_{23} = \frac{\partial_\phi}{r\sin\theta}L_{(3)} + \partial_r L_{(2)},$$

$$\bar{\Phi}_{31} = -\frac{1}{r}L_{(1)} + \partial_r L_{(1)} + \frac{1}{r}\partial_\theta L_{(3)}, \quad \bar{\Phi}_{12} = -\frac{\cos\theta}{r\sin\theta}L_{(2)} + \frac{1}{r}\partial_\theta L_{(2)} + \frac{1}{r\sin\theta}L_{(1)}.$$

7 The structure of the Lorentzian generators

To handle the matrix equation, we need the explicit expressions for the involved Lorentzian generators. For the 4-vector, the generators are given by the formulas

(7.1)
$$j^{12} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, j^{31} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, j^{23} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$j^{01} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, j^{02} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, j^{03} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

The generators for the symmetric tensor are defined as

$$\Phi' = (j^{ab} \otimes I + I \otimes j^{ab})\,\Phi = j^{ab}\Phi + \Phi\tilde{j}^{ab}, \quad (\Phi_{(ab)}) = \begin{pmatrix} f_0 & d_1 & d_2 & d_3 \\ d_1 & f_1 & c_3 & c_2 \\ d_2 & c_3 & f_2 & c_1 \\ d_3 & c_2 & c_1 & f_3 \end{pmatrix}.$$

Further, we derive its (10×10)-representation:

$$J_2^{01} = \begin{pmatrix} . & . & . & . & . & . & 2 & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & 1 & . \\ . & . & . & . & . & . & . & 1 & . & . \\ 1 & . & . & . & . & . & . & . & . & 1 \\ . & . & . & . & . & 1 & . & . & . & . \\ . & . & . & . & 1 & . & . & . & . & . \\ . & . & . & . & . & . & 2 & . & . & . \end{pmatrix}, J_2^{02} = \begin{pmatrix} . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & 2 & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & 1 & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & 1 & . & . & . \\ . & . & . & . & . & 1 & . & . & . & . \\ . & 1 & . & . & . & . & . & . & . & 1 \\ . & . & . & 1 & . & . & . & . & . & . \\ . & . & . & . & . & . & . & 2 & . & . \end{pmatrix},$$

$$
J_2^{03} = \begin{pmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 2 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 2 & \cdot
\end{pmatrix}, \quad
J_2^{23} = \begin{pmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{pmatrix},
$$

$$
J_2^{31} = \begin{pmatrix}
\cdot & \cdot & \cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -2 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot \\
-1 & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{pmatrix},
$$

$$
J_2^{12} = \begin{pmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & -2 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{pmatrix}.
$$

The 3-rank tensor may be presented by means of four skew-symmetric matrices $\Phi_{[ps]t} = \varphi_t,\ t = 0, 1, 2, 3$; so the generators act in accordance with the formula

$$
\varphi'_l = j^{ab}\varphi_l + \varphi_l \tilde{j}^{ab} + (j^{ab})_l{}^t \varphi_t \,. \tag{7.2}
$$

Taking into account the intermediate relations

$$
ab = 12, \quad
\begin{aligned}
\varphi'_0 &= j^{12}\varphi_0 + \varphi_0 \tilde{j}^{12} \,, \\
\varphi'_1 &= j^{12}\varphi_1 + \varphi_1 \tilde{j}^{12} - \varphi_2 \,, \\
\varphi'_2 &= j^{12}\varphi_2 + \varphi_2 \tilde{j}^{12} + \varphi_1 \,, \\
\varphi'_3 &= j^{12}\varphi_3 + \varphi_3 \tilde{j}^{12} \,;
\end{aligned}
\qquad
ab = 31, \quad
\begin{aligned}
\varphi'_0 &= j^{31}\varphi_0 + \varphi_0 \tilde{j}^{31} \,, \\
\varphi'_1 &= j^{31}\varphi_1 + \varphi_1 \tilde{j}^{31} + \varphi_3 \,, \\
\varphi'_2 &= j^{31}\varphi_2 + \varphi_2 \tilde{j}^{31} \,, \\
\varphi'_3 &= j^{31}\varphi_3 + \varphi_3 \tilde{j}^{31} - \varphi_1 \,;
\end{aligned}
$$

$$
ab = 23, \quad
\begin{aligned}
\varphi'_0 &= j^{23}\varphi_0 + \varphi_0 \tilde{j}^{23} \,, \\
\varphi'_1 &= j^{23}\varphi_1 + \varphi_1 \tilde{j}^{23} \,, \\
\varphi'_2 &= j^{23}\varphi_2 + \varphi_2 \tilde{j}^{23} - \varphi_3 \,, \\
\varphi'_3 &= j^{23}\varphi_3 + \varphi_3 \tilde{j}^{23} + \varphi_2 \,;
\end{aligned}
\qquad
ab = 01, \quad
\begin{aligned}
\varphi'_0 &= j^{01}\varphi_0 + \varphi_0 \tilde{j}^{01} + \varphi_1 \,, \\
\varphi'_1 &= j^{01}\varphi_1 + \varphi_1 \tilde{j}^{01} + \varphi_0 \,, \\
\varphi'_2 &= j^{01}\varphi_2 + \varphi_2 \tilde{j}^{01} \,, \\
\varphi'_3 &= j^{01}\varphi_3 + \varphi_3 \tilde{j}^{01} \,;
\end{aligned}
$$

$$ab = 02, \quad \begin{aligned} \varphi_0' &= j^{02}\varphi_0 + \varphi_0\tilde{j}^{02} + \varphi_2\,, \\ \varphi_1' &= j^{02}\varphi_1 + \varphi_1\tilde{j}^{02}\,, \\ \varphi_2' &= j^{02}\varphi_2 + \varphi_2\tilde{j}^{02} + \varphi_0\,, \\ \varphi_3' &= j^{02}\varphi_3 + \varphi_3\tilde{j}^{02}\,; \end{aligned} \qquad ab = 03, \quad \begin{aligned} \varphi_0' &= j^{03}\varphi_0 + \varphi_0\tilde{j}^{03} + \varphi_3\,, \\ \varphi_1' &= j^{03}\varphi_1 + \varphi_1\tilde{j}^{03}\,, \\ \varphi_2' &= j^{03}\varphi_2 + \varphi_2\tilde{j}^{03}\,, \\ \varphi_3' &= j^{03}\varphi_3 + \varphi_3\tilde{j}^{03} + \varphi_0\,. \end{aligned}$$

with

$$\varphi_c = \left(\begin{array}{cccc} 0 & E_{1c} & E_{2c} & E_{3c} \\ -E_{1c} & 0 & B_{3c} & -B_{2c} \\ -E_{2c} & -B_{3c} & 0 & B_{1c} \\ -E_{3c} & B_{2c} & -B_{1c} & 0 \end{array} \right), \quad c = 0, 1, 2, 3\,,$$

we can find the explicit form of six (24×24) generators; because they are rather cumbersome we describe them in the Appendix.

8 Relativistic invariance, additional checking

In order to verify expressions for all blocks in the matrices

$$(\Gamma^a\,\partial_a - m)\,\Phi = 0\,, \quad \Gamma^a = \left(\begin{array}{cccc} 0 & G^a & 0 & 0 \\ \frac{1}{2}\Delta^a & 0 & -\frac{1}{3}K^a & 0 \\ 0 & \Lambda^a & 0 & \frac{1}{2}B^a \\ 0 & 0 & F^a & 0 \end{array} \right),$$

and the expressions for the Loretzian generators, let us examine the relativistic invariance condition

$$\Psi' = S\Psi, \quad \Psi = S^{-1}\Psi', \quad (S\Gamma^a S^{-1}\partial_a - m)\,\Phi' = 0\,;$$

whence we derive

$$(\Gamma^b\,\partial_b' - m)\,\Phi' = 0, \quad S\Gamma^a S^{-1} = \Gamma^b L_b{}^{\;a}\,.$$

By specifying this for the infinitesimal transformations

$$(1 + \omega_{mn}J^{mn})\Gamma^a(1 - \omega_{kl}J^{kl}) = \Gamma^b\,[\delta_b^a + \omega_{ps}(j^{ps})_b{}^{a}]\,,$$

we derive the needed relation

$$J^{mn}\Gamma^a - \Gamma^a J^{mn} = \Gamma^b\,(j^{mn})_b{}^{a}\,. \tag{8.1}$$

In detailed form, this reads (the indices $1, 2, 3$, related to the tensors of 1, 2, and 3-rd ranks are omitted here):

$$\begin{array}{lll} J^{23}\Gamma^0 - \Gamma^0 J^{23} = 0, & J^{31}\Gamma^0 - \Gamma^0 J^{31} = 0, & J^{12}\Gamma^0 - \Gamma^0 J^{12} = 0, \\ J^{23}\Gamma^1 - \Gamma^1 J^{23} = 0, & J^{31}\Gamma^1 - \Gamma^1 J^{31} = -\Gamma^3, & J^{12}\Gamma^1 - \Gamma^1 J^{12} = \Gamma^2, \\ J^{23}\Gamma^2 - \Gamma^2 J^{23} = \Gamma^3, & J^{31}\Gamma^2 - \Gamma^2 J^{31} = 0, & J^{12}\Gamma^2 - \Gamma^2 J^{12} = \Gamma^1, \\ J^{23}\Gamma^3 - \Gamma^3 J^{23} = -\Gamma^2, & J^{31}\Gamma^3 - \Gamma^3 J^{31} = \Gamma^1, & J^{12}\Gamma^3 - \Gamma^3 J^{12} = 0, \end{array}$$

$$\begin{array}{lll} J^{01}\Gamma^0 - \Gamma^0 J^{01} = \Gamma_1, & J^{02}\Gamma^0 - \Gamma^0 J^{02} = \Gamma_2, & J^{03}\Gamma^0 - \Gamma^0 J^{03} = \Gamma_3, \\ J^{01}\Gamma^1 - \Gamma^1 J^{01} = \Gamma_0, & J^{02}\Gamma^1 - \Gamma^1 J^{02} = 0, & J^{03}\Gamma^1 - \Gamma^1 J^{03} = 0, \\ J^{01}\Gamma^2 - \Gamma^2 J^{01} = 0, & J^{02}\Gamma^2 - \Gamma^2 J^{02} = \Gamma_0, & J^{03}\Gamma^2 - \Gamma^2 J^{03} = 0, \\ J^{01}\Gamma^3 - \Gamma^3 J^{01} = 0, & J^{02}\Gamma^3 - \Gamma^3 J^{02} = 0, & J^{03}\Gamma^3 - \Gamma^3 J^{03} = \Gamma_0. \end{array}$$

By considering the block structure of generators J^{mn} and matrices Γ^a:

$$J^{mn} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & J_1^{mn} & 0 & 0 \\ 0 & 0 & J_2^{mn} & 0 \\ 0 & 0 & 0 & J_3^{mn} \end{pmatrix}, \quad \Gamma^a = \begin{pmatrix} 0 & G^a & 0 & 0 \\ \frac{1}{2}\Delta^a & 0 & -\frac{1}{3}K^a & 0 \\ 0 & \Lambda^a & 0 & \frac{1}{2}B^a \\ 0 & 0 & F^a & 0 \end{pmatrix},$$

we derive 144 constraints (which are divided into six groups): $mn = 23$,

$$-G^0 J_1^{23} = 0, \quad -G^1 J_1^{23} = 0, \quad -G^2 J_1^{23} = G^3, \quad -G^3 J_1^{23} = -G^2,$$
$$J_1^{23}\Delta^0 = 0, \quad J_1^{23}\Delta^1 = 0, \quad J_1^{23}\Delta^2 = \Delta^3, \quad J_1^{23}\Delta^3 = -\Delta^2,$$
$$-J_1^{23}K^0 + K^0 J_2^{23} = 0, -J_1^{23}K^1 + K^1 J_2^{23} = 0, -J_1^{23}K^2 + K^2 J_2^{23} = -K^3, -J_1^{23}K^3 + K^3 J_2^{23} = K^2,$$
$$J_2^{23}\Lambda^0 - \Lambda^0 J_1^{23} = 0, J_2^{23}\Lambda^1 - \Lambda^1 J_1^{23} = 0, J_2^{23}\Lambda^2 - \Lambda^2 J_1^{23} = \Lambda^3, J_2^{23}\Lambda^3 - \Lambda^3 J_1^{23} = -\Lambda^2 g^{33},$$
$$J_2^{23}B^0 - B^0 J_3^{23} = 0, J_2^{23}B^1 - B^1 J_3^{23} = 0, J_2^{23}B^2 - B^2 J_3^{23} = B^3, J_2^{23}B^3 - B^3 J_3^{23} = -B^2,$$
$$J_3^{23}F^0 - F^0 J_2^{23} = 0, J_3^{23}F^1 - F^1 J_2^{23} = 0, J_3^{23}F^2 - F^2 J_2^{23} = F^3, J_3^{23}F^3 - F^3 J_2^{23} = -F^2;$$

$mn = 31$,

$$-G^0 J_1^{31} = 0, \quad -G^1 J_1^{31} = -G^3, \quad -G^2 J_1^{31} = 0, \quad -G^3 J_1^{31} = G^1,$$
$$J_1^{31}\Delta^0 = 0, \quad J_1^{31}\Delta^1 = -\Delta^3, \quad J_1^{31}\Delta^2 = 0, \quad J_1^{31}\Delta^3 = \Delta^1,$$
$$-J_1^{31}K^0 + K^0 J_2^{31} = 0, -J_1^{31}K^1 + K^1 J_2^{31} = K^3, -J_1^{31}K^2 + K^2 J_2^{31} = 0, -J_1^{31}K^3 + K^3 J_2^{31} = -K^1,$$
$$J_2^{31}\Lambda^0 - \Lambda^0 J_1^{31} = 0, J_2^{31}\Lambda^1 - \Lambda^1 J_1^{31} = -\Lambda^3, J_2^{31}\Lambda^2 - \Lambda^2 J_1^{31} = 0, J_2^{31}\Lambda^3 - \Lambda^3 J_1^{31} = \Lambda^1,$$
$$J_2^{31}B^0 - B^0 J_3^{31} = 0, J_2^{31}B^1 - B^1 J_3^{31} = -B^3, J_2^{31}B^2 - B^2 J_3^{31} = 0, J_2^{31}B^3 - B^3 J_3^{31} = B^1,$$
$$J_3^{31}F^0 - F^0 J_2^{31} = 0, J_3^{31}F^1 - F^1 J_2^{31} = -F^3, J_3^{31}F^2 - F^2 J_2^{31} = 0, J_3^{31}F^3 - F^3 J_2^{31} = F^1;$$

$mn = 12$,

$$-G^0 J_1^{12} = 0, \quad -G^1 J_1^{12} = G^2, \quad -G^2 J_1^{12} = -G^1, \quad -G^3 J_1^{12} = 0,$$
$$J_1^{12}\Delta^0 = 0, \quad J_1^{12}\Delta^1 = \Delta^2, \quad J_1^{12}\Delta^2 = -\Delta^1, \quad J_1^{12}\Delta^3 = 0,$$
$$-J_1^{12}K^0 + K^0 J_2^{12} = 0, -J_1^{12}K^1 + K^1 J_2^{12} = -K^2, -J_1^{12}K^2 + K^2 J_2^{12} = K^1, -J_1^{12}K^3 + K^3 J_2^{12} = 0,$$
$$J_2^{12}\Lambda^0 - \Lambda^0 J_1^{12} = 0, J_2^{12}\Lambda^1 - \Lambda^1 J_1^{12} = \Lambda^2, J_2^{12}\Lambda^2 - \Lambda^2 J_1^{12} = -\Lambda^1, J_2^{12}\Lambda^3 - \Lambda^3 J_1^{12} = 0,$$
$$J_2^{12}B^0 - B^0 J_3^{12} = 0, J_2^{12}B^1 - B^1 J_3^{12} = B^2, J_2^{12}B^2 - B^2 J_3^{12} = -B^1, J_2^{12}B^3 - B^3 J_3^{12} = 0,$$
$$J_3^{12}F^0 - F^0 J_2^{12} = 0, J_3^{12}F^1 - F^1 J_2^{12} = F^2, J_3^{12}F^2 - F^2 J_2^{12} = -F^1, J_3^{12}F^3 - F^3 J_2^{12} = 0;$$

$mn = 01$,

$$-G^0 J_1^{01} = G^1, \quad -G^1 J_1^{01} = G^0, \quad -G^2 J_1^{01} = 0, \quad -G^3 J_1^{01} = 0,$$
$$J_1^{01}\Delta^0 = \Delta^1, \quad J_1^{01}\Delta^1 = \Delta^0, \quad J_1^{01}\Delta^2 = 0, \quad J_1^{01}\Delta^3 = 0,$$
$$-J_1^{01}K^0 + K^0 J_2^{01} = -K^1, -J_1^{01}K^1 + K^1 J_2^{01} = -K^0, -J_1^{01}K^2 + K^2 J_2^{01} = 0, -J_1^{01}K^3 + K^3 J_2^{01} = 0,$$
$$J_2^{01}\Lambda^0 - \Lambda^0 J_1^{01} = \Lambda^1, J_2^{01}\Lambda^1 - \Lambda^1 J_1^{01} = \Lambda^0, J_2^{01}\Lambda^2 - \Lambda^2 J_1^{01} = 0, J_2^{01}\Lambda^3 - \Lambda^3 J_1^{01} = 0,$$
$$J_2^{01}B^0 - B^0 J_3^{01} = B^1, J_2^{01}B^1 - B^1 J_3^{01} = B^0, J_2^{01}B^2 - B^2 J_3^{01} = 0, J_2^{01}B^3 - B^3 J_3^{01} = 0,$$
$$J_3^{01}F^0 - F^0 J_2^{01} = F^1, J_3^{01}F^1 - F^1 J_2^{01} = F^0, J_3^{01}F^2 - F^2 J_2^{01} = 0, J_3^{01}F^3 - F^3 J_2^{01} = 0;$$

$mn = 02$,

$$-G^0 J_1^{02} = G^2 g, \quad -G^1 J_1^{02} = 0, \quad -G^2 J_1^{02} = G^0, \quad -G^3 J_1^{02} = 0,$$

$$J_1^{02}\Delta^0 = \Delta^2, \quad J_1^{02}\Delta^1 = 0, \quad J_1^{02}\Delta^2 = \Delta^0, \quad J_1^{02}\Delta^3 = 0,$$
$$-J_1^{02}K^0 + K^0 J_2^{02} = -K^2, -J_1^{02}K^1 + K^1 J_2^{02} = 0, -J_1^{02}K^2 + K^2 J_2^{02} = -K^0, -J_1^{02}K^3 + K^3 J_2^{02} = 0,$$
$$J_2^{02}\Lambda^0 - \Lambda^0 J_1^{02} = \Lambda^2, J_2^{02}\Lambda^1 - \Lambda^1 J_1^{02} = 0, J_2^{02}\Lambda^2 - \Lambda^2 J_1^{02} = \Lambda^0, J_2^{02}\Lambda^3 - \Lambda^3 J_1^{02} = 0,$$
$$J_2^{02}B^0 - B^0 J_3^{02} = B^2, J_2^{02}B^1 - B^1 J_3^{02} = 0, J_2^{02}B^2 - B^2 J_3^{02} = B^0, J_2^{02}B^3 - B^3 J_3^{02} = 0,$$
$$J_3^{02}F^0 - F^0 J_2^{02} = F^2, J_3^{02}F^1 - F^1 J_2^{02} = 0, J_3^{02}F^2 - F^2 J_2^{02} = F^0, J_3^{02}F^3 - F^3 J_2^{02} = 0;$$

$mn = 03$,

$$-G^0 J_1^{03} = G^3, \quad -G^1 J_1^{03} = 0, \quad -G^2 J_1^{03} = 0, \quad -G^3 J_1^{03} = G^0,$$
$$J_1^{03}\Delta^0 = \Delta^3, \quad J_1^{03}\Delta^1 = 0, \quad J_1^{03}\Delta^2 = 0, \quad J_1^{03}\Delta^3 = \Delta^0,$$
$$-J_1^{03}K^0 + K^0 J_2^{03} = -K^3, -J_1^{03}K^1 + K^1 J_2^{03} = 0, -J_1^{03}K^2 + K^2 J_2^{03} = 0, -J_1^{03}K^3 + K^3 J_2^{03} = -K^0,$$
$$J_2^{03}\Lambda^0 - \Lambda^0 J_1^{03} = \Lambda^3, J_2^{03}\Lambda^1 - \Lambda^1 J_1^{03} = 0, J_2^{03}\Lambda^2 - \Lambda^2 J_1^{03} = 0, J_2^{03}\Lambda^3 - \Lambda^3 J_1^{03} = \Lambda^0,$$
$$J_2^{03}B^0 - B^0 J_3^{03} = B^3, J_2^{03}B^1 - B^1 J_3^{03} = 0, J_2^{03}B^2 - B^2 J_3^{03} = 0, J_2^{03}B^3 - B^3 J_3^{03} = B^0,$$
$$J_3^{03}F^0 - F^0 J_2^{03} = F^3, J_3^{03}F^1 - F^1 J_2^{03} = 0, J_3^{03}F^2 - F^2 J_2^{03} = 0, J_3^{03}F^3 - F^3 J_2^{03} = F^0.$$

Taking into account the explicit form of all the matrix blocks and all the generators, we can prove that all these equations are satisfied. This indicates that the expressions for all the involved matrices are correct.

9 Conclusion

We have extended the first order system in matrix form to pseudo-Riemannian space-time models, by applying the Tetrode-Weyl-Fock-Ivanenko tetrad method. The basic matrices of the equation and the Lorentzian generators for tensors are presented in block form. They are found in explicit form, with the symmetries taken into account, while the set of tensors $\Psi(x) = \{\Phi, \Phi_c, \Phi_{(ab)}, \Phi_{[ab]c}\}$ is presented by an (1+4+10+24)-component function. All the intrinsic constraints on tensors are contained in the structure of the basic matrices. It is shown that the relativistic invariance requirement provides 144 constraints on blocks and generators, which become identities when taking into account the explicit expressions for all the block matrices.

The introduced tetrad matrix equation is specified in cylindrical and spherical coordinates for the flat Minkowski space. The case of massless field is separately addressed, and we focus in this theory on a detailed matrix representation of the gauge symmetry.

References

[1] A.A. Bogush, B.V. Krylov, F.I. Fedorov, *On matrices of the equations for spin 2 particles*, Proceedings of the National Academy of Sciences of Belarus, Physics and Mathematics Series 1 (1968), 74-81.

[2] A.A. Bogush, V.V. Kisel, N.G. Tokarevskaya, V.M/ Red'kov, *On equations for a spin 2 particle in external electromagnetic and gravitational fields*, Proceedings of the National Academy of Sciences of Belarus, Physics and Mathematics Series, 1 (2003), 62-67.

[3] H.A. Buchdahl, *On the compatibility of relativistic wave equations for particles of higher spin in the presence of a gravitational field*, Nuovo Cimento 10 (1958), 96-103.

[4] H.A. Buchdahl, *On the compatibility of relativistic wave equations in Riemann spaces*, Nuovo Cimento 25 (1962), 486-496.

[5] W. Cox, *First-order formulation of massive spin-2 field theories*, J. Phys. A, 15 (1982), 253-268.

[6] L. De Broglie*Sur l'interprétation de certaines équations dans la thórie des particules de spin 2*, C. R. Acad. Sci. Paris 212 (1941), 657-659.

[7] F.I. Fedorov, *To the theory of particles with spin 2*, Proceedings of Belarus State University, Ser. Phys.-Math. 12 (1951), 156-173.

[8] F.I. Fedorov, *Equations of the first order for gravitational field*, Doklady of the Academy of Sciences of USSR 179, 4 (1968), 802-805.

[9] F.I. Fedorov, A.A. Kirilov, *The first order equations for graviational field in vacuum*, Acta Physica Polonica B, 7, 3 (1976), 161-167.

[10] V.Ya. Feinberg, *On the theory of interaction of particles with higher spins with electromagnetic and meson fields*, Proceedings of the Lebedev Physics Institute of the Academy of Sciences of the USSR, 6 (1955), 269-332.

[11] M. Fierz, W. Pauli, *On relativistic wave equations for particles of arbitrary spin in an electromagnetic field*, Proc. Roy. Soc. London. A. 173 (1939), 211-232.

[12] E.S. Fradkin, *To the theory of particles with higher spins*, Journal of Experimental and Theoretical Physics, 20, 1 (1950), 27-38.

[13] I.M. Gel'fand, A.M. Yaglom, *General relativistically invariant equations and infinite-dimensional representations of the Lorentz group*, Journal of Experimental and Theoretical Physics, 18, 8 (1948), 703-733.

[14] K. Johnson, E.C.G. Sudarshan, *Inconsistency of the local field theory of charged spin 3/2 particles*, Ann. Phys. 13, 1 (1961), 121-145.

[15] K. Johnson, E.C.G. Sudarshan, *The impossibility of a consistent theory of a charged higher spin Fermi fields*, Ann. Phys. 13, 1 (1961), 126-145.

[16] V.V. Kisel, *On relativistic wave equations for a spin 2 particle*, Proceedings of the National Academy of Sciences of Belarus. Physics and Mathematics Series, 5 (1986), 94-99.

[17] V.V. Kisel, E.V. Ovsiyuk, O.V. Veko, V.M. Red'kov, *Contribution of gauge degrees of freedom in the energy-momentum tensor of the massless spin 2 field*, Proceedings of the National Academy of Sciences of Belarus, Physics and Mathematics Series, 2 (2015), 58-63.

[18] V.V. Kisel, E.V. Ovsiyuk, O.V. Veko, V.M. Red'kov, *Nonrelartivistic approximation in the theory of a spin two particle*, Doklady of the National Academy of Sciences of Belarus, 59, 3 (2015), 21-27.

[19] B.V. Krylov, F.I. Fedorov, *Equations of the first order for graviton*, Doklady of the National Academy of Sciences of Belarus 11, 8 (1967), 681-684.

[20] B.V. Krylov, *On the systems of the first order for graviton*, Proceedings of the National Academy of Sciences of Belarus, Physics and Mathematics Series, 6 (1972), 82-89.

[21] R.K. Loide, *On conformally covariant spin-3/2 and spin-2 equations*, J. Phys. A, 19, 5 (1986), 827-829.

[22] E.M. Ovsiyuk, V.V. Kisel, V.M. Red'kov, *Maxwell Electrodynamics and boson fields in spaces of constant curvature*, Nova Science Publishers Inc., New York, 2014.

[23] W. Pauli, M. Fierz, *Über relativistische Feldleichungen von Teilchen mit beliebigem Spin im elektromagnetishen Feld*, Helv. Phys. Acta 12 (1939), 297-300.

[24] V.M. Red'kov, N.G. Tokarevskaya, V.V. Kisel, *Graviton in a curved space-time background and gauge symmetry*, Nonlinear Phenomena in Complex Systems, 6, 3 (2003), 772-778.

[25] T. Regge, *On properties of the particle with spin 2*, Nuovo Cimento 5, 2 (1957), 325-326.

[26] G. Velo, D. Zwanziger, *Noncausality and other defects of interaction Lagrangians for particles with spin one and higher*, Phys. Rev. 188, 5 (1969), 2218-2222.

Appendix.

$$G^0 = \begin{pmatrix} +1 & 0 & 0 & 0 \end{pmatrix}, \quad G^1 = \begin{pmatrix} 0 & -1 & 0 & 0 \end{pmatrix},$$

$$G^2 = \begin{pmatrix} 0 & 0 & -1 & 0 \end{pmatrix}, \quad G^3 = \begin{pmatrix} 0 & 0 & 0 & -1 \end{pmatrix};$$

$$(\Delta^a)_{4\times 1}, \quad \Delta^0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \Delta^1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \Delta^2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \Delta^3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix};$$

$$K^0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$$K^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$K^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$K^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix};$$

$$\Lambda^0 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{3}{2} & 0 & 0 & 0 \end{pmatrix}, \quad \Lambda^1 = \begin{pmatrix} 0 & \frac{3}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \end{pmatrix},$$

$$\Lambda^2 = \begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}, \quad \Lambda^3 = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix};$$

$$B^0 = \left(\begin{array}{cccccccccccccccccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{3}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{3}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{3}{2} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot
\end{array}\right),$$

$$B^1 = \left(\begin{array}{cccccccccccccccccccccccc}
\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot \\
\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{3}{2} & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot \\
\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \frac{3}{2} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\frac{3}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot
\end{array}\right),$$

$$
B^2 = \left(\begin{array}{cccccccccccccccccccccccc}
\cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{3}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot \\
\cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot \\
\cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{3}{2} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \frac{3}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot
\end{array}\right),
$$

$$
B^3 = \left(\begin{array}{cccccccccccccccccccccccc}
\cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{3}{2} & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \frac{3}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \frac{3}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}\right).
$$

$$
F^0 = \left(\begin{array}{cccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{2}{3} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{2}{3} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{2}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{3} \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{3} \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{3} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{3} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{3} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}\right),
$$

$$F^1 = \begin{pmatrix}
1/3 & . & . & . & . & . & . & . & . & -1 \\
. & . & . & . & . & 1/3 & . & . & . & . \\
. & . & . & . & 1/3 & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & -1 & . \\
. & . & . & . & . & . & . & 1 & . & . \\
. & . & . & . & . & . & -2/3 & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & -2/3 & . & . & . & . & . \\
. & . & . & . & . & 2/3 & . & . & . & . \\
. & . & . & . & . & . & . & -1 & . & . \\
. & . & . & . & . & . & 1/3 & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & -1/3 & . & . & . & . & . \\
. & . & . & -1 & . & . & . & . & . & . \\
1/3 & 1 & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & -1 & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & 1/3 & . & . & . \\
. & . & . & . & . & 1/3 & . & . & . & . \\
-1/3 & . & -1 & . & . & . & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . & .
\end{pmatrix},$$

$$F^2 = \begin{pmatrix}
. & . & . & . & . & 1/3 & . & . & . & . \\
. & 1/3 & . & . & . & . & . & . & . & -1 \\
. & . & . & 1/3 & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & 1 & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & -1 & . & . & . \\
. & . & . & . & . & . & . & 1/3 & . & . \\
. & . & . & . & . & . & -1 & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & 1 & . & . & . & . & . \\
. & . & . & 1/3 & . & . & . & . & . & . \\
-1 & -1/3 & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & -2/3 & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & 2/3 & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & -2/3 & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & -1 & . \\
. & . & . & . & . & . & . & 1/3 & . & . \\
. & 1/3 & 1 & . & . & . & . & . & . & . \\
. & . & . & . & . & -1/3 & . & . & . & . \\
. & . & . & . & -1 & . & . & . & . & .
\end{pmatrix},$$

$$
F^3 = \left(\begin{array}{cccccccccc}
. & . & . & . & 1/3 & . & . & . & . & . \\
. & . & . & 1/3 & . & . & . & . & . & . \\
. & . & 1/3 & . & . & . & . & . & . & -1 \\
. & . & . & . & . & . & . & -1 & . & . \\
. & . & . & . & . & . & 1 & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & 1/3 & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & -1 & . & . & . \\
. & . & . & . & . & -1 & . & . & . & . \\
1 & . & 1/3 & . & . & . & . & . & . & . \\
. & . & . & -1/3 & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & 1/3 & . \\
. & . & . & . & . & . & . & -1 & . & . \\
. & -1 & -1/3 & . & . & . & . & . & . & . \\
. & . & . & . & . & 1 & . & . & . & . \\
. & . & . & . & 1/3 & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & -2/3 & . \\
. & . & . & -2/3 & . & . & . & . & . & . \\
. & . & . & . & 2/3 & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & .
\end{array} \right).
$$

$$
J_3^{12} =
$$

$$
\left(\begin{array}{cccccccccccccccccccccccc}
. & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & -1 & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & 1 & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & -1 & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . \\
. & . & . & . & . & . & 1 & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & 1 & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & 1 & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & .
\end{array} \right),
$$

$$
J_3^{31} =
$$

$$
\left(\begin{array}{cccccccccccccccccccccccc}
. & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
-1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . \\
. & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & 1 & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . \\
. & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . \\
. & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 \\
. & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & -1 & . & .
\end{array} \right),
$$

$$J_3^{23} =$$

$$\begin{pmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & -1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot
\end{pmatrix},$$

$$J_3^{01} =$$

$$\begin{pmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
-1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot
\end{pmatrix},$$

$$J_3^{02} =$$

$$\begin{pmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
-1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
-1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot
\end{pmatrix},$$

$$J_3^{03} =$$

$$
\left(\begin{array}{cccccccccccccccccccccccc}
. & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . \\
. & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . \\
. & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . \\
1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 \\
. & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
-1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . \\
. & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . \\
. & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . \\
. & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . \\
. & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & .
\end{array}\right).
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